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# CONSTRUCTION OF A MATHEMATICAL MODEL OF AN UNBALANCED VIBRATING SEPARATOR ON A SPRING SUSPENSION

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**Abstract.** In the article, a mathematical model of the oscillating motion of a vibrating separator is constructed. The methods of nonlinear mechanics and the calculation scheme of the vibration separator with an eccentric and a spring suspension, which is presented in the form of a flat mechanical system with four degrees of freedom, were used for its construction. The amplitude of oscillations of the vibration separator capacities in the vertical plane is greater than the amplitude of its oscillations in the horizontal plane. It is believed that the containers of the vibroseparator move only in the vertical plane, that is, they are in planar motion. The obtained mathematical model makes it possible to investigate the influence of the separator parameters with their arbitrary combination on the productivity of its work with the aim of its optimization.

**Keywords:** vibratory oscillations, mathematical model, separator, motion.

## Introduction and Problem Statement

The field of use of vibrating separators for separating bulk materials into fractions tends to further growth in machine-building, mining, construction, and other industries. Despite the wide range of applications of vibration separators, the dynamic processes in such complex systems were described mainly in a linear formulation, which mostly inadequately reflects the physical phenomena of the separation process [1]. The needs of practice require the prediction of separation results depending on the structure, physical and mechanical properties of the components of the medium to be separated, and therefore the appropriate selection of the type and characteristics of the vibrating separator, taking into account the nonlinear forces that occur during its interaction with the medium. Solving such complex problems, which take into account the influence of the drive on the dynamics of various types of environment and the separator, makes it possible to predict the intensity of interaction between them, to choose the geometric and kinematic parameters of the separator accordingly, and ultimately to reduce their metal and energy consumption and increase the intensity separation process. It explains the relevance of the study of dynamic processes in vibration separators.

The magnitude of the vibration amplitude of the container of the vibrating separator is affected by the parameters of the unbalanced mass of the unbalances, the mass of the working containers of the vibrating separator, the separated medium, the stiffness of the suspension, the rotation frequency of the unbalances, etc. The study of the complex influence of these parameters on the intensity of vibration separation of bulk materials, as well as the geometric parameters of the vibration separator – the dimensions of the working container, the place of attachment of the imbalance, the location of the suspension, the influence of various combinations of these parameters -, is an important task. It is possible to make a comprehensive impact of the above parameters on the intensity of vibration separation of bulk materials by studying dynamic processes in the vibration separator based on the construction of mathematical models of the movement of the working body of the vibration separator. With the help of built models at the design stage, it is possible to select such parameters of the vibrating separator that would ensure the maximum possible intensity of separation of bulk materials.

## Review of Modern Information Sources on the Subject of the Paper

The necessity to apply the phenomenon of separation determines the use of a wide range of separators, which can be classified into separators with movable and stationary sieves [1]. Separators of the first type have significantly higher productivity and, accordingly, the level of application due to a better level of interaction of the sieve surface with the mixture to be separated. The level of rationality of the separation process also increases with an increase in the number of sifting surfaces (sieves) in the design of the separator [2], [3]. Concentric [4] placement of sieves in drum-type separators, or sequential placement of round or U-shaped cross-section sieves in separator structures enable their effective use as separators - conveyors with simultaneous provision of separation phenomena and transfer of separated fractions in space.

Separators with moving sieves are quite complex dynamic systems, the effective design of which and subsequent operation are possible only based on their thorough theoretical research, particularly with mathematical modeling. There are several attempts to theoretically describe the dynamics of separators, but all of them have their limited application. Thus, in [5], [6] only the dynamics of separate parts of the separator are theoretically investigated, there are no studies of the entire separator as a single dynamic system. In [6], the motion of oscillating surfaces is studied only in the linear formulation of the problem, and the obtained theoretical models are linear, which narrows the possibility of their application. In [7], the dynamics of separators is studied by numerical methods.

The authors of the article, based on asymptotic methods of nonlinear mechanics [4], developed several models of vibration processing systems [4, 8], including separators, in the form of sets of analytical expressions, which included the key parameters of these systems - kinematic, geometric and power. These models are non-linear and make it possible to describe the dynamic processes occurring oscillating system under study. According to this technique, the presented vibration separator with the horizontal layering of sieves was studied.

## Main Material Presentation

The schematic diagram of the vibration separator selected for modeling is presented in Fig. 1. The following designations are used in the figure: 1 – working container of the vibrating separator; 2 – unbalanced vibrator; 3 – belt coupling; 4 – vibration separator frame; 5 – a set of sieves for separation.

The vibrating separator consists of three working containers 1, in which there are separation sieves 5 with different cell sizes. The largest cell is in the upper sieve of the upper container, the smallest is in the lower sieve of the lower container. From 0 to 6 sieves can be installed in working containers (2 sieves in each container). Thus, this separator can separate the mixture into a maximum of 9 fractions (three fractions in each container – two accumulate on the sieves, the third – on the bottom of the container). Containers are connected by nozzles in which separation sieves are also installed. The mixture to be separated is loaded into the upper container. Under the action of imbalances 2, which are set in motion by asynchronous electric motors through belt couplings 3, the working containers begin to vibrate making a planar movement. The working mixture is thrown up and moved (due to the different amplitude of left and right unbalance oscillations with different unbalanced masses) along the upper container. The largest particles remain on the upper sieve, and the smaller ones fall on the lower sieve. Particles of a certain size remain on it, which do not pass through the cell of the corresponding sieve. The rest of the mixture falls to the bottom of the upper container and, passing through the sieve of the connecting pipe, which also separates one fraction, falls on the upper sieve of the second (middle container), etc. The separation process on the middle and lower containers are similar to that on the upper one. As a result, the mixture is separated into 9 parts. At the end of the separation process, the operator turns off the drive, opens the side openings on the containers (there are 3 of them on each container – a circle of sieves and a bottom), turns on the drive and the separated mixture is discharged from the vibrating separator under the influence of vibration to the outside - each fraction is separated by size into a separate tray. This is how the investigated vibration separator works.

*Calculation scheme of the generalized vibration separator.* The amplitude of oscillations of the containers of the vibrating separator in the vertical plane (the plane of rotation of imbalances) is much greater than the amplitude of its oscillations in the horizontal plane. Therefore, it can be assumed that the containers of the vibrating separator move only in the vertical plane (in the plane of rotation of imbalances), that is, they are in planar motion. The calculation scheme of the vibrating separator can be represented as a flat mechanical system that has four degrees of freedom (three degrees have containers and one is an imbalance that rotates around a horizontal axis, in one plane). In this case, the calculation scheme will look like this (see Fig. 2).

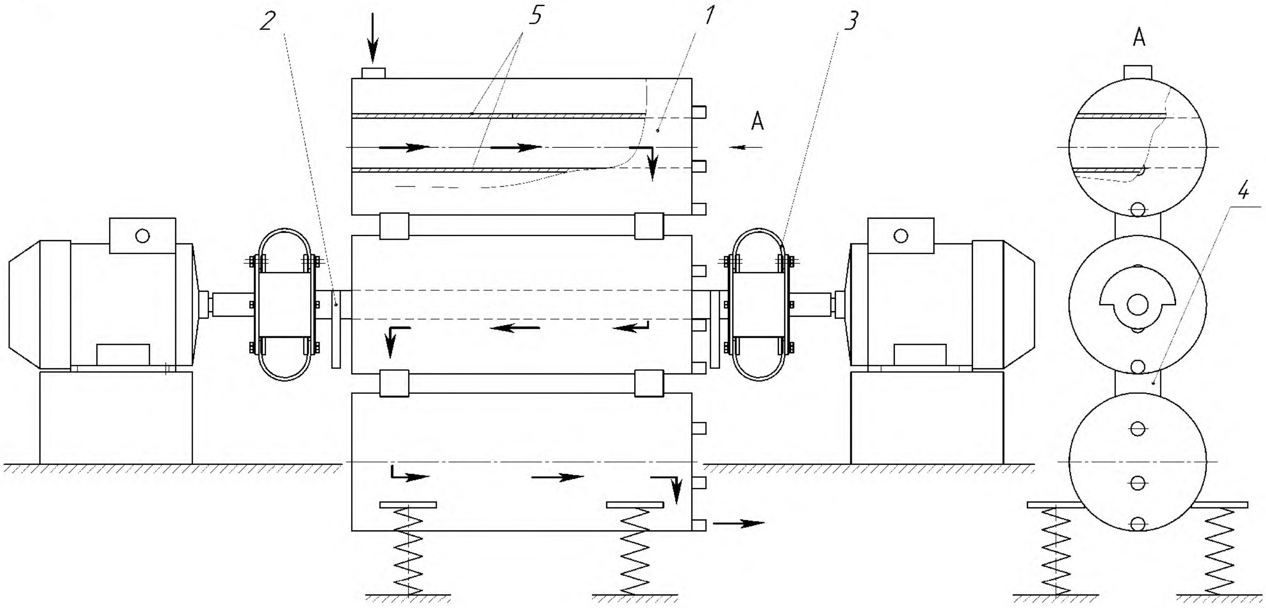


**Fig. 1.** The schematic diagram of the vibration separator selected for modeling.

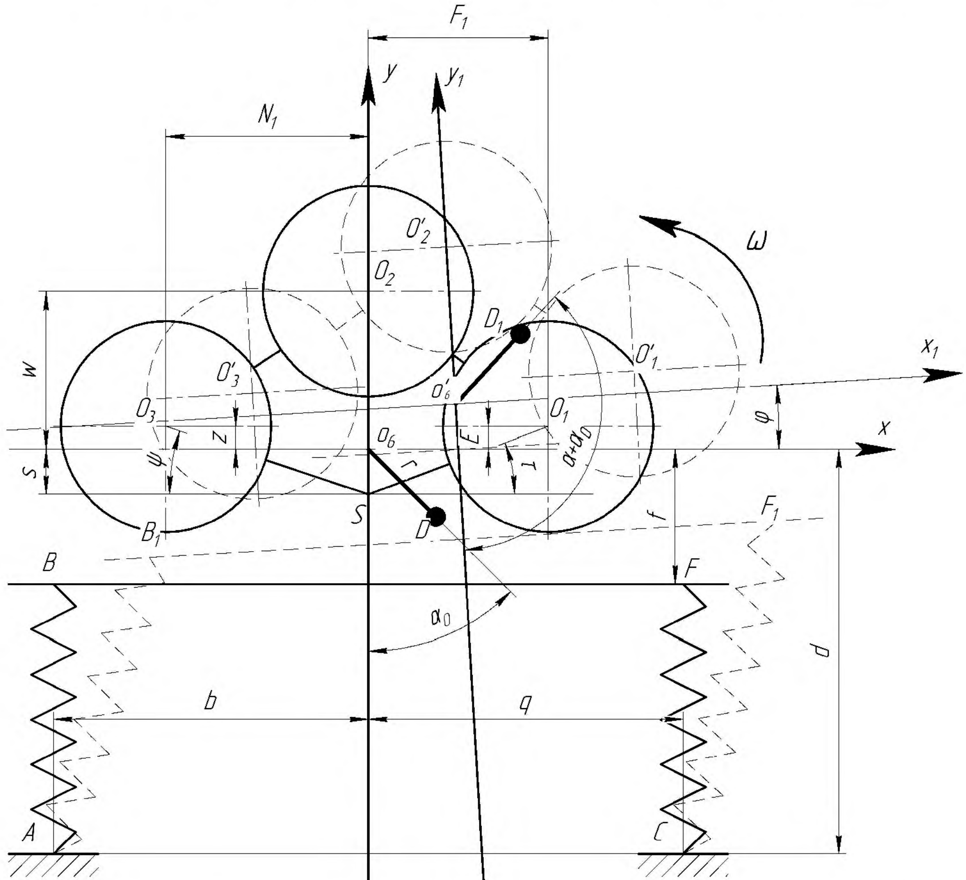


**Fig. 2.** Calculation scheme of the generalized vibration separator with three containers of arbitrary location.

We will introduce the following notations and assumptions: working containers have masses $M_{k1}$, $M_{k2}$, $M_{k3}$, $XO_6Y$ - stationary coordinate system; $X_1O_6'Y_1$ - the moving coordinate system associated with the center of rotation of the imbalance and the vertical axis of symmetry of the containers moves together with them, and its coordinate origin is the point $O_6'$ coincides with the center of rotation of the imbalance; $\phi$ – the rotation angle of the containers relative to the initial position during their movement (the rotation angle of the moving coordinate system relative to the stationary one). At the initial moment, the centers of the moving and stationary coordinate systems coincide, the geometric center of the container is at the point $O_6'$, and its center of mass is a point S, lies on the axis Y. Let $O_1$, $O_2$, $O_3$ be the geometric centers of the containers of the vibrating separator, $O_6D$ = r – the radius of rotation of the balance (the amount of eccentricity of the balance). We assume that mass $M_D$ – unbalanced mass concentrated in one point, $\omega$ – the angular speed of rotation of the imbalance (we take it as a constant value for a certain processing mode), $\alpha_0$ and $\alpha+\alpha_0$ – the initial phases of the imbalance position; $\alpha = \omega t$ – the angle of rotation of the imbalance at an arbitrary moment relative to the initial position; C – total stiffness of elastic elements; $C_1$ and $C_2$ – stiffness of the right and left suspension, respectively; *Lspr* – the length of the undeformed suspension (springs); $AB_1$ and $CF_1$ – the length of the springs at any moment of the movement of the container; AB and CF – the length of the spring at the initial moment of time, b and q – distance from the left and right suspension supports to the axis $O_6Y$ – fixed coordinate system.

The developed scheme generally reflects the structural and kinematic parameters of the investigated vibration separator. It will make it possible to build an adequate parameterized unified mathematical model of the movement of a generalized vibrating separator, which will make it possible to investigate the influence of design parameters and technological modes on the intensity of separation of bulk materials in it.

**Results and discussion**

*Mathematical modeling of the motion of the vibrating separator.* The movement of the vibrating separator (the movement of any point of the working containers during an arbitrary time interval of the separation of bulk materials) can be described by a system of analytical expressions, which includes all its necessary parameters (solutions of the system of differential equations that describe the movement of the vibrating separator). The mathematical model of the movement of the container (working body) of the vibrating separator is the law of movement of its geometric center (point $O_6$) or it's center of mass, and the angle of rotation of the container around its center of mass. By substituting here the coordinates of any point of the container, the movement of which must be investigated, and the necessary parameters of the vibration separator, it is possible to obtain expressions for constructing the trajectories of the movement of the selected point of the container and determine its amplitude, amplitude-frequency characteristics.

The differential equations of the oscillatory motion of this mechanical system can be derived from Lagrange's equations of the II kind:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0, \tag{1}$$

where $L = T - (P + P_p)$ – the Lagrangian function, $T = T_k + T_D$ – kinetic energy of the system, consisting of the sum of the kinetic energy of the containers and the energy of imbalance; $P$ – the potential energy of the system (containers and imbalance*)*, $P_p$ – the potential energy of the elastic suspension of containers; $q_j$ – generalized coordinates, ie $q_1 = x_{0_6}$, $q_2 = y_{0_6}$, $q_3 = \phi$, a $\dot{q}_1 = \dot{x}_{0_6}$, $\dot{q}_2 = \dot{y}_{0_6}$, $\dot{q}_3 = \dot{\phi}$, respectively, their generalized velocities.

An imbalance, as mentioned above, is considered a material point in which the mass of the imbalance is concentrated, and therefore the expression for finding the kinetic energy of a material point is used to record the kinetic energy of the imbalance.

The Lagrangian of this mechanical system, taking into account the above, took the form:

$$
\begin{aligned}
L = {} & \frac{M_D}{2}((\dot{x}_{0_6} + r\cos(\omega t + \phi + \alpha_0)(\omega + \dot{\phi}))^2 + (\dot{y}_{0_6} + r\sin(\omega t + \phi + \alpha_0)(\omega + \dot{\phi}))^2) + \\
& + \frac{M_{K_1}}{2}((\dot{x}_{0_6} - \dot{\phi}F_1\sin\phi + \dot{\phi}(F_1 tg\tau + S)\cos\phi)^2 + (\dot{y}_{0_6} + \dot{\phi}F_1\cos\phi + \\
& + \dot{\phi}(F_1 tg\tau + S)\sin\phi)^2) + \frac{M_{K_2}}{2}((x_{0_6} - \dot{\phi}W\cos\phi)^2 + \\
& + (\dot{y}_{0_6} - \dot{\phi}W\sin\phi)^2 + \frac{M_{K_3}}{2}((\dot{x}_{0_6} + \dot{\phi}N_1\sin\phi + \dot{\phi}(N_1 tg\psi + S)\cos\phi)^2 + \\
& + (\dot{y}_{0_6} - \dot{\phi}N_1\cos\phi + \dot{\phi}(N_1 tg\psi + S)\sin\phi)^2) + \frac{1}{2}j\dot{\phi}^2 - \\
& - \left[\frac{C_1}{2}((x_{0_6} - b\cos\phi + d\sin\phi + b)^2 + (y_{0_6} - b\sin\phi - \right. \\
& \left. - d\cos\phi + d)^2 - (d-f)^2) + \frac{C_2}{2}((x_{0_6} + q\cos\phi + f\sin\phi - q)^2 + \atop + (y_{0_6} + q\sin\phi - \cos\phi + d)^2 - (d-f)^2 \right] + \\
& + \left( M_{K_1} g(y_{0_6} + F_1\sin\phi - (F_1 tg\tau + S)\cos\phi - (F_1 tg\tau + S)) + \atop + M_{K_2} g(y_{0_6} + W\cos\phi - W) + + M_{K_3} g \times \right. \\
& \left. \times(y_{0_6} - N_1\sin\phi - \cos\phi(N_1 tg\psi + S) - (N_1 tg\psi + S)) + \atop + M_D g(r\cos\alpha_0 + y_{0_6} - r\cos(\omega t + \phi + \alpha_0)) \right).
\end{aligned}
\tag{2}
$$

We substitute the derived expressions for each generalized coordinate into Lagrange equations of the II kind, reduce them to such a form that only the sum of the second derivative of the generalized coordinate and the product of this coordinate by a certain coefficient remains in the left part, and we transfer the other terms to the right side of the equation. In this way, the equation is reduced to a system of perturbed (from a mathematical point of view – they have a time-varying function on the right-hand side, from a physical point of view – the presence of an external periodic disturbing force (or several forces), which sets the system in motion – the driving force of the vibrating separator) of nonlinear differential equations, which will be the mathematical model of the movement of the vibrating separator container:

$$
\begin{cases}
\ddot{x}_c + \omega_c{}^2 x_c = \varepsilon f_x(\phi, \dot{\phi}, \ddot{\phi}, \omega t + \alpha_0); \\
\ddot{y}_c + \omega_c{}^2 y_c = \varepsilon f_y(\phi, \dot{\phi}, \ddot{\phi}, \omega t + \alpha_0);, \\
\ddot{\phi} + \omega_\phi^2(t)\phi = \varepsilon' f_\phi(\phi, \dot{\phi}, \ddot{x}_c, \ddot{y}_c),
\end{cases}
\tag{3}
$$

where $\varepsilon = \frac{1}{M}$, $\varepsilon' \approx \varepsilon$, $\omega_c = \sqrt{\frac{c}{M}}$ – natural frequency of oscillations of the container; c – total stiffness of the suspension; $M$ – total mass of containers, imbalance, separating medium; $\omega_\phi(t)$ – “frequency” of circular oscillations of containers, taking into account unbalance.

Consider the third equation. The angle of rotation of the container $\varphi$ changes by an insignificant amount – from $0°$ to $4°$. Therefore, it can be argued that $\sin\phi \approx \phi$, and the third equation of the system can be written as $\ddot{\phi} + k1^2\phi = 0$,

where $k1 = \sqrt{\dfrac{b_1}{d_1}}$, $b_1 = C_1(b^2 + d^2) + C_2q^2 + df - M_{K_1}g(F_1tg\tau + S) + M_{K_2}gW - M_{K_3}g(N_1tg\psi + S)),$

$$d_1 = M_Д r^2 + J + M_{K_1}F_1^2 + M_{K_2}W^2 + M_{K_3}N_1^2 .$$

Its solution can be represented as

$$\phi(t) = L_1 \sin kt + L_2 \cos kt,$$

where $L_1$, $L_2$ – are determined by the initial parameters of the mechanical system.

In this case, taking into account the above, the final system of equations will have the form:

$$\ddot{x}_{0_6} + \frac{C}{M} = \frac{1}{\varepsilon}\left[\begin{array}{l} k^2(L_1\sin kt + L_2\cos kt)(M_D r\cos(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + M_{K_1}\times \\ \times\left[(F_1tg\tau + S)\cos(L_1\sin kt + L_2\cos kt) - F_1\sin(L_1\sin kt + L_2\cos kt)\right] - \\ -M_{K_2}W\cos(L_1\sin kt + L_2\cos kt) + M_{K_3}\left(N_1\sin(L_1\sin kt + L_2\cos kt) + \right. \\ \left. +(N_1tg\psi + S)\cos(L_1\sin kt + L_2\cos kt)\right) - (L_1k\cos kt - L_2k\sin kt)^2\times \\ \times\left[\begin{array}{l} -M_{K_1}(F_1\cos(L_1\sin kt + L_2\cos kt) + (F_1tg\tau + S)\sin(L_1\sin kt + L_2\cos kt) + \\ +M_{K_2}W\sin(L_1\sin kt + L_2\cos kt) + M_{K_3}(N_1\cos(L_1\sin kt + L_2\cos kt) - \\ -(N_1tg\psi + S)\sin(L_1\sin kt + L_2\cos kt)) + (Ptg\gamma + S)\sin(L_1\sin kt + L_2\cos kt)) \end{array}\right] + \\ +M_D r(\omega + L_1k\cos kt - L_2k\sin kt)(-C_2q + C_1b) - \sin(L_1\sin kt + L_2\cos kt)\times \\ \times(C_1d - C_2f) + C_2q - C_1b \end{array}\right]$$

$$\ddot{y}_{0_6} + \frac{C}{M} = \frac{1}{\varepsilon}\left[\begin{array}{l} k^2(L_1\sin kt + L_2\cos kt)(M_D(r\sin(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + \\ M_{K_1}(F_1\cos(L_1\sin kt + L_2\cos kt) + (F_1tg\tau + S)\sin(L_1\sin kt + L_2\cos kt)) - \\ -M_{K_2}W\sin(L_1\sin kt + L_2\cos kt) + M_{K_3}((N_1tg\psi + S)\sin(L_1\sin kt + L_2\cos kt) - \\ -N_1\cos(L_1\sin kt + L_2\cos kt)) + (Ptg\gamma + S)\sin(L_1\sin kt + L_2\cos kt))) - \\ -(L_1k\cos kt - L_2k\sin kt)^2\times(M_{K_1}((F_1tg\tau + S)\cos(L_1\sin kt + L_2\cos kt) - \\ -F_1\sin(L_1\sin kt + L_2\cos kt)) - M_{K_2}W\cos(L_1\sin kt + L_2\cos kt) + M_{K_3}\times \\ \times(N_1\sin(L_1\sin kt + L_2\cos kt) + (N_1tg\psi + S)\times\cos(L_1\sin kt + L_2\cos kt)) - \\ -P\sin(L_1\sin kt + L_2\cos kt))) - M_D r(\omega + L_1k\cos kt - L_2k\sin kt)^2\times \\ \times\cos(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + \cos(L_1\sin kt + L_2\cos kt)(C_1d + C_2f) + \\ \times\sin(L_1\sin kt + L_2\cos kt)\times(C_1b - C_2q) - d(C_1 + C_2) + (M_{K_1} + M_{K_2} + M_{K_3} + M_Д)g \end{array}\right] \quad (4)$$

$$\ddot{\phi} + k1^2\phi = 0 .$$

The next step is the construction of the solution of this system of equations to obtain analytical solutions - the movement model of the generalized vibration separator.

Using the Poincaré method, the solution of a perturbed system of nonlinear differential equations, which describes the motion (small oscillations) of a given mechanical system (the vibrating separator operates in a stationary mode), can be represented in the form of a series of powers $\varepsilon$, which are convergent if sufficiently small $\varepsilon$):

$$x_{0_6}(t) = x_0\sin\left(\sqrt{\frac{C}{M}}t + \alpha_x\right) + \varepsilon\chi_x\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \varepsilon^2\chi_x'\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \ldots,$$

$$y_{0_б}(t) = y_0 \sin(\sqrt{\frac{C}{M}}t + \alpha_y) + \varepsilon\chi_y\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \varepsilon^2\chi'_y\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + ...,$$

where $\varepsilon = \dfrac{1}{M} << 0$ - the condition of smallness is fulfilled;

$\chi_i^j\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right)$ – some functions that are determined according to the Poincaré method:

Analyzing the system according to the weight of the parameters $\varepsilon^i = \left(\dfrac{1}{M}\right)^i$, where $i \in [0;\infty)$, we limit ourselves in constructing the solution to the first two members of the series - the first represents the solution of an undisturbed system, the second – the effect of an external perturbation.

Therefore, the solutions of the perturbed system (3) (the system of equations defining the generalized coordinates) in the first approximation (taking into account the first two terms of the series) in the general case took the form (5):

$$x_{0_б}(t) = x_0 \sin(\sqrt{\frac{c}{M}}t + \alpha_x) + \varepsilon\int_0^t f_x(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)\sin\left(\sqrt{\frac{c}{M}}(t-u)\right)du,$$
$$y_{0_б}(t) = y_0 \sin(\sqrt{\frac{c}{M}}t + \alpha_y) + \varepsilon\int_0^t f_y(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)\sin\left(\sqrt{\frac{c}{M}}(t-u)\right)du. \quad (5)$$

where $f_x(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)$ and $f_y(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)$ – functions of the right-hand side of equations in generalized coordinates $x_{0_б}$ and $y_{0_б}$ system of equations of the data above.

Therefore, the solution of the system in the first approximation will have the form (6):

$$\begin{aligned}
x_{0_б}(t) &= x_0 \sin\left(\sqrt{\frac{C}{M}}t + \alpha_x\right) + \\
&+\varepsilon\int_0^t \left[\begin{array}{l}
k^2(L_1\sin kt + L_2\cos kt)(M_D r\cos(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + \\
+M_{K_1}\left[(F_1 tg\tau + S)\cos(L_1\sin kt + L_2\cos kt) - F_1\sin(L_1\sin kt + L_2\cos kt)\right] - \\
-M_{K_2}W\cos(L_1\sin kt + L_2\cos kt) + \\
+M_{K_3}\left[N_1\sin(L_1\sin kt + L_2\cos kt) + (N_1 tg\psi + S)\cos(L_1\sin kt + L_2\cos kt)\right]\Big] - \\
-(L_1 k\cos kt - L_2 k\sin kt)^2 \times \\
\times\left[\begin{array}{l}
M_{K_1}(F_1\cos(L_1\sin kt + L_2\cos kt) + (F_1 tg\tau + S)\sin(L_1\sin kt + L_2\cos kt) + \\
+M_{K_2}W\sin(L_1\sin kt + L_2\cos kt) + M_{K_3}(N_1\cos(L_1\sin kt + L_2\cos kt) - \\
-(N_1 tg\psi + S)\sin(L_1\sin kt + L_2\cos kt)) + (Ptg\gamma + S)\sin(L_1\sin kt + L_2\cos kt))
\end{array}\right] + \\
+M_D r(\omega + L_1 k\cos kt - L_2 k\sin kt)(-C_2 q + C_1 b) - \sin(L_1\sin kt + L_2\cos kt)\times \\
\times(C_1 d - C_2 f) + C_2 q - C_1 b
\end{array}\right]\times \\
&\times\sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du,
\end{aligned} \quad (6)$$

$$y_{0_б}(t) = y_0 \sin\left(\sqrt{\frac{C}{M}}t + \alpha_y\right) +$$

$$+\varepsilon\int_0^t \left[\begin{array}{l} k^2(L_1 \sin kt + L_2 \cos kt)(M_D(r\sin(\omega t + L_1 \sin kt + L_2 \cos kt + \alpha_0) + \\ +M_{K_1}(F_1 \cos(L_1 \sin kt + L_2 \cos kt) + \\ +(F_1 tg\tau + S)\sin(L_1 \sin kt + L_2 \cos kt) - M_{K_2} W \sin(L_1 \sin kt + L_2 \cos kt) + \\ +M_{K_3}((N_1 tg\psi + S)\sin(L_1 \sin kt + L_2 \cos kt) - N_1 \cos(L_1 \sin kt + L_2 \cos kt)) - \\ -K\cos(L_1 \sin kt + L_2 \cos kt))) - (L_1 k \cos kt - L_2 k \sin kt)^2 \times \\ \times(M_{K_1}((F_1 tg\tau + S)\cos(L_1 \sin kt + L_2 \cos kt) - F_1 \sin(L_1 \sin kt + L_2 \cos kt)) - \\ -M_{K_2} W \cos(L_1 \sin kt + L_2 \cos kt) + M_{K_3}(N_1 \sin(L_1 \sin kt + L_2 \cos kt) + \\ +(N_1 tg\psi + S)\cos(L_1 \sin kt + L_2 \cos kt)) - P\sin(L_1 \sin kt + L_2 \cos kt))) - \\ -M_D r(\omega + L_1 k \cos kt - L_2 k \sin kt)^2 \cos(\omega t + L_1 \sin kt + \\ +L_2 \cos kt + \alpha_0) + \cos(L_1 \sin kt + L_2 \cos kt)(C_1 d + C_2 f) + \sin(L_1 \sin kt + L_2 \cos kt)(C_1 b - C_2 q) - \\ -d(C_1 + C_2) + (M_{K_1} + M_{K_2} + M_{K_3} + M_{K_4} + M_Д)g \end{array}\right] \times$$

$$\times \sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du,$$

$$\phi(t) = L_1 \sin kt + L_2 \cos kt.$$

The obtained system of equations describes the movement of the vibrating separator during an arbitrary time interval of its operation. The connection between the coordinates of the points of the containers relative to the two reference systems – fixed and moving (connection with the center of rotation of the imbalance) has the form:

$$x_{i_б} = x_{0_б} + x_i \cos\phi(t) - y_i \sin\phi(t),$$

$$y_{i_б} = y_{0_б} + x_i \sin\phi(t) + y_i \cos\phi(t).$$

Using the last ratio, it is possible to determine the horizontal and vertical component of the vibration amplitude of any point of the container of the vibrating separator in the plane of its movement - in the plane of unbalanced oscillations at an arbitrary moment in time or during the period of operation of the vibrating separator during the separation of bulk materials. It is also possible to construct the trajectory of the movement of a given point during the investigated time interval of processing, to find the influence of all kinematic and geometric parameters of the vibration separator on the amplitude of oscillations of any point of the working containers, to investigate their importance in terms of influence on the magnitude of the amplitude and the nature of the oscillations of the containers, and also, at arbitrary their variations among themselves, over the entire interval of the possible change of these parameters.

The obtained dependencies are solutions of the system of differential equations - the investigated vibration separator. The built model is parameterized and contains all the necessary geometric and kinematic parameters of the vibrating separator.

## Conclusions

A unified parameterized model of a vibrating separator with an unbalanced drive and a spring suspension has been developed, which can be used (by changing its parameters or setting them to zero) for a wide range of designs of vibrating separators with three working containers, as well as for separators with two or one working containers, with by their various arrangement relative to each other and the frame or suspension of the separator. The built model of the vibrating separator covers all the dynamic phenomena that occur in it and allows to study of the influence of its basic parameters on the amplitude of vibrations of the containers of the vibrating separator (parameters that can be changed during the operation of the separator), to obtain graphical dependences of the amplitude of vibrations of the separator containers on its parameters and trajectory movements of arbitrary points of the containers in the plane of their oscillations. These dependencies can be used to select the required operating modes of the separator.

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