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Further results on the regulation problem for linear systems with constraints on control and its increment

Abdelhak A., Ou-azzou R.

Department of Mathematics, Laboratory Partial Differential Equations, Algebra and Spectral Geometry, Faculty of Sciences, Ibn Tofail University, B.P. 133, Kénitra, Morocco

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The regulator problem for both discrete-time and continuous-time linear systems is considered. The control and its increments are under non-symmetrical constraints and the domain of constraints includes the origin on its boundary. We derive necessary and sufficient conditions which ensure the satisfaction of all the constraints and also the asymptotic stability by a state feedback. An illustrative example shows the application of our method.

Keywords: *linear systems; non-symmetrical constraints; control; positive invariance.*

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1. Introduction

A constraint is anything that prevents the system from achieving its goal. There are three major types of constraints frequently encountered in applications, i.e., constraints on amplitude variation, constraints on incremental variation, and constraints on output or state [1]. If constraints of the considered system are not paid attention to, the closed loop control performance could be severely deteriored [2] in the presence of constraints. Many authors have studied the problem of regulation [3–8], and there are several approaches proposed in the literature, we can cite, not exhaustively, the positive invariance concept [9], predictive control [10,11], the maximal admissible sets concept [12,13] and other approaches.

Incremental input constraints are serious challenges in many automatic control application like Aerospace systems [14, 15], wastewater treatment [16], chemical processes [17, 18], climatic variables inside buildings [19], mechatronics [18], automatic drug dosing [20], sterilization processes [16], high energy physics [21, 22].

Problems of designing stabilizing regulators for linear systems subject to control saturations and asymmetric constraints on its increment or rate are solved in [23, 24], the authors have considered an asymmetric domain of control constraints that contains the zero of $\mathbb{R}^m$ in its interior. However, in many engineering problems, like obstacle avoidance problem, the regulation around an equilibrium situated on the boundary of the domain of attraction is necessary [25,26]. In our paper we consider a regulation problem for linear systems with asymmetric constraints on both control variables and its increments and with the zero on the boundary of control domain constraints. From a practical point of view, probably the most successful approach that makes possible to consider simultaneously control and increment constraints is the predictive control approach [27]. Unfortunately the implementation of Predictive Controllers is complex. In this paper we select the positive invariance approach [23, 28] because it proposes simple methods to calculate constant state feedback controllers, in both the continuous and the discrete-time cases. This approach is based on constraint avoidance [29]: preventing the saturation, the closed-loop system, therefore, stays in a region of linear behavior.

The paper is organized as follows. The problem set-up is introduced in section 2. Some preliminaries, that consist of necessary and sufficient conditions of positive invariance of incremental domain with respect to autonomous systems are given in section 3. Section 4 is devoted to state the main result. In this section we obtain sufficient conditions that ensure the determination of a stabilizing linear state feedback controllers that respect both constraints on control and its increment. A simulation example is given in section 5.

Notations. For a scalar $a \in \mathbb{R}$, we define $a^+ = \sup(a, 0)$ and $a^- = \sup(-a, 0)$. Furthermore, for a matrix $A = (a_{ij})$, $1 \leqslant i, j \leqslant n$, the tilde transforms are defined by

$$\tilde{A} = \begin{bmatrix} A^+ & A^- \\ A^- & A+ \end{bmatrix},$$

where $A^+ = (a_{ij}^+)$, $A^- = (a_{ij}^-)$ and

$$\tilde{A}_c = \begin{bmatrix} A_1 & A_2 \\ A_2 & A_1 \end{bmatrix}$$

with

$$A_1 = \begin{cases} a_{ii} & \text{for } i = j, \\ a_{ij}^+ & \text{for } i \neq j, \end{cases}, \quad A_2 = \begin{cases} 0 & \text{for } i = j, \\ a_{ij}^- & \text{for } i \neq j. \end{cases}$$

We denote by $\sigma(A)$ the spectrum of the matrix A and by D_s the stability domain for eigenvalues (that is, the left half plane in the continuous-time case or the unit disk in the discrete-time case). For any two vectors $x, y \in \mathbb{R}^n$, $x \leqslant y$ (respectively $x < y$) if $x_i \leqslant y_i$ (respectively $x_i < y_i$), $i = 1, \ldots, n$.

2. Problem statement

Consider a linear time invariant system represented in the state space by

$$\delta x(\cdot) = Ax(\cdot) + Bu(\cdot), \tag{1}$$

where $x(\cdot) \in \mathbb{R}^n$ is the state of system, $\delta x(\cdot)$ represents its derivative with respect to time in the continuous-time case or $x(t+1)$ in the discrete-time case. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $u(\cdot) \in \mathbb{R}^m$ is the input variable subject to constraints

$$-q_2 \leqslant u(\cdot) \leqslant q_1, \tag{2}$$

where q_1 and q_2 are vectors of $\mathbb{R}^m_+$.

The control increment is constrained as follows

$$-d_2 \leqslant u(i+1) - u(i) \leqslant d_1, \qquad \text{for discrete-time systems,} \tag{3}$$

$$-d_2 \leqslant \dot{u}(t) \leqslant d_1, \qquad \text{for continuous-time systems,} \tag{4}$$

where d_2 and d_1 are elements of $\mathbb{R}^m_+$.

We suppose that the equilibrium $u = 0$ is on the boundary of the domain of control constraints, that is, the vector $q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ has at least one component null. This implies that there exists a permutation matrix P and an integer p such that

$$Pq = \gamma = \begin{pmatrix} \gamma_0 \\ 0 \end{pmatrix},$$

where $\gamma_0 > 0$ is a vector in $\mathbb{R}^p$ that contains all the non negative components of q. The inverse of P is its transpose, that is $P^{-1} = P^T$.

The problem studied in this paper is the following: find a stabilizing state feedback as

$$u(\cdot) = Gx(\cdot), \quad G \in \mathbb{R}^{n\times m}$$

ensuring closed-loop asymptotic stability of the system with non saturating control that also respects incremental constraints.

3. Preliminary results

Consider the following linear autonomous system

$$\delta v(\cdot) = Kv(\cdot), \quad v(t_0) = v_0, \tag{5}$$

where $v(\cdot) \in \mathbb{R}^m$ is constrained by

$$-q_2 \leqslant v(\cdot) \leqslant q_1 \tag{6}$$

and its increments constrained by

$$-d_2 \leqslant v(i+1) - v(i) \leqslant d_1, \qquad \text{for discrete-time systems,} \tag{7}$$

$$-d_2 \leqslant \dot{v}(t) \leqslant d_1, \qquad \text{for continuous-time systems.} \tag{8}$$

Define the domain $D(q_1, q_2)$ by

$$D(q_1, q_2) = \{v \in \mathbb{R}^m, -q_2 \leqslant v \leqslant q_1\}. \tag{9}$$

Definition 1. *A domain D is positively invariant with respect to motion of system (5) if for any initial condition $v_0 \in D$, the corresponding trajectory $v(t, t_0, v_0)$ remains in D for all $t > t_0$.*

Define the augmented state space variables

$$Y(\cdot) = \begin{pmatrix} v(\cdot) \\ -v(\cdot) \end{pmatrix} \quad \text{and} \quad X(\cdot) = PY(\cdot).$$

Then

$$\delta Y(\cdot) = \tilde{K} Y(\cdot) \quad \text{and} \quad \delta X(\cdot) = T^K X(\cdot),$$

where

$$T^K = P\tilde{K}P^T. \tag{10}$$

Define the matrix

$$\Phi^K = P\tilde{K_c}P^T. \tag{11}$$

Let decompose the matrices T^K and Φ^K as follows

$$T^K = \begin{pmatrix} T_1^K & T_2^K \\ T_3^K & T_4^K \end{pmatrix}, \qquad \Phi^K = \begin{pmatrix} \Phi_1^K & \Phi_2^K \\ \Phi_3^K & \Phi_4^K \end{pmatrix},$$

where T_1^K, $\Phi_1^K \in \mathbb{R}^{p\times p}$; T_2^K, $\Phi_2^K \in \mathbb{R}^{p\times(2m-p)}$; T_3^K, $\Phi_3^K \in \mathbb{R}^{(2m-p)\times p}$ and T_4^K, $\Phi_4^K \in \mathbb{R}^{(2m-p)\times(2m-p)}$.

Lemma 1. *The domain $D(q_1, q_2)$ given by (9) is positively invariant with respect to system (5) if and only if*

$$T_1^K \gamma_0 \leqslant \gamma_0 \text{ and } T_3^K = 0, \textit{ for the discrete-time case,} \tag{12}$$

$$\Phi_1^K \gamma_0 \leqslant 0 \text{ and } \Phi_3^K \gamma_0 \leqslant 0, \textit{ for the continuous-time case.} \tag{13}$$

Proof. For discrete-time case, it is known, see [9], that the domain (9) is positively invariant with respect to system (5) if and only if $\tilde{K}q \leqslant q$, or equivalently

$$P\tilde{K}P^{-1}\gamma \leqslant \gamma \Leftrightarrow T^K\gamma \leqslant \gamma \Leftrightarrow \begin{cases} T_1^K\gamma_0 \leqslant \gamma_0, \\ T_3^K\gamma_0 \leqslant 0. \end{cases}$$

Since $T^K \geqslant 0$ (and also T_3^K) and $\gamma_0 > 0$ then $T_3^K = 0$. For continuous-time case, it follows from [30] that $D(q_1, q_2)$ is positively invariant with respect to system

$$\dot{v}(t) = Kv(t) \tag{14}$$

if and only if $\tilde{K_c}q \leqslant 0$, which is equivalent to

$$\Phi^K\gamma \leqslant 0 \Leftrightarrow \begin{cases} \Phi_1^K\gamma_0 \leqslant \gamma_0, \\ \Phi_3^K\gamma_0 \leqslant 0. \end{cases}$$

∎

Lemma 2. *Suppose that the domain (9) is positively invariant with respect to system (5). The increment condition is verified iff matrix K satisfies:*

$$\widetilde{(K - I)}q \leqslant d, \quad \textit{for discrete-time systems,} \tag{15}$$

$$\tilde{K}q \leqslant d, \quad \textit{for continous-time systems,} \tag{16}$$

where $d = \binom{d_1}{d_2}$.

Proof. It is easy to verify that the proof of this result given in [31], in the case $q > 0$, remains also valid when some components of q are null. ∎

Using Lemma 1 and Lemma 2 we can derive the following result.

Lemma 3. *Domain $D(q_1, q_2)$ given by (9) is positively invariant with respect to motion of system (5) and the increment constraints (7) are respected if and only if*

$$\begin{cases} T_1^K \gamma_0 \leqslant \gamma_0, \\ T_3^K = 0, \\ \widetilde{(K-I)}q \leqslant d, \end{cases} \quad \text{for the discrete-time case;} \qquad \begin{cases} \Phi_1^K \gamma_0 \leqslant 0, \\ \Phi_3^K \gamma_0 \leqslant 0, \\ \tilde{K} q \leqslant d, \end{cases} \quad \text{for the continuous-time case.}$$

4. Main results

Consider the system (1) with control variable $u(\cdot)$ subject to constraints (2) and (3) (or (4) for continuous-time case), and the feedback law

$$u(\cdot) = Gx(\cdot), \quad G \in \mathbb{R}^{m\times m} \tag{17}$$

with $\operatorname{rank} G = m$ and

$$\sigma(A + BG) \subset D_S. \tag{18}$$

Using state feedback (17) we can write system (1) as

$$\delta x(\cdot) = (A + BG)x(\cdot).$$

It follows that

$$\begin{aligned} \delta u(\cdot) &= G\delta x(\cdot) \\ &= G(A + BG)x(\cdot). \end{aligned}$$

If there exists a matrix K such that

$$G(A + BG) = KG$$

then

$$\delta u(\cdot) = Ku(\cdot).$$

With this transformation and results of the preceding section it is possible to derive the following result.

Theorem 1. *System (1) with state feedback (17)–(18) and subject to constraints (2) and (3) (or (4) for continuous systems) is asymptotically stable at the origin if there exists a matrix $K \in \mathbb{R}^{m\times m}$ such that*

$$GA + GBG = KG \tag{19}$$

$$\begin{cases} T_1^K \gamma_0 \leqslant \gamma_0, \\ T_3^K = 0, \\ \widetilde{(K-I)}q \leqslant d, \end{cases} \quad \text{for the discrete-time case;} \tag{20}$$

$$\begin{cases} \Phi_1^K \gamma_0 \leqslant 0, \\ \Phi_3^K \gamma_0 \leqslant 0, \\ \tilde{K} q \leqslant d, \end{cases} \quad \text{for the continuous-time case,} \tag{21}$$

where T^K and Φ^K are given respectively by (10), (11), $I \in \mathbb{R}^{m\times m}$ is the identity matrix.

Proof. Let $u(\cdot) = Gx(\cdot)$, where x is the trajectory of system (1) and suppose that the conditions of Theorem 1 are verified. Since $\delta u(\cdot) = Ku(\cdot)$ and application of Lemma 3 we deduce that both the constraints on $u(\cdot)$ and on its increments are satisfied. Bearing in mind that $\sigma(A + BG) \subset D_s$ one can conclude to the asymptotic stability of the closed-loop system. ∎

Remark 1. The resolution of equation (19) is described in [32].

5. Example

Consider the system

$$x(k+1) = Ax(k) + Bu(k), \quad x_0 \in \mathbb{R}^4,$$

where $x(k) \in \mathbb{R}^4$, $u(k) \in \mathbb{R}^3$

$$A = \begin{pmatrix} 0.4 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ -1 & 2 & 7 & 0 \\ 4 & 3 & 0 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -1 \\ 8 & 1 & 2 \\ -5 & -3 & 0 \\ 0 & 5 & 3 \end{pmatrix}.$$

The constraints on the input $u(\cdot)$ are given by

$$-q_2 \leqslant u(\cdot) \leqslant q_1$$

with $q_1 = (0,1,0)^T$ and $q_2 = (0,0,1)^T$ or equivalently

$$u_1(\cdot) = 0, \quad 0 \leqslant u_2(\cdot) \leqslant 1, \quad -1 \leqslant u_3(\cdot) \leqslant 0. \tag{22}$$

The increments on the control are subject to constraints described by

$$-d_2 \leqslant u(i+1) - u(i) \leqslant d_1 \tag{23}$$

with $d_2 = (1,1,1)^T$ and $d_1 = (1,1,1)^T$.

Let $q = \left(\begin{smallmatrix} q_1 \\ q_2 \end{smallmatrix}\right) = (0,1,0,0,0,1)^T$. Consider the matrix

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

then we have $Pq = (1,1,0,0,0,0)^T = \left(\begin{smallmatrix} \gamma_0 \\ 0 \end{smallmatrix}\right)$ with $\gamma_0 = \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right) \in \mathbb{R}^2$. Denote $K = (k_{ij})$ and $\tilde{K} = (\tilde{k}_{ij})$, then

$$\tilde{k}_{ij} = \begin{cases} k_{ij}^+ & \text{if } \ 1 \leqslant i,j \leqslant 3; \\ k_{i(j-3)}^- & \text{if } \ 1 \leqslant i \leqslant 3 \text{ and } 4 \leqslant j \leqslant 6; \\ k_{(i-3)j}^- & \text{if } \ 4 \leqslant i \leqslant 6 \text{ and } 1 \leqslant j \leqslant 3; \\ k_{(i-3)(j-3)}^+ & \text{if } \ 4 \leqslant i \leqslant 6 \text{ and } 4 \leqslant j \leqslant 6. \end{cases}$$

Let $T = \left(\begin{smallmatrix} T_1 & T_2 \\ T_3 & T_4 \end{smallmatrix}\right) = P\tilde{K}P^T$, where $T_1 \in \mathbb{R}^{2\times 2}$. By identification we obtain that

$$T_1 = \begin{pmatrix} \tilde{k}_{66} & \tilde{k}_{62} \\ \tilde{k}_{26} & \tilde{k}_{22} \end{pmatrix} = \begin{pmatrix} k_{33}^+ & k_{32}^- \\ k_{23}^- & k_{22}^+ \end{pmatrix}, \qquad T_3 = \begin{pmatrix} \tilde{k}_{36} & \tilde{k}_{32} \\ \tilde{k}_{46} & \tilde{k}_{42} \\ \tilde{k}_{56} & \tilde{k}_{52} \\ \tilde{k}_{16} & \tilde{k}_{12} \end{pmatrix} = \begin{pmatrix} k_{33}^- & k_{32}^+ \\ k_{13}^+ & k_{12}^- \\ k_{23}^+ & k_{32}^- \\ k_{13}^- & k_{12}^+ \end{pmatrix}.$$

First, we will search K such that conditions (20) of Theorem 1 are satisfied. Then we will determine the gain matrix G by using the procedure described in [32].

By simple calculations we can establish that the matrix K will verify the conditions (20) in Theorem 1 if and only if

$$\begin{cases} k_{12} = k_{13} = k_{32} = 0; \\ 0 \leqslant k_{33} \leqslant 1; \\ k_{23} \leqslant 0; \ k_{22}^- \leqslant 1; \\ k_{22}^+ - k_{23} \leqslant 1. \end{cases}$$

Consider the asymptotically stable matrix K

$$K = \begin{pmatrix} 0.2 & 0 & 0 \\ -4 & 0.3 & 0 \\ 0 & 0 & 0.7 \end{pmatrix},$$

which verifies all the requirement conditions. The resolution of

$$GA + GBG = KG$$

gives

$$G = \begin{pmatrix} 0.1253609 & 0.0893536 & 0.5329934 & 0.4601572 \\ -0.4011306 & 0.3942382 & 0.3926859 & -0.3530406 \\ -0.5407766 & -2.0535016 & -2.8614331 & -1.0192376 \end{pmatrix}.$$

The eigenvalues of $A + BG$ are $\{0.2; 0.3; 0.4; 0.7\}$, which shows that $A + BG$ is asymptotically stable. In the following we give numerical results corresponding to the initial state $x(0) = (-6.277059\text{e} - 01, 2.649606\text{e} - 01, 1.032181\text{e} - 01, 0.000000\text{e} + 00)$. We have $u(0) = Gx(0) \in D(q_1, q_2)$. Figure 1 shows the evolution of the second and third components of the control while the first component obtained is zero. In this figure we see that the calculated feedback $u(\cdot) = Gx(\cdot)$ respects the constraints (22) and converges to the origin as expected.

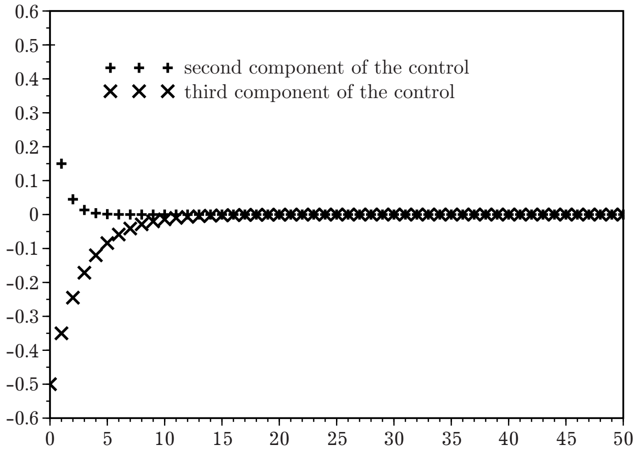

Fig. 1. Evolution of the second and third components of the control.

In Table 1 we see the satisfaction of the increment constraints given by (23) and in Table 2 we observe the asymptotic stability of the state. Further numerical simulations with any arbitrary initial state $x(0)$ such that $Gx(0) \in D(q_1, q_2)$ also confirm the theoretical results.

Table 1. Evolution of the increment components.

i	$u_1(i+1) - u_1(i)$	$u_2(i+1) - u_2(i)$	$u_3(i+1) - u_3(i)$
0	$-6.106227\text{e} - 16$	$-3.500000\text{e} - 01$	$1.500000\text{e} - 01$
1	$1.443290\text{e} - 15$	$-1.050000\text{e} - 01$	$1.050000\text{e} - 01$
2	$1.3877779\text{e} - 16$	$-3.150000\text{e} - 02$	$7.350000\text{e} - 02$
3	$-1.665335\text{e} - 16$	$-9.450000\text{e} - 03$	$5.145000\text{e} - 02$
4	$-2.636780\text{e} - 16$	$-2.835000\text{e} - 03$	$3.601500\text{e} - 02$
5	$-2.220446\text{e} - 16$	$-8.505000\text{e} - 04$	$2.521050\text{e} - 02$
6	$-1.387779\text{e} - 16$	$-2.551500\text{e} - 04$	$1.764735\text{e} - 02$
7	$-5.204170\text{e} - 17$	$-7.654500\text{e} - 05$	$1.235315\text{e} - 02$
8	$-4.857226\text{e} - 17$	$-2.296350\text{e} - 05$	$8.647202\text{e} - 03$
9	$-2.428613\text{e} - 17$	$-6.889050\text{e} - 06$	$6.053041\text{e} - 03$
10	$-1.301043\text{e} - 17$	$-2.066715\text{e} - 06$	$4.237129\text{e} - 03$
...	...	...	...
29	$-1.185846\text{e} - 20$	$-2.397730\text{e} - 16$	$4.829859\text{e} - 06$
30	$-1.694066\text{e} - 20$	$-7.179705\text{e} - 17$	$3.380901\text{e} - 06$

Table 2. Evolution of the state components.

i	$x_1(i)$	$x_2(i)$	$x_3(i)$	$x_4(i)$
0	$-6.277059e - 01$	$2.649606e - 01$	$1.032181\text{e} - 01$	$0.000000\text{e} + 00$
1	$1.248918e + 00$	$-3.328241e - 01$	$3.801536\text{e} - 01$	$-7.159417\text{e} - 01$
2	$1.149567e + 00$	$-2.995548e - 01$	$2.965095\text{e} - 01$	$-5.984521\text{e} - 01$
3	$7.948268e - 01$	$-1.940973e - 01$	$1.918899\text{e} - 01$	$-4.011084\text{e} - 01$
4	$5.164307e - 01$	$-1.169650e - 01$	$1.197080\text{e} - 01$	$-2.566352\text{e} - 01$
5	$3.347223e - 01$	$-7.051426e - 02$	$7.544512\text{e} - 02$	$-1.648831\text{e} - 01$
6	$2.203539e - 01$	$-4.367549e - 02$	$4.872006\text{e} - 02$	$-1.079819\text{e} - 01$
7	$1.476951e - 01$	$-2.795704e - 02$	$3.224203\text{e} - 02$	$-7.215340\text{e} - 02$
8	$1.004739e - 01$	$-1.842101e - 02$	$2.175699\text{e} - 02$	$-4.899596\text{e} - 02$
9	$6.907917e - 02$	$-1.240436e - 02$	$1.488464\text{e} - 02$	$-3.365125\text{e} - 02$
10	$4.782815e - 02$	$-8.477679e - 03$	$1.027506\text{e} - 02$	$-2.328510\text{e} - 02$
...	...	...	...	...
49	$4.282060\text{e} - 08$	$-7.447060\text{e} - 09$	$9.161066\text{e} - 09$	$-2.083069\text{e} - 08$
50	$2.997442\text{e} - 08$	$-5.212942\text{e} - 09$	$6.412746\text{e} - 09$	$-1.458148\text{e} - 08$

6. Conclusion

In this paper, we have considered the regulator problem for linear systems with non-symmetrical constraints on control and its increments and with the origin on the boundary of the constraint domain of control variable. Both continuous and discrete systems are studied. On each case, sufficient conditions that allow the existence and the determination of a state feedback which ensure the satisfaction of all the constraints and also the asymptotic stability of the system are given.

[1] Wang L. Model Predictive Control System Design and Implementation Using MATLAB. London, Springer (2009).

[2] Chen H. Stability and Robustness Considerations in Nonlinear Model Predictive Control. Ph.D. Thesis. Germany, University of Stuttgart (1997).

[3] Gutman P. O., Hagander P. A new design of constrained controllers for linear systems. IEEE Transactions on Automatic Control. **30** (1), 22–33 (1985).

[4] Benzaouia A., Hmamed A., Tadeo F. Stabilization of controlled positive delayed continuous time systems. International Journal of Systems Science. **41** (12), 1473–1479, (2010).

[5] Hmamed A., Ait Rami M., Benzaouia A., Tadeo F. Stabilization under constrained states and controls of positive systems with time delays. European Journal of Control. **18** (2), 182–190 (2012).

[6] Bensalah H., Baron L. Positive invariance of constrained linear continuous-time delay system with delay dependence. Proceedings of the international conference of control, dynamic systems and robotics. Ottawa, Ontario, Canada. 179 (2015).

[7] El Bhih A., Benfatah Y., Ghazaoui A., Rachik M. On the maximal output set of fractional-order discrete-time linear systems. Mathematical Modeling and Computing. **9** (2), 262–277 (2022).

[8] Dórea C. E., Olaru S., Niculescu S.-I. Delay-dependent polyhedral invariant sets for continuous-time linear systems. IFAC-PapersOnline. **55** (34), 108–113 (2022).

[9] Benzaouia A., Burgat C. Regulator problem for linear discrete-time systems with non-symmetrical constrained control. International Journal of Control. **48** (6), 2441–2451 (1988).

[10] Clarcke D. W., Mohtadi C., Tuffs P. S. Generalised predictive control – Part I. The basic algorithm. Automatica. **23** (2), 137–148 (1987).

[11] Clarcke D. W., Mohtadi C., Tuffs P. S. Generalised predictive control – Part II. Extension and Interpretations. Automatica. **23** (2), 149–160 (1987).

[12] Gilbert E. G., Tan K. T. Linear systems with state and control constraints: on the theory of maximal output admissible sets. IEEE Transactions on Automatic Control. **36** (9), 1008–1020 (1991).

[13] Zakary O., Rachik M., Tridane A., Abdelhak A. Identifying the set of all admissible disturbances: discrete-time systems with perturbed gain matrix. Mathematical Modeling and Computing. **7** (2), 293–309 (2020).

[14] Biannic J.-M., Tarbouriech S. Optimization and implementation of dynamic anti-windup compensators with multiple saturations in flight control systems. Control Engineering Practice. **17** (6), 703–713 (2009).

[15] Brieger O., Kerr M., Leissling D., Postlethwaite I., Sofrony J., Turner M. C. Flight testing of a rate saturation compensation scheme on the ATTAS aircraft. Aerospace Science and Technology. **13** (2–3), 92–104 (2009).

[16] Syafiie S., Tadeo F., Villafin M., Alonso A. A. Learning control for batch thermal sterilization of canned foods. ISA Transactions. **50** (1), 82–90 (2011).

[17] Mhaskar P., Keneddy A. B. Robust model predictive control of nonlinear process systems: Handling rate constraints. Chemical Engineering Science. **63** (2), 366–375 (2008).

[18] Valencia-Palomo G., Rossiter J. A. Programmable logic controller implementation of an auto-tuned predictive control based on minimal plant information. ISA Transactions. **50** (1), 92–100 (2011).

[19] Prívara S., Široký J., Ferkl L., Cigler J. Model predictive control of a building heating system: The first experience. Energy and Buildings. **43** (2–3), 564–572 (2011).

[20] Parker R. S., Doyle F. J., Peppas N. A. A model-based algorithm for blood glucose control in Type I diabetic patients. IEEE Transactions on Biomedical Engineering. **46** (2), 148–157 (1999).

[21] Schuster E., Walker M. L., Humphreys D. A., Krstić M. Plasma vertical stabilization with actuation constraints in the DIII-D tokamak. Automatica. **41** (7), 1173–1179 (2005).

[22] Blanco E., De Prada C., Cristea S., Casas J. Nonlinear predictive control in the LHC accelerator. Control Engineering Practice. **17** (10), 1136–1147 (2009).

[23] Mesquine F., Tadeo F., Benzaouia A. Regulator problem for linear systems with constraints on the control and its increments or rate. Automatica. **40** (8), 1387–1395 (2004).

[24] Mesquine F., Tadeo F., Benzaouia A. Constrained Control and Rate or Increment for Linear Systems with Additive Disturbances. Mathematical Problems in Engineering. **2006**, 037591 (2006).

[25] Bitsoris G., Olaru S. Further results on the linear constrained regulation problem. 21st Mediterranean Conference on Control and Automation. 824–830 (2013).

[26] Bitsoris G., Olaru S., Vassilaki M. On the linear constrained regulation problem for continuous-time systems. IFAC Proceedings Volumes. **47** (3), 4004–4009 (2014).

[27] Fernandez-Camacho E., Bordons-Alba C. Model Predictive Control in the Process Industry. Springer Verlag (2004).

[28] Bitsoris G. Positively invariant polyedral sets of discrete-time linear systems. International Journal of Control. **47** (6), 1713–1726 (1988).

[29] Henrion D., Tarbouriech S., Kučera V. Control of linear systems subject to input constraints: a polynomial approach. Automatica. **36** (4), 597–604 (2001).

[30] Benzaouia A., Hmamed A. Regulator problem of linear continuous-time systems with non-symmetrical constrained control. IEEE Transaction on Automatic Control. **38** (10), 1550–1560 (1993).

[31] Mesquine F., Tadeo F., Benzaouia A. Regulator problem for linear systems with constraints on control and its increment. IFAC Proceedings Volumes. **35** (1), 85–90 (2002).

[32] Benzaouia A. Resolution of equation XA+XBX=HX and the pole assignment problem. IEEE Transaction on Automatic Control. **39** (10), 2091–2095 (1994).

Подальші результати щодо задачі регулювання для лінійних систем з обмеженнями на керування та його приріст

Абдельхак А., У-аззу Р.

Кафедра математики, лабораторія диференціальних рівнянь у частинних похідних, алгебри та спектральної геометрії, факультет наук, Університет Ібн Тофаїл, п.с. 133, Кенітра, Марокко

Розглянуто задачу регулятора як для дискретних, так і для неперервних лінійних систем. Елемент керування та його прирости знаходяться під несиметричними обмеженнями, а область обмежень включає початок координат на його межі. Отримано необхідні та достатні умови, які забезпечують виконання всіх обмежень, а також асимптотичну стійкість за допомогою зворотного зв'язку за станом. Наочний приклад демонструє застосування запропонованого методу.

Ключові слова: *лінійні системи; несиметричні обмеження; контроль; додатна інваріантність.*