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ANALYSIS OF THE INFLUENCE OF THE DISPLACEMENT OF THE GEAR RACK PROFILE OF THE PAIR EVOLUTION GEARING ON THE QUALITY INDICATORS OF THE TRANSMISSION

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Abstract. The article is devoted to the study of the influence of the modification of the paired evolute gearing with convex-concave contact on the quality indicators of transmission, namely, the contact pressure and the speed of slippage in the engagement. Previous studies of evolute gears showed the presence of characteristic zones on the side surface of the tooth, which have worse values of quality indicators than in similar involute gear. One of the ways to remove these zones or to reduce their impact on the load capacity of the transmission is to modify the tooth profile, which will be implemented by shifting the profile of the processing gear rack. A displacement factor is added to the equation of the rail profile curve, which has the same value for the gear and wheel teeth, but the opposite sign. On the basis of the developed equation, the side profiles of the teeth of the gear and the wheel were constructed. Several values of the rack displacement coefficient with a positive and negative sign for the gear teeth are considered. The results of the research allow us to evaluate the influence of the amount and direction of the rail displacement on the curvature of the tooth profiles, the contact pressure in engagement along the height of the tooth, and the relative and absolute speed of sliding. The modifications of the profile of the processing toothed rack proposed in this work will expand the existence of paired evolute gears with a different set of quality indicators. This will make it possible to design gears with rational values of contact pressures and slip speeds when they are used in heavily loaded transmissions created for specific operating conditions. Gears with relatively high slip speeds can be used in road and construction equipment transmissions, which are characterized by low shaft rotation frequency, in which the speed of slip between the teeth does not have a significant effect on the life or efficiency of the transmission. Otherwise, gears with relatively low slip speeds are very relevant when they are implemented in the transmission of modern and promising vehicles with a hybrid or fully electric power unit, especially in the case of passenger cars with high-speed electric motors.

Keywords: gearing, rack displacement, convex-concave contact, paired evolute gearing, profile curvature, contact pressure.

Introduction

When designing modern transmissions, it is necessary to achieve rational indicators of such parameters as its load capacity, weight, dimensions, service life and price. One of the ways to increase the load capacity of the transmission without increasing the dimensions and weight is the use of gears with convex-concave contact [1]. Existing Novikov gears, which have become the most widespread among gears with convex-concave contact in mechanical engineering, have certain design limitations [2, 3]. The most significant is the feature of the engagement geometry, which requires gear wheels to be made only with helical teeth [4].

In recent decades, already known gears with convex-concave contact have gained further development and new ones have appeared. The latter include transmissions developed by M. Vereš, M. Bošanský, J. Gaduš [5] and S. Radzevich [1]. It should be noted that gears with convex-concave contact, especially those created recently, have certain disadvantages associated with some quality indicators. A detailed analysis of such defects can only be realized with the help of a large volume of research. One of the methods of elimination is the correction of the profile of the tooth, which is used in involute gears, namely, the movement of the toothed tool rail in the direction from the center of the axis of the gear wheel or in the direction of the center.

Proposed by the Ukrainian scientist A. I. Pavlov's new engagement, which he called evolute, can be straight or oblique. Due to the implementation of a convex-concave contact in the evolute gearing, such gears have lower contact pressure compared to standard involute gears and are promising for their use in heavily loaded transmissions.

In the works [3, 4, 6–8] it is shown that the contact crushing of the tooth occurs in the zone of the engagement pole and begins below the pitch diameter of the gear tooth. This is due to the joint action of contact pressures and the sliding speed of one profile relative to the other. Also, the sliding speed is the most important parameter that affects the wear resistance of the transmission and the wear of the contact surfaces of the teeth. Therefore, the study of the speed of sliding in evolute engagement is an important scientific and practical task.

Evolution gears refer to parts with a complex profile of the working surface. As mentioned above, the manufacturing processability and cost have a great influence on the speed of wide introduction into the production of promising gears with increased load capacity. But in recent times, many improved methods of processing parts with a complex profile and with high geometric accuracy have been developed. Some of them are described in works [9–11].

It is also possible to note very promising methods of increasing the contact strength of the surfaces of parts that work under high pressure, in the form of applying a thin hard coating. Such technologies are successfully used, including in gear transmissions [12, 13].

Problem Statement

The most frequent types of damage to gear teeth are tooth breakage, peeling and chipping of the material on the working surfaces of the teeth, abrasive wear, and scratching of the contact surfaces resulting from the rupture of the oil film. The last three types of damage depending on the contact pressures between the working surfaces of the teeth and the speeds of mutual sliding.

Previous studies of evolute gears showed the presence of a biconvex contact in the engagement pole and higher sliding speeds in the area of the leg and head of the tooth compared to the indicators in the indicated areas in similar involute gear. Applying the modification of the profile of the involute tooth, namely the displacement of the profile of the processing rack, qualitative indicators of the involute transmission will be obtained, which will be compared with the indicators without modification.

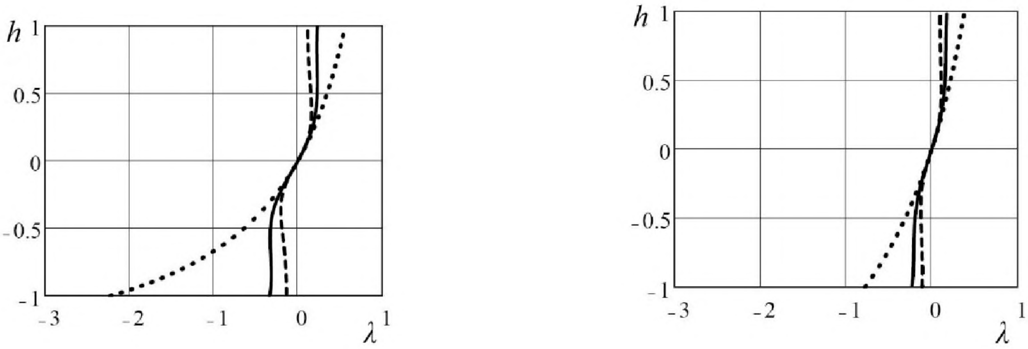
In Fig. 1, *a* shows the general view of the evolute gear, and Fig. 1, *b* shows the gear and wheel engagement zone.

The conducted studies showed that in evolute transmissions, the index of slip speed is slightly worse compared to standard involute transmission [14].

Also, in some types of evolute gears, there is a biconvex contact in the near-pole zone (Fig. 3), which puts spur gears on the same level as standard involute gears.

a *b*

Fig. 1. Evolute gear:
a – general view; b – gear and wheel engagement zone



a *b*

Fig. 2. Specific sliding speed:
a – $\alpha = 15^{\circ}$; b – $\alpha = 20^{\circ}$; - - - – $k = 2$, —— – $k = 5$, ········ – involute

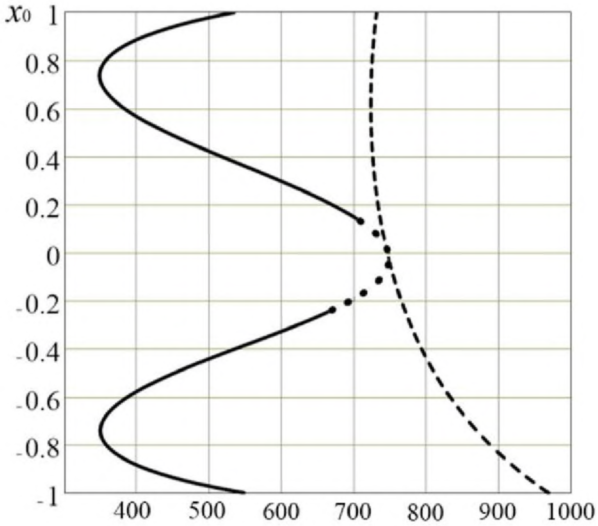


Fig. 3. Change in contact stresses σ_H, MPa, along the height of the tooth:
—— – evolute gearing; ········ – zone of biconvex contact in evolute gearing; ---- – involute gearing

Review of Modern Information Sources on the Subject of the Paper

The experience of the synthesis of standard evolute gears shows that the implementation of rational indicators, including contact pressure and sliding speed, is achieved by the use of teeth adjustment (displacement of the instrumental toothed rack). From this it can be concluded that the specified indicators can be improved by introducing the adjustment of the teeth of the evolute gearing. Due to the presence of a convex-concave contact, it is possible to synthesize evolute transmissions with a larger range of values of parameters of qualitative indicators, namely, to implement transmissions with a low sliding speed due to some deterioration of contact pressure values. But at the same time, the load capacity of the evolute transmission will still be higher than the similar involute due to the convex-concave contact. Also, it can partially eliminate one of the disadvantages of the involute engagement, namely, reduce the height of the biconvex contact zone and its effect on the load capacity of the transmission.

Several notable works [2–4] are devoted to the correction of the standard involute transmission. The result of these studies is the creation of areas of existence of gear transmissions and graphs of the dependence of correction parameters on transmission load parameters. Also, from the point of view of gear wheel manufacturing technology, this technique is very simple, so the analysis of its use for involute engagement is very relevant.

Objectives and Problems of Research

The purpose of the study is to analyze the influence of the movement of the tool gear rack on the quality indicators of the evolute gear transmission, such as the contact pressure along the height of the tooth and the sliding speed. The research will be conducted on the basis of known analytical formulas and dependencies, which are used in the synthesis of both evolute engagement and standard involute engagement and analysis of its quality indicators.

The obtained results in the form of graphs will make it possible to analyze the nature of the change in the qualitative indicators of the evolutionary transmission from the modification of the initial profile and to preliminarily determine the range of rational values of the positive and negative movement of the tool rail.

Main Material Presentation

The equation of the tool rail profile of the involute engagement was obtained on the basis of Bobillier's construction in work [1] and has the form of a 2nd-order differential equation:

$$y'' = \frac{y'(1 + y'^2)}{-ky' + x}, \tag{1}$$

where k is the coefficient of the type of transmission, $k = h \sin\alpha_0$ (h is the distance between the engagement pole and the center of rotation of the connecting rod of the replacement mechanism, α_0 is the angle of engagement in the pitch point).

The solution of the differential equation is obtained as an n-degree polynomial [1, 15]. In turn, by substituting the equation of the parametric curve into the polynomial, you can get the profile of the instrument rail:

$$\left.\begin{aligned} x_1 &= x_0 \cdot m; \\ y_1 &= ((C_1 x_0 + C_2 x_0^2 + C_2 x_0^3 + \ldots + C_n x_0^n) + \pi/4) \cdot m, \end{aligned}\right\}, \tag{2}$$

where x_0 is the coefficient of the height of the head and foot of the tooth, in this work it takes the value from -1 to $+1$, m is the transmission module, $\pi/4$ is the displacement of the profile of the instrument rail relative to the axis of symmetry of the tooth.

To construct the profile of the gear tooth, a system of transition equations from the profile of the rail, which is engaged with the wheel being processed, was used [4, 5]:

$$\left.\begin{aligned} x_2 &= (x_1 - a)\cos\phi_2 + (y_1 + r_2)\sin\phi_2; \\ y_2 &= -(x_1 - a)\sin\phi_2 + (y_1 + r_2)\cos\phi_2. \end{aligned}\right\}, \tag{3}$$

where r_2 is the radius of the dividing circle of the gear, and is the amount of translational movement of the tool rail, φ is the angle of rotation of the gear being processed.

The tooth profile of the coupled wheel was obtained in a similar way, but taking into account the gear ratio, which is expressed in the radius of the dividing circle of the wheel r_3:

$$\left.\begin{aligned} x_3 &= (x_1 - a)\cos\phi_3 + (y_1 + r_3)\sin\phi_3\,; \\ y_3 &= -(x_1 - a)\sin\phi_3 + (y_1 + r_3)\cos\phi_3\,. \end{aligned}\right\} \tag{4}$$

Adding the displacement factor χ to the tool rail equation will further use two separate calculations to obtain the positive and negative displacement on the gear tooth and the wheel respectively:

$$\left.\begin{aligned} x_1 &= x_0 \cdot m \pm \chi \cdot m; \\ y_1 &= ((C_1 x_0 + C_2 x_0^2 + C_2 x_0^3 + \ldots + C_n x_0^n) + \pi/4) \cdot m. \end{aligned}\right\}, \tag{5}$$

where χ is the displacement coefficient of the instrument rail.

At the same time, if the "+" sign is used for the gear, then "-" is used for the wheel and vice versa.

The work used a pair of gear wheels with the following parameters: number of gear teeth $z_1 = 21$, number of wheel teeth $z_2 = 51$, transmission module $m = 2$, correction factor χ: -0.4, -0.2, +0.2, +0 ,4.

Conjugated profiles in relative motion roll one after the other with slip at the point of contact, and the sliding speed:

$$V_y = |V_{Fy1}| = |V_{Fy2}|, \tag{6}$$

is equal to the difference of the tangential components of the velocities of the points that are in contact with each other:

$$V_y = V_{Fy1} = V_{Fy2}, \tag{7}$$

The absolute speed of sliding of the profiles that are caught is equal to:

$$V_y = PY(\omega_1 + \omega_1). \tag{8}$$

In order to give a qualitative assessment that characterizes the degree of mutual sliding of the profiles, it is necessary to introduce an objective indicator (dimensionless value). This indicator is the sliding coefficient λ – the ratio of the absolute sliding speed of the point of the profile, which is currently in contact, to the tangential component of this speed.

We write the expressions for the slip coefficient in the following form:

- for pinion $\lambda = V_y / V_{Fy1}$,
- for gear $\lambda = V_y / (V_{Fy1} \cdot u)$

the tangential component of the sliding speed:

$$V_{Fy} = \omega_1 r_y sin\alpha_y, \tag{9}$$

Contact stresses are determined by the following formula:

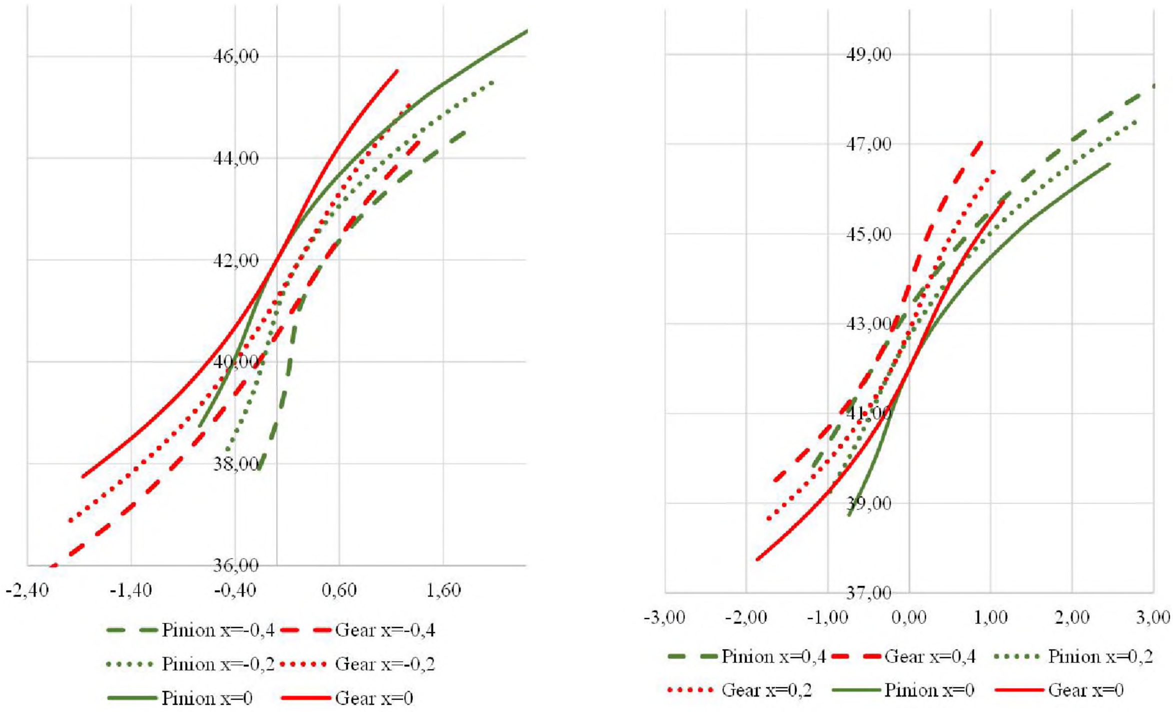
$$\sigma_H = 0{,}418\sqrt{\frac{F_n E_{\text{пр}}}{b_w \rho_{\text{пр}}}}, \tag{10}$$

where ρ_{pr} is the reduced radius of curvature in the contact of two surfaces, which is the main geometric factor that affects the number of contact stresses (here the sign is "+" for biconvex contact, and "-" for convex-concave contact); E_{pr} – the reduced modulus of elasticity of the material; b_w – working tooth width; F_n – the normal force in engagement.

When calculating the contact pressure, the torque was set at 530 Nm, and the width of the gears was 30 mm.

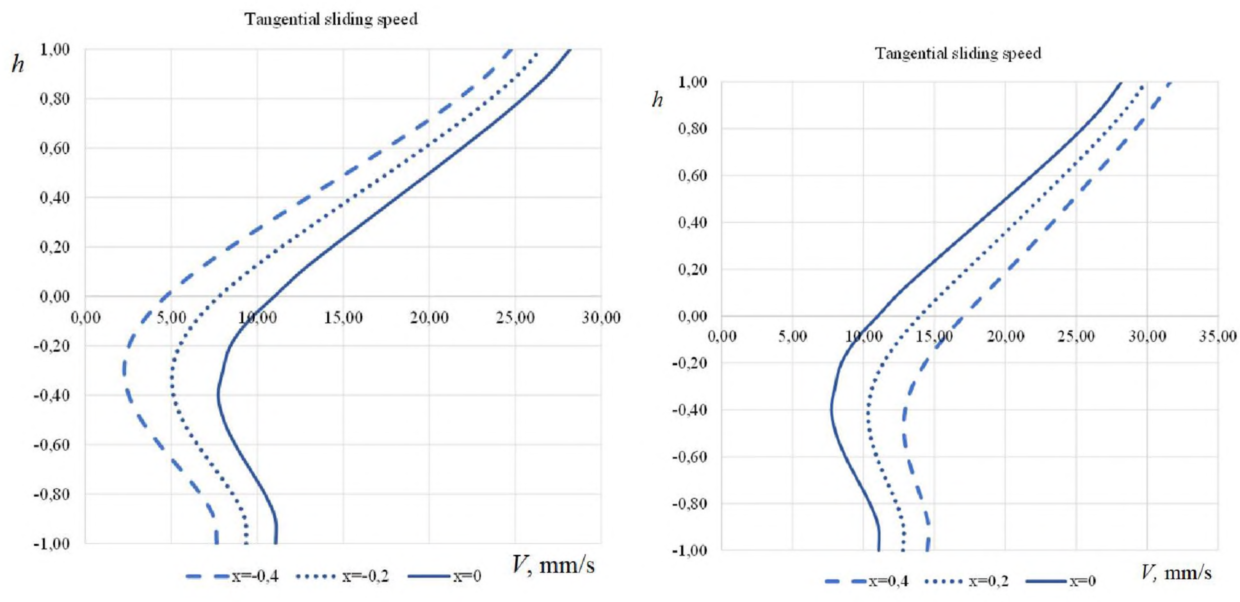
Fig. 4 shows the profiles of gears and wheels with involute engagement, and Fig. 4, *a* shows profiles with a negative displacement, and Fig. 4, *b* profiles with a positive displacement.

Fig. 5 show tangential sliding speeds. It can be seen from the Fig. 5, *a* that with a negative displacement of the rail, the amount of sliding decreases. But when moving the rail in the positive direction, the tangential sliding speed increases.



a *b*

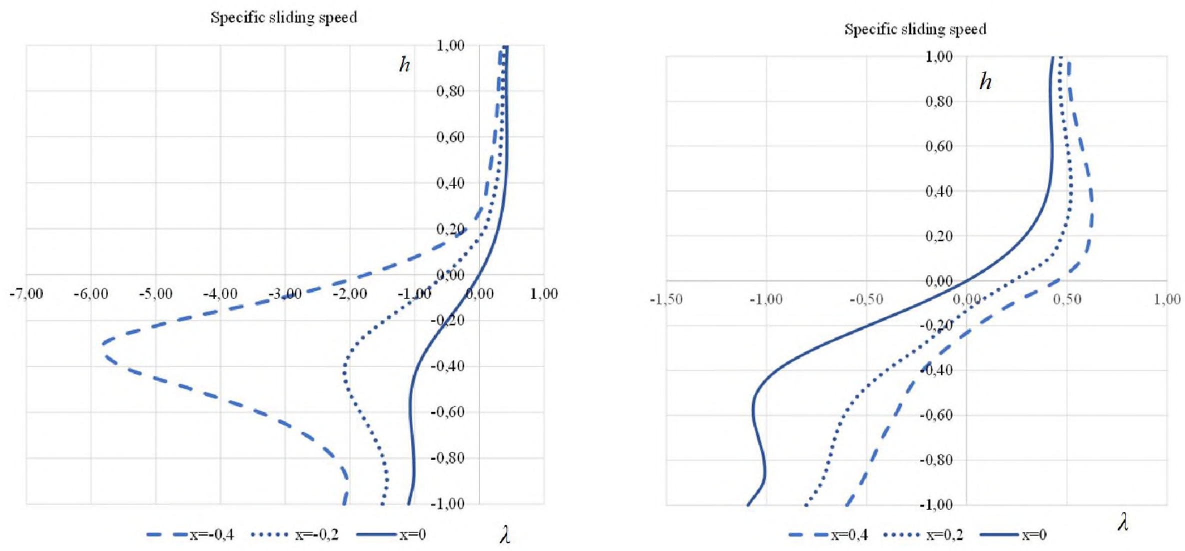
Fig. 4. Conjugated profiles of the teeth of the gear and the wheel of the involute transmission:
a – positive bias; b – negative bias



a *b*

Fig. 5. Tangential sliding speed:
a – positive bias; b – negative bias

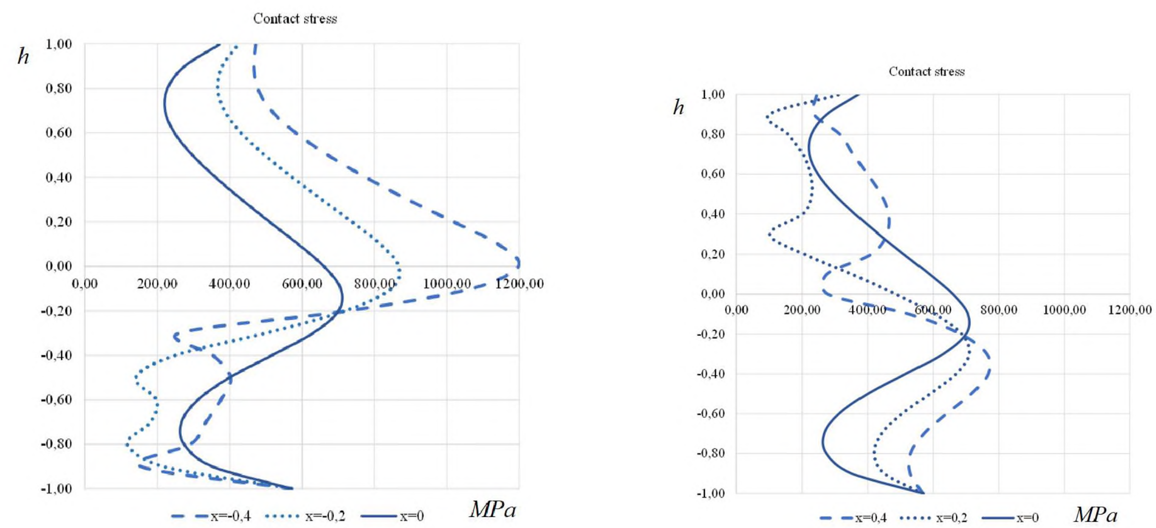
Fig. 6 shows the specific sliding speed of the evolutionary transmission, which takes into account normal and tangential speeds. From the analysis of the graphs given in example 6, and it can be concluded that with a correction greater than "-0.2" the specific sliding speed increases significantly, so the use of such values should be justified by other parameters of quality indicators. In turn, the positive correction has almost no effect on the specific sliding speed at values up to at least 0.4.



a *b*

Fig. 6. Specific sliding speed:
a – positive bias; b – negative bias

Fig. 7 shows the contact pressure along the tooth height of the involute gear, taking into account that only one pair of teeth transmits the torque. It can be seen from the Fig. 7, *a* that the negative correction at the value of -0.4 almost doubles the contact pressure in the area of the engagement pole. Analyzing the sample 7, *b* we can conclude that the positive correction, although it increases the contact pressure in the zone below the pole, but in general the growth of this parameter is not significant.



a *b*

Fig. 7. Contact pressure along the height of the tooth:
a – positive bias; b – negative bias

Conclusions

Analyzing the obtained results, it is possible to draw conclusions about the influence of the displacement of the instrumental evolution rail on the quality and loading parameters of the transmission:

– the absolute sliding speed in the negative and positive direction changes by 25 % percent with each movement by 0.2;

– the specific speed of sliding in the negative direction of movement increases by two times at the value of 0.2 and almost six times at the value of the absolute movement of 0.4;

– the specific speed of sliding in the positive direction of movement is reduced by two times at a value of 0.2 and by 50 % at a value of 0.4;

– the contact pressure in engagement with the negative direction of movement in the pole zone increases by approximately 17 %, at "-0.2" and at "-0.4" it increases almost twice. In the area of the tooth head, the contact pressure is 12–17 % higher than in the transmission without correction for each absolute displacement value of 0.2.

– the contact pressure in the positive direction of movement in the pole zone almost does not change. But in the area of the tooth leg, there is an increase of this parameter by 25–30 % for each absolute displacement value of 0.2.

On the basis of the conclusions, it can be stated that the introduction of a negative correction during the manufacture of an evolutionary transmission will significantly worsen its quality indicators, especially the contact certainty. Although the positive correction has slightly worse slip speed indicators, at the same time the contact pressure in the pole zone increases by only 12–17 %, so further research in this direction is promising.

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