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Thermally stressed state of the layer under the influence of currents periodic with respect to the longitudinal coordinate

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To evaluate the impact of the important non-uniformity technological factor of the external electric current distribution on the low-temperature heating parameters of flat ferrite elements, taking into account the stressed state (strength characteristics), a model problem is considered to determine and study the thermomechanical behavior of the ferrite layer under the influence of a quasi-steady electromagnetic field created by the current flowing in the current-carrying plane, the density of which changes sinusoidally along the longitudinal coordinate. In accordance with the earlier results regarding the dependence of the heating process on the frequency of external electromagnetic influence, it is accepted that the circular carrier frequency of the electromagnetic field is outside the vicinity of the resonant ones (when the thermally stressed state has an almost quasi-static character). In this case, the calculation scheme consists of three stages for sequentially determining the parameters that describe the electromagnetic, temperature, and mechanical fields.

Keywords: *thermally stressed state; ferrite solid; quasi-steady electromagnetic field; complex amplitude method; heat generation; ponderomotive force.*

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1. Introduction

Ferrite materials have a wide range of applications in functional devices across various fields such as radio engineering, electronics, magnetic recording technology, automation, television, computing, high-frequency (HF), ultra-high-frequency (microwave) and pulse technology. They are primarily used for creating magnetic links and in the manufacture of circuit coils cores with constant and variable inductance, filters in radio and wire communication equipment, cores of pulse and broadband as well as flyback transformers and deflection systems of electrovacuum devices, magnetic modulators and amplifiers, rod antennas, inductive delay lines and other components of electronic equipment.

A common heat treating method of complex-shaped parts or structural elements in many industries is the non-contact heating of electrically conductive materials by electric currents induced by an alternating magnetic field of an inductor. Various issues of the industrial application of induction heating, as well as the calculation and design of induction units, are quite fully described in [1–3].

The specificity of ferrite products manufacturing technologies is often associated with the need to use heat treatment. In a wide enough temperature range (practically up to Curie temperatures) the electromagnetic method of heat treatment (low-temperature processing) of ferrite products is effective. The heating process occurs similarly to the induction heat treatment of electrically conductive products. However, it has a specificity due to the material's ability to remagnetise and repolarise with heat generation in a time-periodic electromagnetic field (EMF). This occurs at low Joule heat production due to low electrical conductivity. For this heat treatment, it is necessary to reach a solid temperature of at least 150°C. In order to develop effective methods for using EMF to construct rational modes for the mentioned technological heating (in particular, at training and low-temperature annealing of ferrite products to remove structural, assembly or other residual stresses, etc.), a comprehensive study of thermally stressed state in specific ferrite elements is necessary. The study should be based on developed thermomechanics models of ferrite solids under the influence of quasi-steady EMF (QSEMF), while considering the mentioned conditions of heat treatment.

The impact of EMF on the material environment results in a range of interrelated physical processes. The study of their interrelation enables to describe more accurately and completely the behavior of material and structural elements under electromagnetic, thermal, and mechanical loads. This allows to discover a number of useful for practice regularities in ongoing physical and mechanical processes, and to evaluate the applicability limits of theories that neglect the coupling of processes. On the other hand, the interconnection of varying physical processes in deformable media can negatively impact the functional properties of structural elements in devices, resulting in a significant reduction in material functionality and lifespan.

The influence of EMF on the solid is typically accompanied by the appearance of a temperature field, deformations, and stresses in it. These factors significantly affect both the parameters of corresponding technological processes in products and the conditions of their operation. Therefore, to adequately describe physical and mechanical processes in deformable solids under EMF action, it is necessary to consider the interrelation of electromagnetic, thermal, and mechanical processes within appropriate models.

In [4–9], a mathematical model for a quantitative description and a methodology for investigating the thermally stressed state of ferrite solids, caused by the influence of low external QSEMF of high carrier frequency, were constructed. They are based on the general theory of interaction between EMF and the material continuum (which can be simultaneously magnetised and polarised in certain parts of the frequency spectrum of the QSEMF), in which the influence of electromagnetic radiation is taken into account by heat dissipation (associated with both remagnetisation and repolarization, as well as Joule heat) and ponderomotive forces (when describing the force factors of the radiation influence on the considered solid using physical statistics), and also on the experimentally determined characteristics known from the literature for such materials – complex magnetic and dielectric permeabilities as well as the corresponding tangents of loss angles. Solutions have been obtained for new problems concerning the determination of electromagnetic, temperature and mechanical fields in a layer made from ferrite material under the influence of EMF caused by a given quasi-steady electric current uniformly distributed in the current-carrying plane parallel to the layer base. New regularities in the mechanical behavior of ferrite solids under the influence of external electromagnetic loads, in particular related to resonance phenomena, have been identified.

Below, in order to evaluate the non-uniformity influence of electric current distribution on the process heating parameters of flat ferrite elements, taking into account the stressed state (strength characteristics), the thermomechanical behavior of the ferrite layer under the influence of external QSEMF, created by the current flowing in the current-carrying plane, the density of which has a sinusoidal variation along the coordinate y, is determined and investigated.

2. Initial subproblems. Solution methodology

Consider an infinite elastic layer of thickness h (see Figure 1) made of ferrite material. The layer is under the influence of an EMF created in the external environment $z_* > h$ by a system of quasi-steady electric currents of the radio and microwave ranges with amplitude modulation. These currents are periodically distributed along the coordinate y_* in a thin layer located at a distance $l - h$ from the considered ferrite layer surface.

The layer with a spatial distribution of currents (inducing EMF in the studied ferrite layer), as well as in [4–9], is modelled by an infinitely thin current-carrying plane ($z_* = l$), parallel to the ferrite layer surface, with a current density

$$\mathbf{J}^{(0)}_{**} = \left\{ J_x(y_*, z_*, t) = \mathrm{Re}\left[J^{0*}(y_*, z_*, t) \right]; 0; 0 \right\}. \tag{1}$$

Here: $J^{0*}(y_*, z_*, t) = J^{(0)}_* e^{i\omega t}$ is complex electric current density, $J^{(0)}_* = j_0 J_0(t) e^{-i\alpha_* y_*} \delta(z_* - l)$, j_0 is amplitude of the carrier signal currents, $J_0(t)$ is a function ($0 \leqslant J_0(t) \leqslant 1$) that varies slowly over the period $f_* = 2\pi/\omega$ of electromagnetic oscillations (satisfies the quasi-steady approximation condition [4–9, 12, 13]), α_* is frequency of current density sinusoidal variation with respect to the coordinate, $\delta(z)$ is Dirac delta function [10, 11].

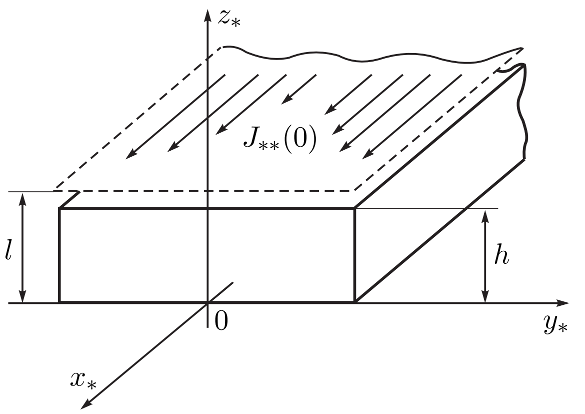

Fig. 1. Geometric scheme.

The objective of this investigation is to determine the influence of distribution periodicity along the coordinate of external electric current on the thermally stressed state of the ferrite layer caused by the impact of an EMF induced by such a current. The conditions for fixing and heating the layer are: the lower base of the layer is rigidly coupled at $z_* = 0$ (it means that displacements $u_z = 0$) with the dielectric half-space (the electrophysical characteristics of which are considered in the vacuum approximation); the upper base $z_* = h$ is not subjected to any mechanical loads. It experiences convective heat exchange with the external environment (which maintains a constant temperature equal to the layer's initial one T_0), and the lower base $z_* = 0$ is thermally insulated.

It is a well-established fact that external influences with a harmonic or quasi-harmonic nature can cause resonance in solids, associated with the natural frequencies of free harmonic oscillations of a dynamic system. The studies conducted on the thermally stressed state of the layer have shown that for ferrite solids (like for electrically conductive non-ferromagnetic non-polarisable, ferromagnetic non-polarisable and non-ferromagnetic solids of low electrical conductivity) there are such frequencies of external EMF at which the levels of temperature fields and stresses significantly increase [5, 9, 12, 13]. There is a mechanical resonance caused by the periodic change in time of heat generations and ponderomotive forces. Each resonant frequency of the EMF is approximately half of the corresponding natural frequency of thermoelastic (thickness) vibrations of the layer. This is true when the coupling parameter of strain and temperature fields is small, resulting in insignificant thermoelastic energy dissipation.

The numerical studies conducted in [8,9] indicate that the considered oscillatory system, corresponding to mechanical resonance, has a high resonant (selective) response, characterized by a quality factor $Q_f \approx 10^9$ of the oscillatory system, and very narrow vicinities of resonant frequencies $\Delta\omega = 10^{-5}\,\mathrm{s}^{-1}$ (angular half-power bandwidth). Furthermore, it is shown in [5–9] that the influence of a dynamic nature of the external impact on the thermally stressed state of the ferrite layer is negligible for all EMF frequencies, except in the vicinities of resonant ones. Therefore, to evaluate the influence and potential of utilising the non-uniformity technological factor of the external electric current distribution during the heat treatment process, we propose an approach that studies the thermally stressed state in a quasi-static formulation.

Thus, to ensure the stability (monotonicity) of the heating and deformation processes from the frequency of electromagnetic influence, we limit ourselves to frequencies located outside the vicinities of resonant ones (at which the thermally stressed state can be considered quasi-static). In this case, a solution to the original unrelated problem of electromagneto-thermomechanics is reduced to sequentially determining the parameters that describe the electromagnetic, temperature, and mechanical fields. These parameters are obtained from the corresponding problems in electrodynamics, heat conduction, and temperature stresses theory.

A dimensionality of the considered thermoelasticity problem is determined by the above mentioned loading and fastening conditions, which make it two-dimensional (plane). Therefore, the research and analysis have been conducted for such a case.

The analysis of Maxwell's equations reveals the presence of non-zero quasi-steady components of the complex electric and magnetic field intensities in the layer and external medium regions:

$$E_x^*(y_*, z_*, t), \quad H_y^*(y_*, z_*, t), \quad H_z^*(y_*, z_*, t) \quad (0 \leqslant z_* \leqslant h);$$
$$E_x^{*1}(y_*, z_*, t), \quad H_y^{*1}(y_*, z_*, t), \quad H_z^{*1}(y_*, z_*, t) \quad (z_* > h);$$
$$E_x^{*2}(y_*, z_*, t), \quad H_y^{*2}(y_*, z_*, t), \quad H_z^{*2}(y_*, z_*, t) \quad (z_* < 0).$$

This paper considers the case of low EMFs at high frequencies, which is important for engineering applications. The scientific and technical literature for this case provides data on the values of magnetic and dielectric permeabilities, as well as the corresponding tangents of loss angles, and their dependences on frequency for various types of ferrite materials. An approximate methodology for investigating the physical and mechanical fields present under such electromagnetic influences is proposed. The shape ellipticity of hysteresis loops in low EMFs of high frequency makes it possible for such fields, using the method of complex amplitudes (when describing EMF parameters in the first harmonic approximation), to linearise the initial problem of electrodynamics by introducing approximate complex representations of the electric and magnetic field intensity vectors [4–9, 12, 13]:

$$\mathbf{E}_{**} = \operatorname{Re} \mathbf{E}^*, \quad \mathbf{H}_{**} = \operatorname{Re} \mathbf{H}^*;$$
$$\mathbf{E}_{**}^{(0)} = \operatorname{Re} \mathbf{E}^{*(0)}, \quad \mathbf{H}_{**}^{(0)} = \operatorname{Re} \mathbf{H}^{*(0)},$$

where

$$\mathbf{E}^* = \mathbf{E}_* e^{i\omega t} = J_0(t)\mathbf{E}(\mathbf{r})e^{i\omega t},$$
$$\mathbf{H}^* = \mathbf{H}_* e^{i\omega t} = J_0(t)\mathbf{H}(\mathbf{r})e^{i\omega t};$$
$$\mathbf{E}^{*(0)} = \mathbf{E}_*^{(0)} e^{i\omega t} = J_0(t)\mathbf{E}^{(0)}(\mathbf{r})e^{i\omega t},$$
$$\mathbf{H}^{*(0)} = \mathbf{H}_*^{(0)} e^{i\omega t} = J_0(t)\mathbf{H}^{(0)}(\mathbf{r})e^{i\omega t};$$

$\mathbf{E}_*(\mathbf{r}, t)$, $\mathbf{H}_*(\mathbf{r}, t)$; $\mathbf{E}_*^{(0)}(\mathbf{r}, t)$, $\mathbf{H}_*^{(0)}(\mathbf{r}, t)$ are modulated complex amplitudes satisfying the conditions defining a quasi-steady character of the field [4–9, 12, 13]:

$$\left|\frac{\partial \mathbf{E}_*}{\partial t}\right| \ll \omega \left|\mathbf{E}_*\right|, \quad \left|\frac{\partial \mathbf{H}_*}{\partial t}\right| \ll \omega \left|\mathbf{H}_*\right|;$$
$$\left|\frac{\partial \mathbf{E}_*^{(0)}}{\partial t}\right| \ll \omega \big|\mathbf{E}_*^{(0)}\big|, \quad \left|\frac{\partial \mathbf{H}_*^{(0)}}{\partial t}\right| \ll \omega \big|\mathbf{H}_*^{(0)}\big|.$$

To describe the elliptic dependences between electric $\mathbf{D}_{**}$ and magnetic $\mathbf{B}_{**}$ flux densities and their corresponding field intensities $\mathbf{E}_{**}$, $\mathbf{H}_{**}$, we introduce the field frequency dependent complex permeabilities:

$$\varepsilon'_*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega), \quad \mu'_*(\omega) = \mu'(\omega) - i\mu''(\omega)$$

and the corresponding tangents of the hysteresis loss angles:

$$\tan \delta_{\mathrm{d}} = \frac{\varepsilon''}{\varepsilon'}, \quad \tan \delta_{\mathrm{m}} = \frac{\mu''}{\mu'},$$

found in the literature.

The problem of electrodynamics is formulated with respect to the y-component of the magnetic field intensity vector when decisive functions are $H_y^*(y_*, z_*, t)$ and $H_y^{*j}(y_*, z_*, t)$ $(j = 1, 2)$. Considering a structure of the approximate complex representations of the electric and magnetic field intensity vectors, these functions are expressed as:

$$\begin{aligned} H_y^*(y_*, z_*, t) &= J_0(t)\, H_y(z_*)\, e^{-i(\alpha_* y_* - \omega t)}, \\ H_y^{j*}(y_*, z_*, t) &= J_0(t)\, H_y^{(j)}(z_*)\, e^{-i(\alpha_* y_* - \omega t)}, \quad (j = 1, 2). \end{aligned} \tag{2}$$

Based on Maxwell's equations for complex amplitudes [9], we obtain that in each of the available regions, the values of $H_y(z)$, $H_y^{(1)}(z)$ and $H_y^{(2)}(z)$ satisfy, respectively, the equations:

$$\frac{d^2 H_y}{dz^2} + (k^2 - \alpha^2) H_y = 0, \quad 0 \leqslant z \leqslant 1; \tag{3}$$

$$\frac{d^2 H_y^{(1)}}{dz^2} + (k_1^2 - \alpha^2) H_y^{(1)} = -\frac{d\delta(z-\xi)}{dz}, \quad z > 1; \tag{4}$$

$$\frac{d^2 H_y^{(2)}}{dz^2} + (k_2^2 - \alpha^2) H_y^{(2)} = 0, \quad z < 0, \tag{5}$$

where $k^2 = h^2 k_*^2$, $k_1^2 = k_2^2 = h^2 k_0^2$, $\alpha = \alpha_* h$, $z = z_*/h$, $\xi = l/h$.

The boundary conditions based on the conjugation terms for complex amplitudes at the interface between media and radiation conditions [4–9, 12, 13] will be:

$$\begin{aligned}
&H_y^{(1)}(1) = H_y(1), && \frac{1}{i\omega\varepsilon_0}\frac{k_1^2}{k_1^2-\alpha^2}\frac{dH_y^{(1)}(1)}{dz} = \frac{1}{\sigma_{\mathrm{e}*}}\frac{k^2}{k^2-\alpha^2}\frac{dH_y(1)}{dz};\\
&H_y(0) = H_y^{(2)}(0), && \frac{1}{\sigma_{\mathrm{e}*}}\frac{k^2}{k^2-\alpha^2}\frac{dH_y(0)}{dz} = \frac{1}{i\omega\varepsilon_0}\frac{k_2^2}{k_2^2-\alpha^2}\frac{dH_y^{(2)}(0)}{dz};\\
&\lim_{z\to\infty}\left[\frac{dH_y^{(1)}}{dz} + ik_1 H_y^{(1)}\right] = 0, && \lim_{z\to\infty}\left[-\frac{dH_y^{(2)}}{dz} + ik_2 H_y^{(2)}\right] = 0.
\end{aligned} \tag{6}$$

Here: $\sigma_{\mathrm{e}*} = \sigma_{\mathrm{e}} + \omega\,\varepsilon_0\,\varepsilon'' + i\,\omega\,\varepsilon_0\,\varepsilon'$, σ_{e} is the electrical conductivity coefficient. At known $H_y(z)$, $H_y^{(1)}(z)$, and $H_y^{(2)}(z)$, functions $H_z(z)$, $H_z^{(1)}(z)$, $H_z^{(2)}(z)$, $E_x(z)$, $E_x^{(1)}(z)$, and $E_x^{(2)}(z)$ are determined from the relations:

$$\begin{aligned}
&H_z = -\frac{i\alpha}{k^2-\alpha^2}\frac{dH_y}{dz}, && E_x = -\frac{1}{\sigma_{\mathrm{e}*}h}\frac{k^2}{k^2-\alpha^2}\frac{dH_y}{dz},\\
&H_z^{(1)} = -\frac{i\alpha}{k_1^2-\alpha^2}\left(\frac{dH_y^{(1)}}{dz} + \delta(z-\xi)\right), && H_z^{(2)} = -\frac{i\alpha}{k_2^2-\alpha^2}\frac{dH_y^{(2)}}{dz},\\
&E_x^{(1)} = -\frac{1}{i\omega\varepsilon_0 h}\frac{k_1^2}{k_1^2-\alpha^2}\left(\frac{dH_y^{(1)}}{dz} + \delta(z-\xi)\right), && E_x^{(2)} = -\frac{1}{i\omega\varepsilon_0 h}\frac{k_2^2}{k_2^2-\alpha^2}\frac{dH_y^{(2)}}{dz}.
\end{aligned} \tag{7}$$

We reduce the coupled electrodynamics problem (3)–(6) for three regions by simple transformations [5, 9, 12, 13] to a boundary value problem only for the layer (region $0 \leqslant z \leqslant 1$), described by equation (3) and equivalent to the original boundary conditions:

$$H_y(0) - \frac{1}{\sigma_{\mathrm{e}*}h}\frac{k^2\sqrt{k_2^2-\alpha^2}}{k_2(k^2-\alpha^2)}\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{dH_y(0)}{dz} = 0 \tag{8}$$

at surface $z = 0$ and

$$H_y(1) + \frac{1}{\sigma_{\mathrm{e}*}h}\frac{k^2\sqrt{k_1^2-\alpha^2}}{k_1(k^2-\alpha^2)}\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{dH_y(1)}{dz} = e^{i\sqrt{k_1^2-\alpha^2}(1-\xi)} \tag{9}$$

at $z = 1$.

The solution to equation (3) that satisfies conditions (8) and (9) will be

$$H_y(z) = \sigma_{\mathrm{e}*}k_1^2\sqrt{k^2-\alpha^2}\,\frac{\gamma_+ e^{i\sqrt{k^2-\alpha^2}z} - \gamma_- e^{-i\sqrt{k^2-\alpha^2}z}}{\gamma_+^2 e^{i\sqrt{k^2-\alpha^2}} - \gamma_-^2 e^{-i\sqrt{k^2-\alpha^2}}}\,e^{i\sqrt{k_1^2-\alpha^2}(1-\xi)}, \tag{10}$$

where $\gamma_\pm = \sigma_{\mathrm{e}*}k_1^2\sqrt{k^2-\alpha^2} \pm i\omega\varepsilon_0 k^2\sqrt{k_1^2-\alpha^2}$, $k_1^2 = k_2^2 = h^2k_0^2$. Hence, using relations (7), we obtain such expressions for functions $H_z(z)$ and $E_x(z)$:

$$H_z(z) = \alpha\sigma_{\mathrm{e}*}k_1^2\,\frac{\gamma_+ e^{i\sqrt{k^2-\alpha^2}z} + \gamma_- e^{-i\sqrt{k^2-\alpha^2}z}}{\gamma_+^2 e^{i\sqrt{k^2-\alpha^2}} - \gamma_-^2 e^{-i\sqrt{k^2-\alpha^2}}}\,e^{i\sqrt{k_1^2-\alpha^2}(1-\xi)}, \tag{11}$$

$$E_x(z) = -i\frac{k_1^2k^2}{h}\,\frac{\gamma_+ e^{i\sqrt{k^2-\alpha^2}z} + \gamma_- e^{-i\sqrt{k^2-\alpha^2}z}}{\gamma_+^2 e^{i\sqrt{k^2-\alpha^2}} - \gamma_-^2 e^{-i\sqrt{k^2-\alpha^2}}}\,e^{i\sqrt{k_1^2-\alpha^2}(1-\xi)}. \tag{12}$$

For uniform current distribution (when $\alpha = 0$), expressions (10) and (12) respectively take the following form [5, 9]:

$$H(z) = \sigma_{\mathrm{e}*} h \frac{(\sigma_{\mathrm{e}*} h + ik\varepsilon_0 c)\, e^{ikz} - (\sigma_{\mathrm{e}*} h - ik\varepsilon_0 c)\, e^{-ikz}}{(\sigma_{\mathrm{e}*} h + ik\varepsilon_0 c)^2\, e^{ik} - (\sigma_{\mathrm{e}*} h - ik\varepsilon_0 c)^2\, e^{-ik}} e^{ik_1(1-\xi)},$$

$$E(z) = -ik \frac{(\sigma_{\mathrm{e}*} h + ik\varepsilon_0 c)\, e^{ikz} + (\sigma_{\mathrm{e}*} h - ik\varepsilon_0 c)\, e^{-ikz}}{(\sigma_{\mathrm{e}*} h + ik\varepsilon_0 c)^2\, e^{ik} - (\sigma_{\mathrm{e}*} h - ik\varepsilon_0 c)^2\, e^{-ik}} e^{ik_1(1-\xi)},$$

and the H_z component of the magnetic field intensity is zero.

Based on expressions (10)–(12) for the complex amplitudes of the field intensities, heat generations and ponderomotive forces are determined using the corresponding formulas from [4–9, 12, 13], which in the considered case have the form:

$$Q = \frac{1}{2} J_0^2(t)\, \omega \left[\varepsilon_0\, \varepsilon' \tan\delta_{\mathrm{e}} E_x \bar{E}_x + \mu_0\, \mu' \tan\delta_{\mathrm{m}} (H_y \bar{H}_y + H_z \bar{H}_z)\right], \tag{13}$$

$$F = \left\{0; F_y(z,t); F_z(z,t)\right\},$$

$$F_y = \frac{1}{4} J_0^2(t) \left\{-\sigma_{\mathrm{e}} \mu_0 (\bar{\mu}'_* E_x \bar{H}_z + \mu'_* \bar{E}_x H_z) + i\frac{\alpha}{h}\left[\varepsilon_0 (\varepsilon'_* - \bar{\varepsilon}'_*) E_x \bar{E}_x + \mu_0 (\mu'_* - \bar{\mu}'_*)\left(H_y \bar{H}_y + H_z \bar{H}_z\right)\right]\right\},$$

$$F_z = \frac{1}{4} J_0^2(t) \Bigg\{\sigma_{\mathrm{e}} \mu_0 \left(\bar{\mu}'_* E_x \bar{H}_y + \mu'_* \bar{E}_x H_y\right) + \frac{1}{h}\Bigg[\varepsilon_0 \left(\left(\bar{\varepsilon}'_* - 1\right) \bar{E}_x \frac{dE_x}{dz} + \left(\varepsilon'_* - 1\right) E_x \frac{d\bar{E}_x}{dz}\right)$$
$$+ \mu_0 \left(\mu'_* (\bar{\mu}'_* - 1)\left(\bar{H}_y \frac{dH_y}{dz} + \bar{H}_z \frac{dH_z}{dz}\right) + \bar{\mu}'_* (\mu'_* - 1)\left(H_y \frac{d\bar{H}_y}{dz} + H_z \frac{d\bar{H}_z}{dz}\right)\right)\Bigg]\Bigg\}. \tag{14}$$

Given the known heat generations and ponderomotive forces, the temperature and stresses in the layer are described by the following independent equations system of the unrelated quasi-static problem of thermoelasticity [12, 13]:

$$\begin{aligned}
&\Delta T - \frac{1}{a}\frac{\partial T}{\partial t} = -\frac{1}{\lambda} Q,\\
&\Delta \Psi = -\frac{1}{1-\nu}\left(\alpha_{\mathrm{T}} E_{\mathrm{p}}\, \Delta T + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right),\\
&\Delta \sigma_{yz} = -\frac{\partial^2 \Psi}{\partial y \partial z} - \frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y},\\
&\frac{\partial}{\partial y}\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + F_y\right) = 0,\\
&\sigma_{zz} = \Psi - \sigma_{yy}, \quad \sigma_{xx} = \nu \Psi - \alpha_{\mathrm{T}} E_{\mathrm{p}} T
\end{aligned} \tag{15}$$

(where $\Psi = \sigma_{yy} + \sigma_{zz}$ is the function related to the first invariant of the stress tensor $I_{1\sigma}$ by the dependence $\sigma_{\alpha\alpha} = I_{1\sigma} = (1+\nu)\Psi - \alpha_{\mathrm{T}} E_{\mathrm{p}} T$) at the initial $T(z,0) = 0$ and boundary conditions [12, 13]:

$$\begin{aligned}
&\frac{\partial T(1,t)}{\partial z} + \mathrm{Bi}\, T(1,t) = 0, \quad \frac{\partial T(0,t)}{\partial z} = 0;\\
&\sigma_{zz}(1,t) = 0, \quad \sigma_{yz}(1,t) = 0;\\
&\nu \frac{\partial \Psi(0,t)}{\partial y} + \frac{\partial \sigma_{yz}(0,t)}{\partial z} = \alpha_{\mathrm{T}} E_{\mathrm{p}} \frac{\partial T(0,t)}{\partial y} - F_y(0,t),\\
&\frac{\partial \sigma_{yz}(0,t)}{\partial y} + (1-\nu)\frac{\partial \Psi(0,t)}{\partial z} = -\alpha_{\mathrm{T}} E_{\mathrm{p}} \frac{\partial T(0,t)}{\partial z} - F_z(0,t),
\end{aligned} \tag{16}$$

corresponding to the accepted ones of heating and fixing of the layer.

To find the temperature and stress fields from the thermoelasticity problem (15)–(16), as in [4–9, 12, 13], we use the method of finite integral transformations along the thickness coordinate, as well as numerical differentiation and integration methods.

3. Analysis of the obtained results

In order to study the influence of a distribution periodicity along the coordinate of an external electric current on the thermally stressed state of the layer caused by the EMF effect induced by such a current, quantitative research was carried out on a layer made of the same material as in [4–9] (nickel-zinc ferrite 1000HH, the characteristics of which are given in [14–17]) at the same values of layer thickness, distance of the current-carrying plane from the layer base, electric current density and EMF frequencies. The corresponding dependences of the dielectric and magnetic permeabilities, as well as tangents of the hysteresis loss angles, on the field frequency are taken into account [18]. The layer's upper base $z_* = h$ is not subject to any mechanical loads. On it occurs convective heat exchange with the external environment, the temperature of which is maintained constant and equal to the initial temperature of the layer. This exchange occurs at the Biot criterion $\mathrm{Bi} = 0.2$, which practically corresponds to heat exchange with air. The lower base $z_* = 0$ is thermally insulated.

The results analysis shows that at different frequencies α_* of sinusoidal variation in a current density the distributions character (linear or wave) of the EMF intensities amplitudes, the volume density of heat generations, and the ponderomotive force components depends on a ratio between the EMF frequency and the ferrite layer thickness (as in the case of a uniform distribution of the external electric current density (at $\alpha_* = 0$) [5–9]). As noted in these works, at the EMF frequency for which a wavelength in the ferrite medium $\lambda_{\mathrm{e/m}}$ is a multiple of the two maximum dimensions of the solid in a direction of its propagation, and in the presence of low hysteresis losses at this frequency, a parametric resonance occurs in it: a significant increase in electric and magnetic field intensities (and, as a consequence, corresponding temperature fields and stresses), due to standing electromagnetic waves.

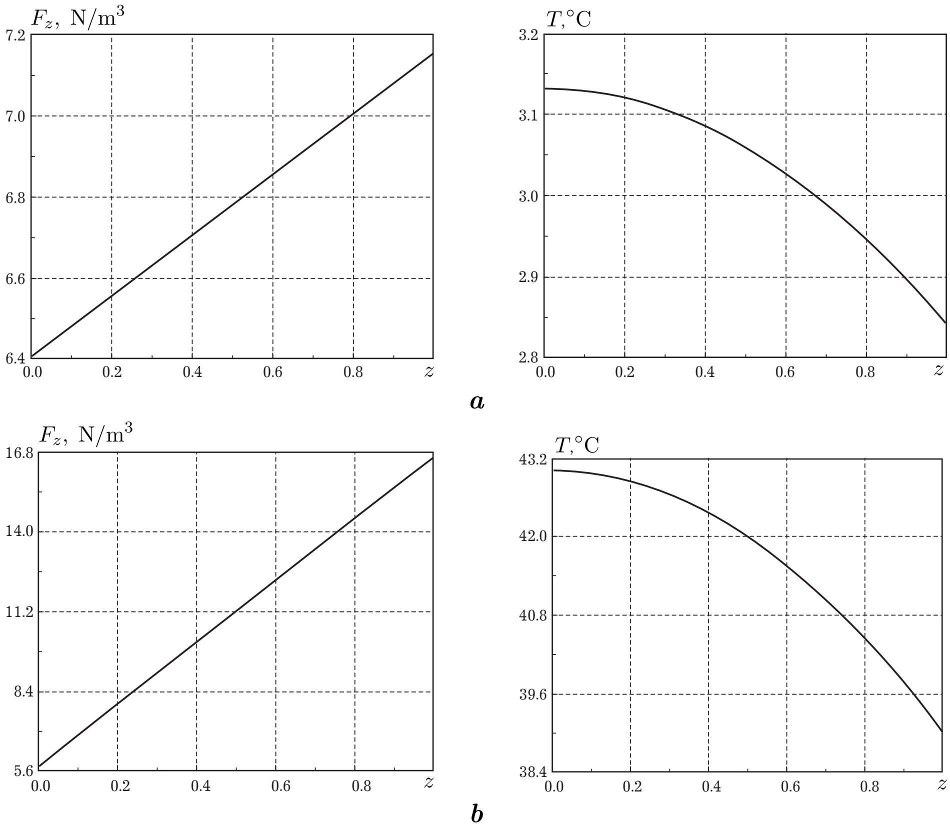

Fig. 2. Distributions of the ponderomotive force F_z component and temperature increment T in the layer during a steady mode for two frequencies α_* of sinusoidal variation in the current density.

Upon analysing the thickness coordinate distributions of the F_y and F_z components of the ponderomotive force, it is evident that the F_y component is consistently smaller than the F_z one for all

α_* values. Additionally, as the external electric current density distribution approaches uniformity (at $\alpha_* \to 0$), the F_y/F_z ratio decreases, eventually reaching zero in the case of a completely uniform current distribution.

Figure 2 shows the ponderomotive force F_z component and temperature deviation T distributions from the initial one T_0 as a function of the layer thickness (for $h = 0.03\,\mathrm{m}$). Calculations were performed using an EMF frequency $\omega = 3.77 \cdot 10^5\,\mathrm{s}^{-1}$ selected from the list of amplitude modulated waves of the radio frequency range (from longwave to microwave bands) authorised for use in industry [19,20], as well as two frequencies α_* of sinusoidal change in current density ($\alpha_* = 0$ — Figure 2a and $\alpha_* = 0.0013\,\mathrm{m}^{-1}$ — Figure 2b), for the steady temperature mode (for a point in time ($t \approx 10\,\mathrm{h}$) when further heating practically does not lead to a change the temperature distribution in the layer).

The solutions analysis and numerical studies allows us to draw conclusions about the influence of a distribution periodicity along the coordinate of an external electric current on the thermally stressed state of a ferrite layer, the main ones are as follows:

- the distribution of electric current density has a significant impact on the values of ponderomotive forces and temperature, which in turn affects the thermally stressed state of the ferrite layer. For example, if for a uniform distribution of current density (for $\alpha_* = 0$) at the EMF frequency $\omega = 3.77 \cdot 10^5\,\mathrm{s}^{-1}$, the maximum values of the ponderomotive force and temperature increment in the layer are $F = 7.14\,\mathrm{N/m}^3$ and $T = 3.13\,^\circ\mathrm{C}$, respectively, then at $\alpha_* = 0.0013\,\mathrm{m}^{-1}$, the largest value of the force increases by more than 2 times, and of the temperature — by more than an order of magnitude, reaching $F = 16.62\,\mathrm{N/m}^3$ and $T = 43.02\,^\circ\mathrm{C}$, respectively;
- the frequency value α_* of the sinusoidal change in current density, at which the largest increments of ponderomotive force and temperature occur, increases as the external EMF frequency ones.

4. Conclusions

1) To evaluate the non-uniformity impact of external electric current distribution on the thermally stressed state of the layer, we solved the problem to determine the thermally stressed state parameters of a layer made from ferrite material under the influence of an EMF created by a given quasi-steady current, the density of which has a sinusoidal character along one of the longitudinal coordinates.
2) The distributions of field intensities, heat generations, ponderomotive forces, and temperature in the layer depending on its thickness, the amplitude-frequency characteristics of the external EMF, and the frequency of sinusoidal variation in the induction current density are investigated.
3) The obtained results analysis allowed us to draw conclusions about the influence of a distribution periodicity along the coordinate of an external electric current on the thermally stressed state of a ferrite layer, the main ones are as follows:
   - the distribution's harmonic character along the coordinate of electric current density has a significant impact on the values of ponderomotive forces and temperature in the layer, which in turn affects its thermally stressed state. In particular, this distribution can cause a significant increase of temperature in subregions of the layer local along the transverse coordinate by more than an order of magnitude compared to the uniform one;
   - the frequency value of the sinusoidal change in current density, at which the largest increments of ponderomotive force and temperature occur in local subregions, increases as the external EMF frequency ones.
4) The possibility of neglecting dynamic effects for all EMF frequencies except for resonant ones, along with the presence of very narrow vicinities of resonant frequencies, makes it possible to effectively control (when applying frequencies located outside the vicinities of resonant ones) the heat treatment of ferrite products using external EMFs, by utilising the important non-uniformity technological factor of the external electric current distribution. In this case, the frequency of electromagnetic influence ensures the stability (monotonicity) of the heating and deformation processes.

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Термонапружений стан шару за дії струмів, періодичних відносно поздовжної координати

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Для оцінки впливу важливого технологічного чинника нерівномірності розподілу зовнішнього електричного струму на параметри низькотемпературного нагрівання пласких феритових елементів з урахуванням напруженого стану (міцнісних характеристик) розглянуто модельну задачу про визначення та дослідження термомеханічної поведінки феритового шару за дії квазіусталеного електромагнітного поля, створюваного струмом, що протікає в струмовідній площині, густина якого має синусоїдальний характер зміни за поздовжньою координатою. Відповідно до результатів, отриманих раніше для залежності процесу нагрівання від частоти зовнішньої електромагнітної дії, прийнято, що кругова несуча частота електромагнітного поля лежить поза околами резонансних (коли термонапружений стан має практично квазістатичний характер). При цьому розрахункова схема складається з трьох етапів послідовного визначення параметрів, які описують електромагнітне, температурне та механічні поля.

Ключові слова: *термонапружений стан; феритове тіло; квазіусталене електромагнітне поле; метод комплексних амплітуд; тепловиділення; пондеромоторна сила.*