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Ihor Vrublevskyi

Department of Engineering Mechanics (Weapons and Equipment of Military Engineering Forces), Hetman Petro Sahaidachnyi National Army Academy, 32, Heroes of Maidan str., Lviv, 79026
email: vrublew@gmail.com, ORCID 0000-0001-5954-7754

INCREASING OF VIBRATORY CONVEYING VELOCITY BY OPTIMIZING THE NORMAL VIBRATION

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Abstract. The paper is dedicated to researching the influence of normal vibration on the vibratory conveying velocity of particles on an inclined track that performs independent longitudinal and normal oscillations (two-component vibration). The study considers the optimizing conditions of the conveying velocity for different laws of oscillating components (harmonic, polyharmonic, oscillations with piecewise constant acceleration) with a limited value of the longitudinal acceleration of conveying track and with maximal normal acceleration that does not exceed the gravitational acceleration (non-hopping modes of moving, when particles slide without detachment from the surface). The optimization criterion is the maximal distance, traveled by the particle during the oscillation period, or the maximal value of dimensionless conveying velocity, depending on several dimensionless parameters. The maximal conveying velocity with polyharmonic normal oscillations is achieved at a certain ratio of the amplitudes of harmonic oscillation, which essentially depends on the track's inclination angle to the horizon. The ratios of the amplitudes of harmonic oscillation, which practically do not reduce conveying velocity at any inclination angles, are proposed.

Two-component vibratory conveying under normal oscillations with piecewise constant acceleration is considered in optimal non-hopping modes of a particle moving with one forward (or upward on an inclined track) sliding stage and one backward (or downward) sliding stage during the oscillation period. The equations for determining the dimensionless conveying velocity are derived for different values of dimensionless parameters, such as the inclination angle parameter (a ratio of an inclination angle tangent to a frictional coefficient) and the intensive vibration parameter (a ratio of the amplitudes of longitudinal and normal oscillations, divided by the frictional coefficient). The effectivity of polyharmonic normal oscillations in two-component vibratory conveying is compared with the effectivity of normal oscillations with piecewise constant acceleration. Maximal conveying velocity is achieved at certain values of phase difference angles between longitudinal and normal oscillations, which are called optimal. The value of dimensionless conveying velocity V increases with the increase of asymmetry of normal oscillations, which is described by the ratio n of the maximal acceleration of the track when moving down to the acceleration of gravity. This ratio n corresponds to the number of harmonics for polyharmonic oscillations. A comparison of values of V for normal oscillations with piecewise constant acceleration shows an advantage in velocity compared to polyharmonic normal oscillations at the same number n of harmonics, especially with increasing inclination angles.

The research was carried out by the numerical step-by-step integration method, which allows for performing calculations with any given accuracy. The obtained results are demonstrated in figures and comparative tables.

Keywords: vibratory conveying, two-component longitudinal and normal vibration.

Introduction

Vibratory conveying devices are widely used in different industrial enterprises, especially for piece goods transportation [1–3]. The most common and simple vibratory conveying devices provide the linear vibration of conveying track, usually using the modes of moving with hopping for reaching the high conveying velocity. For conveying massive heavy goods or fragile and explosive products with high velocity, it is necessary to ensure the moving of products without hopping, when particles slide without detachment from the surface. However, the vibratory conveying devices with linear vibration do not allow for achieving high conveying velocity in the moving modes without hopping, especially when conveying upward on an inclined track. An increase in conveying velocity in non-hopping modes can be reached in vibratory devices with two-component vibration: independent longitudinal (along the conveying track) and normal (in the direction perpendicular to the conveying plane) oscillations of conveying track. They also make it possible to increase the track's inclination angles for conveying upward and provide the reverse of moving.

Problem Statement

To achieve the high conveying velocity, the devices with two-component vibration must ensure certain correspondence of parameters of longitudinal and normal oscillations (ratios of oscillation amplitudes, phase difference angles between oscillations' components, the form of oscillations with a more complex vibration law). The simplest two-component vibration – elliptical vibration, when the both components (longitudinal and normal oscillations) are sinusoidal with phase difference between them, and conveying track vibrates by elliptical trajectory. Elliptical vibration allows us to achieve not only high conveying velocity but to increase the lifting angles of the load. Even greater advantages in increasing of conveying velocity and angles of lifting loads are achieved when using polyharmonic oscillations or oscillations with piecewise constant acceleration. However, these advantages can be achieved only with certain values of oscillations parameters, and failure to observe which may not lead to an increase in velocity and lifting angles, but on the contrary to a significant decrease.

Review of Modern Information Sources on the Subject of the Paper

The theory of vibratory conveying of a material point using two-component independent longitudinal and normal oscillations was studied in many papers. The simplest case – elliptical vibration is considered in [2, 4, 5], and optimization of its parameters is carried out in the author's papers [6, 7]. Vibratory conveying by two-component biharmonic oscillations was considered by A. Dunayevetsky, and V. Yefimov, but optimal parameters of vibration were not determined. The author's research determined them for conveying by harmonic longitudinal and polyharmonic normal oscillations with the number of harmonics from 2 to 7 [8, 9].

Vibratory conveying with piecewise constant acceleration is considered in [10, 11]. The optimal relative to velocity law of two-component vibration with a limited value of acceleration was obtained by E. Lavendelis: both components should be oscillations with piecewise constant acceleration (and piecewise linear velocity). The vibratory conveying with harmonic longitudinal oscillations and normal oscillations with piecewise constant acceleration was studied by the author [10], using the approximate harmonic balance method. In these papers, the optimal values of phase difference angles are not determined, but they significantly affect the conveying velocity.

Objectives and Problems of Research

The investigations show that normal oscillations have a decisive influence on the value of conveying velocity because they affect the distribution of frictional forces throughout the vibration's period. The effectiveness of the vibratory conveying process can be determined by the dimensionless conveying velocity as an optimization criterion – the maximum distance traveled by the particle during the oscillation period divided by the amplitude of oscillation velocity, which depends on several dimensionless parameters.

Numerical methods are used in the article, which ensures any given accuracy of the study of the influence of normal vibration parameters, namely polyharmonic oscillations and oscillations with piecewise-constant acceleration, on the conveying velocity.

Optimal parameters of biharmonic normal vibration determining with conveying by harmonic longitudinal vibration

The mean conveying velocity v is calculated by formula [6]:

$$v = A_x \omega V , \tag{1}$$

where A_x and ω – amplitude and frequency of longitudinal (along the X-axis, in the conveying direction on a conveyor's track, inclined at an angle α to the horizon) oscillations; V – dimensionless conveying velocity – parameter, depending on several dimensionless parameters. In the author's papers [8, 9] the vibratory conveying using harmonic longitudinal and polyharmonic normal oscillations is considered, described by equations:

$$x_1 = A_x \sin(\omega t + \varepsilon),$$

$$y_1 = \sum_{i=1}^{n} A_i \sin[(i\omega t + \frac{\pi}{2}(i-1)], \tag{2}$$

where A_i – amplitude of i-th harmonic of normal (along the Y-axis, perpendicular to the conveying surface) oscillations; n – number of harmonics; ε – phase difference angle, t – time. According to the studies, the value of dimensionless V velocity depends on the next dimensionless parameters: K_α – the inclination angle parameter; K_β – the intensive vibration parameter; G – the gravitational overload parameter, which are calculated as follows:

$$K_\alpha = \frac{\tan\alpha}{\mu}, \qquad K_\beta = \frac{A_x}{A_1 \mu}, \qquad G = \frac{A_m \omega^2}{g\cos\alpha},$$

where g – is the gravitational acceleration; A_m – amplitude of normal oscillations in a moment when its acceleration is maximum; μ – the frictional coefficient. To achieve maximum velocity in non-hopping modes the condition $G = 1$ is necessary.

The equation of a particle's motion without hopping on a track in coordinate system XOY, connected with vibratory track, vibrating according to Eq. (2), using dimensionless values can be represented as follows [8]:

$$\chi'' = l_\pm \sin(\tau - \kappa_\pm) - \vartheta_\pm + \sum_{i=2}^{n} \frac{w_i}{w_1 K_\beta l_\pm} \sin\{i\tau - [i\varepsilon - \frac{\pi}{2}(i-1)]\}, \tag{3}$$

where $l_\pm = \dfrac{\sqrt{1 + K_\beta{}^2 \pm 2K_\beta \cos\varepsilon}}{K_\beta}$; $\kappa_\pm = \arctan\dfrac{\sin\varepsilon}{\cos\varepsilon \pm K_\beta}$; $\vartheta_\pm = \dfrac{K_\alpha \pm 1}{w_1 K_\beta l_\pm}$;

$\tau = \omega t$ – dimensionless time; $\chi = \dfrac{x}{A_x}$ – dimensionless distance; $\chi'' = \dfrac{d^2 x}{d\tau^2}$. The signs "+" and "–" correspond to the particle's sliding forward (or upward on an inclined track) and backward (or downward on an inclined track) respectively. The initial moments of the sliding motion forward $\tau = \tau_+$ and sliding motion backward $\tau = \tau_-$ are determined as the roots of the system of two nonlinear equations [8]:

$$\begin{gathered} \cos(\tau_+ - \kappa_+) - \cos(\tau_- - \kappa_+) + \vartheta_+(\tau_+ - \tau_-) + \\ + \sum_{i=2}^{n} \frac{w_i}{i w_1 K_\beta l_+} \{\cos[i\tau_+ - (i\varepsilon - \frac{\pi(i-1)}{2})] - \cos[i\tau_- - (i\varepsilon - \frac{\pi(i-1)}{2})]\} = 0, \\ \cos(\tau_- - \kappa_-) - \cos(\tau_+ - \kappa_-) + \vartheta_-(2\pi + \tau_- - \tau_+) + \\ + \sum_{i=2}^{n} \frac{w_i}{i w_1 K_\beta l_-} \{\cos[i\tau_- + (i\varepsilon - \frac{\pi(i-1)}{2})] - \cos[i\tau_+ + (i\varepsilon - \frac{\pi(i-1)}{2})]\} = 0. \end{gathered} \tag{4}$$

Dimensionless velocity can be found by the formula:

$$V = \frac{S_+ + S_-}{2\pi}, \quad (5)$$

where S_+ and S_- – dimensionless distances of a particle moving forward and backward respectively during the vibration period, is calculated by integrating Eq. (3), using the methodology, described in [9].

The calculations using MathCAD-programs show that the maximal value of velocity V is reached with a certain ratio of harmonics' amplitudes, which is dependent on the parameter K_α and is practically independent of K_β. The velocity V dependence on parameter K_α with different numbers n of harmonics with their optimal ratios is shown in Fig. 1 by continuous lines. Deviations from the optimal values of harmonic's amplitudes ratio can lead to a significant decrease in velocity. However, studies and calculations have shown that at certain ratios of them, the reduction in velocity is relatively insignificant compared to the optimal values of harmonics' amplitudes for any values of K_α . Usually, these values of harmonics' amplitudes ratios (we called them the fixed values) correspond to their optimal values for K_α=0.9.

The velocity V dependence on parameter K_α with these fixed values is shown in Fig.1 by dashed lines. These values also are shown in Table 1.

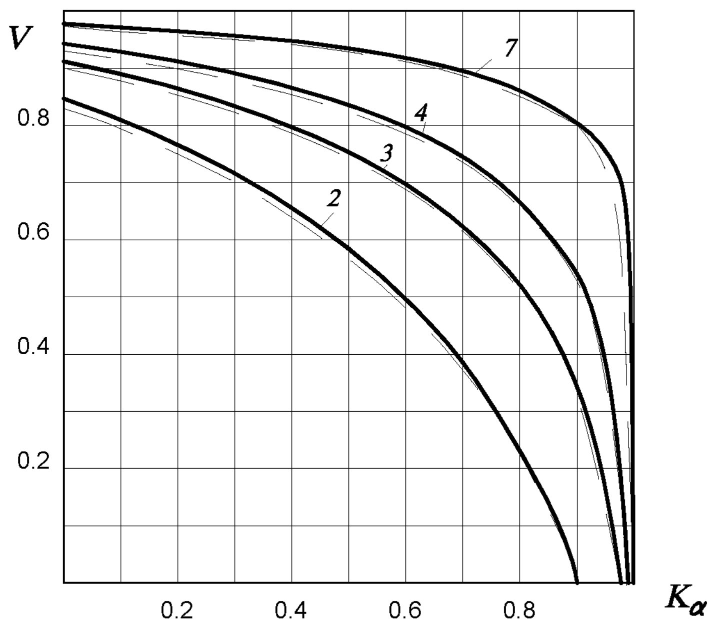

Fig. 1. Dependence of dimensionless velocity V on inclination angle parameter K_α with values of parameters K_β =100 and different number of harmonics n

Table 1

The normal acceleration amplitudes w_i which are suitable for any parameter K_α and K_β

number of harmonics n	w_1	w_2	w_3	w_4	w_5	w_6	w_7	V (at K_α= 0)	
								fixed	*optimal*
2	1.414	0.500	–	–	–	–	–	0.839	0.846
3	1.617	0.916	0.300	–	–	–	–	0.905	0.912
4	1.728	1.199	0.628	0.205	–	–	–	0.938	0.943
5	1.795	1.386	0.898	0.458	0.151	–	–	0.956	0.959
6	1.839	1.514	1.103	0.691	0.350	0.118	–	0.966	0.969
7	1.869	1.606	1.260	0.890	0.549	0.278	0.096	0.973	0.976

The far-right column of the table shows the difference in the value of velocity V at fixed and optimal values of harmonics' amplitudes ratio for horizontal conveying ($K_\alpha = 0$). The difference between these values is insignificant, therefore the ratio of harmonics' amplitudes, which are given in table 1, can be used in practice if it is necessary to frequently change the angles of inclination α.

As seen from Fig. 1 and Table 1, a further increase in the number of harmonics is impractical because a kind of law of change of acceleration of polyharmonic normal oscillations approaches law with piecewise-constant acceleration, which is easier to implement in practice.

Investigation of optimal law of normal oscillations with piecewise constant acceleration

Let us consider the normal oscillations with piecewise constant acceleration (piecewise linear velocity). Their optimal parameters are appropriate for a law with maximum asymmetry in the directions of movement up and down as described in [10]. To correspond to the phase of polyharmonic oscillations, we represent the variation of acceleration in the form (Fig. 2, *a*), described by Eq. 6:

$$y_1'' = \begin{cases} -w, & 0 < \tau < \tau_2 \\ nw, & \tau_2 < \tau < \tau_3 \\ -w, & \tau_3 < \tau < 2\pi \end{cases} \tag{6}$$

where $w = g\cos\alpha$, $\tau_2 = \dfrac{3\pi}{2} - \delta$, $\tau_3 = \dfrac{3\pi}{2} + \delta$, $\delta = \dfrac{\pi}{n+1}$.

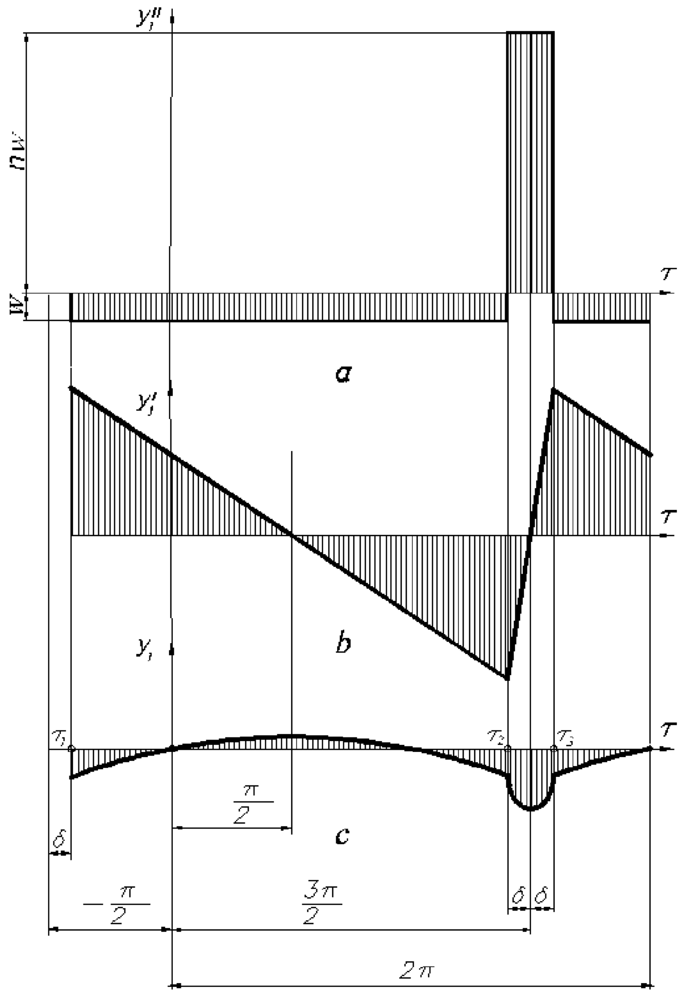

Fig. 2. Normal oscillations with piecewise constant acceleration:
a – acceleration; *b* – velocity; *c* – displacement

By integrating Eq. (6) twice, we obtain the equations for the variation of oscillation velocity (Fig. 2, *b*) and displacement (Fig. 2, *c*) in the normal direction: piecewise linear velocity and displacement in the form of two segments of parabolas during vibration period:

$$y_1 = \begin{cases} -\frac{w}{\omega^2}(\frac{\tau^2}{2} + C_1\tau) + C, & \tau_1 < \tau < \tau_2 \\ \frac{w}{\omega^2}[\frac{n\tau^2}{2} + (C_1 + C_2)\tau + C_3] + C, & \tau_2 < \tau < \tau_3 \end{cases}, \qquad (7)$$

where $\tau_1 = \tau_3 - 2\pi$. The values of the integration constants are determined from the condition of continuity of the function $y_1(\tau)$, namely: $C_1 = \frac{\pi}{2}$, $C_2 = -\frac{3\pi n}{2}$, $C_3 = \frac{\pi^2}{8} \cdot \frac{(3n+1)^2}{n+1}$. The value of constant C does not affect the vibratory conveying velocity; it is selected from the conditions of the best drive operation.

Investigation of dimensionless velocity *V* dependence on vibration parameters with longitudinal harmonic oscillations and normal oscillations with piecewise constant acceleration

Vibratory conveying with longitudinal sinusoidal oscillations and normal oscillations with piecewise constant acceleration was considered in [10] by approximate harmonic balance method. The obtained values of dimensionless conveying velocity *V*, depending on inclination angle parameter K_α, are suitable only when longitudinal oscillation amplitudes are essentially larger than normal amplitudes. The optimal values of phase difference angles are not determined, but these angles have a significant effect on conveying velocity.

Vibratory conveying with longitudinal sinusoidal oscillations and normal oscillations with piecewise constant acceleration is described by equations:

$$x_1 = A_x \cdot \sin(\omega t + \varepsilon),$$

$$y_1 = \begin{cases} -\frac{w}{2\omega^2}(\omega^2 t^2 + \pi\omega t), & -\frac{\pi}{2} + \delta < \omega t < \frac{3\pi}{2} - \delta \\ \frac{w}{2\omega^2}[n\omega^2 t^2 - 3\pi n\omega t + \frac{\pi^2}{8} \cdot \frac{(3n+1)^2}{n+1}], & \frac{3\pi}{2} - \delta < \omega t < \frac{3\pi}{2} + \delta \end{cases}. \qquad (8)$$

As the studies show [6, 8], with intensive vibration ($K_\beta > 10$) the particles move on the vibrating track in the mode with one sliding motion forward and one sliding motion backward during the vibration period without sticks relative to the track or in the mode with one sliding motion forward and one stick. Consider the vibratory conveying process with two-component oscillations of the track, described by Eq. (8), in mode without sticks. The variation of a particle's velocity at four stages during vibration period occurs according to different dependencies (Fig. 3).

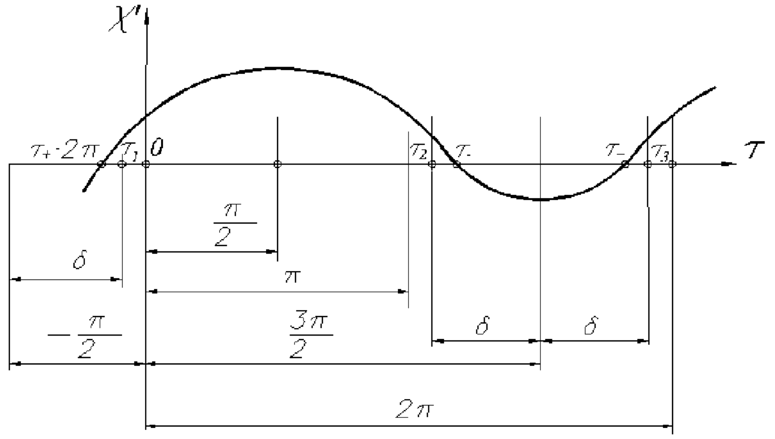

Fig. 3. Conveying velocity variation during vibration period

The equation of a particle's motion without hopping on a track, vibrating according to this law:

$$\chi'' = \sin(\tau + \varepsilon) + \vartheta_{1,2\pm}. \tag{9}$$

The values of $\vartheta_{1,2\pm}$ in Eq. (9) are different for different stages: $\vartheta_1 = qK_\alpha$ when $\tau_1 < \tau < \tau_2$; $\vartheta_{2\pm} = q(1+n\pm K_\alpha)$ when $\tau_2 < \tau < \tau_3$, where $q = 1/K_\beta$. The signs "+" and "–" correspond to the particle's sliding forward and backward respectively.

Integrating Eq. (9), we obtain the dependencies of variation in a particle's velocity at four stages:

$$\begin{aligned}
&\chi_1' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + q[(1+n)(\tau_- - \tau_2) + K_\alpha(\tau_- - \tau)] \quad \textit{if} \quad \tau_3 - 2\pi < \tau < \tau_2;\\
&\chi_2' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + q(1+n+K_\alpha)(\tau_- - \tau) \quad \textit{if} \quad \tau_2 < \tau < \tau_-;\\
&\chi_3' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + q(1+n-K_\alpha)(\tau - \tau_-) \quad \textit{if} \quad \tau_- < \tau < \tau_+;\\
&\chi_4' = \cos(\tau_+ + \varepsilon) - \cos(\tau + \varepsilon) + q(1+n+K_\alpha)(\tau_+ - \tau) \quad \textit{if} \quad \tau_+ < \tau < \tau_3.
\end{aligned} \tag{10}$$

The initial moments of the sliding motion forward $\tau = \tau_+$ and sliding motion backward $\tau = \tau_-$ are determined as the roots of the system of two nonlinear equations:

$$\begin{aligned}
&\cos(\tau_+ + \varepsilon) - \cos(\tau_- + \varepsilon) + q[(1+n+K_\alpha)(\tau_+ - \tau_-) + (1+n)(\tau_2 - \tau_3) - 2\pi K_\alpha] = 0,\\
&\cos(\tau_- + \varepsilon) - \cos(\tau_+ + \varepsilon) + q(1+n-K_\alpha)(\tau_+ - \tau_-) = 0.
\end{aligned} \tag{11}$$

Integrating the Eqs. (10) at the corresponding stages, we obtain the values of dimensionless distances, travelled by a particle at different stages:

$$\begin{aligned}
&S_1 = \cos(\tau_- + \varepsilon)(\tau_2 - \tau_3 + 2\pi) + \sin(\tau_3 + \varepsilon) - \sin(\tau_2 + \varepsilon) +\\
&\quad + q[(1+n)(\tau_- - \tau_2)(\tau_2 - \tau_3 + 2\pi) - K_\alpha \frac{(\tau_- - \tau_2)^2 - (\tau_- - \tau_3 + 2\pi)^2}{2}];\\
&S_2 = \cos(\tau_- + \varepsilon)(\tau_- - \tau_2) + \sin(\tau_2 + \varepsilon) - \sin(\tau_- + \varepsilon) - q(1+n+K_\alpha)\frac{(\tau_- - \tau_2)^2}{2};\\
&S_3 = \cos(\tau_- + \varepsilon)(\tau_+ - \tau_-) + \sin(\tau_- + \varepsilon) - \sin(\tau_+ + \varepsilon) + q(1+n-K_\alpha)\frac{(\tau_+ - \tau_-)^2}{2};\\
&S_4 = \cos(\tau_+ + \varepsilon)(\tau_3 - \tau_+) + \sin(\tau_+ + \varepsilon) - \sin(\tau_3 + \varepsilon) - q(1+n+K_\alpha)\frac{(\tau_+ - \tau_3)^2}{2}.
\end{aligned} \tag{12}$$

Dimensionless velocity V is determined in accordance with Eq. (5) by formula:

$$V = \frac{S_1 + S_2 + S_3 + S_4}{2\pi}. \tag{13}$$

So by setting the values of dimensionless parameters K_α, K_β, n, ε, we can calculate the value of dimensionless velocity V.

The Eqs. (10)–(12) are valid under the condition that the stage of sliding motion backward are inside region ($\tau_2 < \tau < \tau_3$), or $\tau_- > \tau_2$ and $\tau_+ < \tau_3$. If this condition is not fulfilled, the equations take another form, and there are only three stages during the vibration period.

If $\tau_- < \tau_2$ and $\tau_+ < \tau_3$, which may occur when the value of ε increases, then we obtain

– the dependencies of variation in a particle's velocity at different stages:

$$\begin{aligned}
&\chi_1' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + qK_\alpha(\tau_- - \tau) \quad \textit{if} \quad \tau_3 - 2\pi < \tau < \tau_2,\\
&\chi_3' = \cos(\tau_+ + \varepsilon) - \cos(\tau + \varepsilon) + q(1+n-K_\alpha)(\tau - \tau_+) \quad \textit{if} \quad \tau_2 < \tau < \tau_+,\\
&\chi_4' = \cos(\tau_+ + \varepsilon) - \cos(\tau + \varepsilon) + q(1+n+K_\alpha)(\tau_+ - \tau) \quad \textit{if} \quad \tau_+ < \tau < \tau_3,
\end{aligned}$$

– the system of two equations for determining the values of τ_- and τ_+ :

$$\begin{aligned}
&\cos(\tau_+ + \varepsilon) - \cos(\tau_- + \varepsilon) + q[(1+n)(\tau_2 - \tau_+) + K_\alpha(\tau_+ - \tau_-)] = 0,\\
&\cos(\tau_- + \varepsilon) - \cos(\tau_+ + \varepsilon) + q[(1+n)(\tau_3 - \tau_+) + K_\alpha(\tau_- - \tau_+ + 2\pi)] = 0,
\end{aligned}$$

– the values of dimensionless distances, travelled by a particle at different stages:

$$\begin{aligned}
&S_1 = \cos(\tau_- + \varepsilon)(\tau_2 - \tau_3 + 2\pi) + \sin(\tau_3 + \varepsilon) - \sin(\tau_2 + \varepsilon) - qK_\alpha \frac{(\tau_- - \tau_2)^2 - (\tau_- - \tau_3 + 2\pi)^2}{2};\\
&S_2 = 0; \quad S_3 = \cos(\tau_+ + \varepsilon)(\tau_+ - \tau_2) + \sin(\tau_2 + \varepsilon) - \sin(\tau_+ + \varepsilon) - q(1+n-K_\alpha)\frac{(\tau_2 - \tau_+)^2}{2};
\end{aligned}$$

$$S_4 = \cos(\tau_+ + \varepsilon)(\tau_3 - \tau_+) + \sin(\tau_+ + \varepsilon) - \sin(\tau_3 + \varepsilon) - q(1 + n + K_\alpha)\frac{(\tau_+ - \tau_3)^2}{2}.$$

If $\tau_- > \tau_2$ and $\tau_+ > \tau_3$, which may occur when the value of ε decreases, then we obtain

– the dependencies of variation in a particle's velocity at different stages:

$$\chi_1' = \cos(\tau_+ + \varepsilon) - \cos(\tau + \varepsilon) + qK_\alpha(\tau_+ - \tau - 2\pi) \quad if \quad \tau_3 - 2\pi < \tau < \tau_2,$$

$$\chi_2' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + q(1 + n + K_\alpha)(\tau_- - \tau) \quad if \quad \tau_2 < \tau < \tau_-,$$

$$\chi_3' = \cos(\tau_- + \varepsilon) - \cos(\tau + \varepsilon) + q(1 + n - K_\alpha)(\tau - \tau_-) \quad if \quad \tau_+ < \tau < \tau_3,$$

– the system of equations for determining the values of τ_- and τ_+:

$$\cos(\tau_+ + \varepsilon) - \cos(\tau_- + \varepsilon) + q[(1 + n)(\tau_2 - \tau_-) + K_\alpha(\tau_+ - \tau_- - 2\pi)] = 0,$$

$$\cos(\tau_- + \varepsilon) - \cos(\tau_+ + \varepsilon) + q[(1 + n)(\tau_3 - \tau_-) + K_\alpha(\tau_- - \tau_+)] = 0; \qquad (14)$$

– the values of dimensionless distances, travelled by a particle at different stages:

$$S_1 = \cos(\tau_+ + \varepsilon)(\tau_2 - \tau_+ + 2\pi) + \sin(\tau_+ + \varepsilon) - \sin(\tau_2 + \varepsilon) - qK_\alpha\frac{(\tau_+ - \tau_2 - 2\pi)^2}{2};$$

$$S_2 = \cos(\tau_- + \varepsilon)(\tau_- - \tau_2) + \sin(\tau_2 + \varepsilon) - \sin(\tau_- + \varepsilon) - q(1 + n + K_\alpha)\frac{(\tau_- - \tau_2)^2}{2};$$

$$S_3 = \cos(\tau_- + \varepsilon)(\tau_3 - \tau_-) + \sin(\tau_- + \varepsilon) - \sin(\tau_3 + \varepsilon) + q(1 + n - K_\alpha)\frac{(\tau_3 - \tau_-)^2}{2}.$$

Under certain conditions the system of eqs. (14) is not solved, and a mode with one sliding motion forward and one sliding motion backward, and one stick during the oscillation period are possible. In this case, the first stage of moving forward begins at $\tau = \tau_1$. And the dependence of the particle's velocity variation at the first stage:

$$\chi_1' = \cos(\tau_1 + \varepsilon) - \cos(\tau + \varepsilon) + qK_\alpha(\tau_1 - \tau)]$$

The system of equations for determining their roots as the values of τ_- and τ_+:

$$\cos(\tau_+ + \varepsilon) - \cos(\tau_- + \varepsilon) + q(1 + n - K_\alpha)(\tau_- - \tau_+) = 0,$$

$$\cos(\tau_- + \varepsilon) - \cos(\tau_3 + \varepsilon) + q[(1 + n)(\tau_- - \tau_+) + K_\alpha(\tau_- - \tau_3 + 2\pi)] = 0. \qquad (15)$$

The dimensionless distance at the first stage:

$$S_1 = \cos(\tau_1 + \varepsilon)(\tau_2 - \tau_1) + \sin(\tau_1 + \varepsilon) - \sin(\tau_2 + \varepsilon) - qK_\alpha\frac{(\tau_1 - \tau_2)^2}{2}.$$

S_2 and S_3 are determined by Eqs (12). And with an increase in n the system of equations (15) is not solved, and a mode with one sliding motion forward and one stick without sliding motion backward during the oscillation period is possible. In this case $S_3 = 0$.

Table 2

Comparison of dimensionless velocity values V for normal polyharmonic oscillations and oscillations with piecewise constant acceleration for different values of K_α and $K_\beta = 100$

number of harmonics or ratio of acceleration's amplitudes *n*	*Dimensionless velocity V*			
	at $K_\alpha = 0$		at $K_\alpha = 0.9$	
	acceleration of normal oscillations			
	polyharmonic	*piecewise constant*	*polyharmonic*	*piecewise constant*
1	0.636	0.707	-0.297	0.079
2	0.846	0.871	0.001	0.545
3	0.912	0.935	0.343	0.735
4	0.943	0.959	0.548	0.827
5	0.959	0.974	0.672	0.879
6	0.969	0.984	0.751	0.910
7	0.976	0.991	0.807	0.930

Calculations of dimensionless velocity V values with different values of dimensionless parameters K_α, K_β, n, ε were performed using MathCAD-programs. Some results are shown in Fig. 4 and in Table 2. The maximum value of V is reached at certain (optimal) value of phase difference angle $\varepsilon = \varepsilon_o$, which is depend on values of K_α, K_β and n. Graphs of dimensionless velocity V dependence on inclination angle parameter K_α are shown in Fig. 4 for normal oscillations with piecewise constant acceleration by continuous lines, and for normal polyharmonic oscillations are shown by dashed lines for comparison.

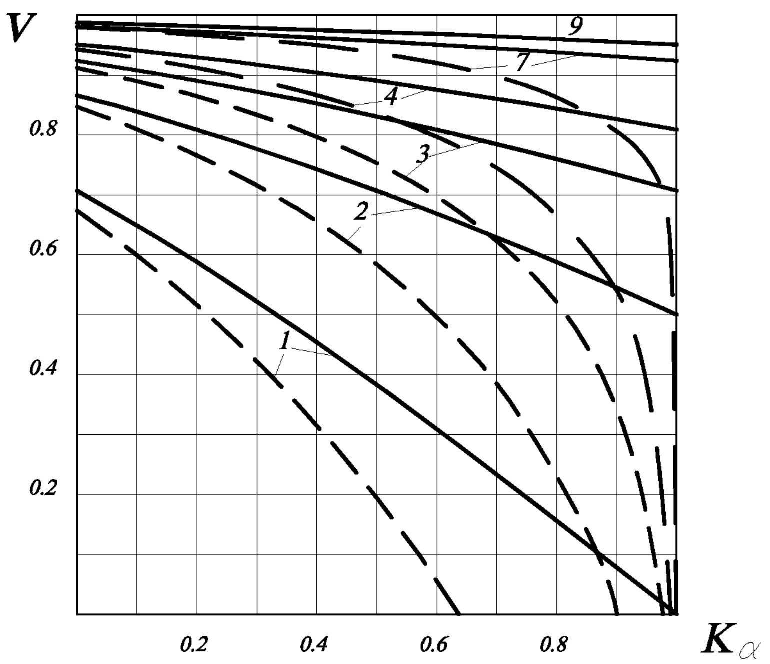

Fig. 4. Dependence of dimensionless velocity V on inclination angle parameter K_α with values of parameters K_β =100 and different ratios of normal acceleration n

Comparison of V values for normal oscillations with piecewise constant acceleration with the values for polyharmonic normal oscillations at the same value of n shows the superiority of the former, especially with increasing K_α. The value of K_β in the range 10< K_β <100 practically does not affect V but significantly affects ε_o. An increase in n gives an increase in the velocity V, especially at values of K_α close to the limiting values. But as n increases, the increase in V decreases significantly, so it is hardly worth using n>10.

Conclusions

The paper studies the vibratory conveying process with normal oscillations with piecewise constant acceleration and harmonic longitudinal oscillations of track for non-hopping modes of particles moving (when particles slide without detachment from the surface). The dimensionless velocity V dependence on several dimensionless parameters is investigated. These dimensionless parameters are the inclination angle parameter (a ratio of an inclination angle tangent to a frictional coefficient) and the intensive vibration parameter (a ratio of the amplitudes of longitudinal and normal oscillations, divided by a frictional coefficient). The velocity V decreases with increasing inclination angle parameter K_α (increasing of inclination angle α when moving upward) and practically does not change in the range of values $10<K_\beta<100$.

The velocity V increases with an increase of the asymmetry of normal oscillations, which is characterized by parameter n, but with a subsequent value of n increasing this increase decreases. The maximum value of velocity is achieved at certain values of different phase angles ε between longitudinal and normal oscillations, which also depend on parameters K_α and K_β.

In further research, it is necessary to investigate in detail the dependence of the optimal phase difference angle ε_o between longitudinal and normal oscillations on the dimensionless parameters, namely n, K_α, and K_β. It is also proposed to develop the experimental vibratory drives of harmonic longitudinal oscillations and normal oscillations with piecewise constant acceleration, which will allow us to verify the obtained theoretical results in practice.

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