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Termostressed state of a three-layer rectangular plate under non-stationary convective heating conditions

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The study considers a rectangular isotropic plate with a layered irregular structure. It is convectively non-stationarily heated by an external environment. The initial relationships of the non-stationary heat conduction and thermoelasticity problem are formulated using a five-mode mathematical model based on the shear deformation theory of thermoelasticity. Using the methods of Fourier and Laplace integral transforms, general solutions have been obtained for the non-stationary heat conduction problem and the quasi-static thermoelasticity problem for a hinge-supported plate along its edges. A numerical analysis of the temperature field, radial deflections, normal forces, bending moments, and normal stresses, depending on geometric parameters and the Bi criterion, has been performed for a three-layer plate. The materials of its layers are made of ceramics and metal. The temperature and mechanical parameters have been analyzed for the layering configuration of the plate: metal-ceramic-metal.

Keywords: *three-layer plate; convective heat transfer; non-stationary heating; temperature; thermoelastic state.*

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1. Introduction

Multilayer materials are frequently employed in various fields of modern technology as components of structures operating under high-temperature conditions. Multilayer structures are used for protection against aggressive environments, reinforcement of constructions, thermal insulation, or enhancement of heat transfer. Therefore, research in this direction remains pertinent.

Scientists have extensively investigated the components of layered structures [1–3]. Precise solutions for thermoelasticity problems of layered plates based on three-dimensional equations have been developed in [4, 5]. Refined models have been proposed using two-dimensional equations [6–8]. Analytical solutions for the bending of inhomogeneous and composite plates under thermomechanical loading have been derived in [9–13]. Interrelated thermoelasticity equations have been utilised to analyse the influence of the coupling coefficient on the nonlinear behaviour of plates [14]. The finite element method has been employed for studying thermomechanical processes in layered plates [14, 15]. Attention has been focused on thermoelastomechanical analysis of non-ferromagnetic plates under electromagnetic impulses in a study [16]. Temperature stability of plates made of composite material has been investigated in [17]. A more detailed overview of various models and research methods for inhomogeneous thin-walled structures is provided in works [1, 2, 18–20].

The aim of this article is to investigate the thermoelastic state of a three-layer rectangular plate with irregular symmetric structure under heating from its surroundings through heat exchange, based on the equations of thermoelasticity theory for plates with five degrees of freedom and two-dimensional heat conduction equations.

2. Determination of the temperature field

Consider a rectangular plate of dimensions $a \times b$ and constant thickness $2h$ made of an isotropic material inhomogeneous in the transverse direction. The points of the plate space belong to the orthogonal coordinate system x, y, z and occupy the region $[0, a] \times [0, b] \times [-h, h]$.

Suppose that at the initial time the temperature of the plate is zero. Starting from $\tau > 0$, the plate is heated by the environment, whose temperature on the surface $z = +h$ is $t_c^+(x, y, \tau) = t_c(x, y)\, t^+(\tau)$, and on the surface $z = -h$ is $t_c^-(x, y, \tau) = 0$. Convective heat transfer with a constant heat transfer coefficient α_z occurs between the environment and the surfaces $z = \pm h$. The temperature field of the plate $t(x, y, z, \tau)$ is determined from the system of two-dimensional heat conduction equations with a linear dependence of temperature on the transverse coordinate [7]:

$$A^\lambda \Delta T_1 - 2\alpha_z T_1 + B^\lambda \Delta T_2 - A^c \frac{\partial T_1}{\partial \tau} - B^c \frac{\partial T_2}{\partial \tau} = -\alpha_z t_c^+,$$
$$B^\lambda \Delta T_1 + D^\lambda \Delta T_2 - \left(\frac{A^\lambda}{h^2} + 2\alpha_z\right) T_2 - B^c \frac{\partial T_1}{\partial \tau} - D^c \frac{\partial T_2}{\partial \tau} = -\alpha_z t_c^+. \tag{1}$$

Here

$$\{A^\lambda, B^\lambda, D^\lambda\} = \int_{-h}^{h} \lambda(z) \{1, z/h, (z/h)^2\} dz, \quad \{A^c, B^c, D^c\} = \int_{-h}^{h} c_v(z) \{1, z/h, (z/h)^2\} dz, \tag{2}$$

$$\Delta = \left(\partial_{11}^2 + \partial_{22}^2\right), \quad T_j = \frac{2j-1}{2h^j} \int_{-h}^{h} t\, z^{j-1} dz, \quad (j = 1, 2)$$

are the integral characteristics of temperatures; $c_v = c\rho$, $\partial_1 = \partial/\partial x$, $\partial_2 = \partial/\partial y$, $\lambda(z)$ is the thermal conductivity coefficient, $c(z)$ is the specific heat capacity, $\rho(z)$ is the specific density, τ is the time variable, $t(x, y, z, \tau)$ is the temperature field function.

The system (1) is subjected to homogeneous boundary and initial conditions:

$$x = 0, a\colon \quad T_1 = T_2 = 0,$$
$$y = 0, b\colon \quad T_1 = T_2 = 0, \tag{3}$$
$$\tau = 0\colon \quad T_1 = T_2 = 0. \tag{4}$$

The solution of the system (1) under the conditions (3) and (4) has been found by the methods of the integral Laplace transform in time and the finite Fourier transform in coordinates. Then we obtain the following integral characteristics of temperatures:

$$T_1 = \frac{1}{2C^*} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{i \neq j=1}^{2} \frac{Q_{nm} Z_j(\tau')\, \mathrm{Bi}}{p_j - p_i} \left[(C_3 p_j - g_4) - (C_2 p_j - g_2)\right] \sin\frac{\pi n}{a} x \sin\frac{\pi m}{b} y,$$
$$T_2 = \frac{1}{2C^*} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{i \neq j=1}^{2} \frac{Q_{nm} Z_j(\tau')\, \mathrm{Bi}}{p_j - p_i} \left[(C_1 p_j - g_1) - (C_2 p_j - g_3)\right] \sin\frac{\pi n}{a} x \sin\frac{\pi m}{b} y. \tag{5}$$

Here

$$g_1 = \Lambda_1 \left(\mu_n^2 + \mu_m^2\right) + \mathrm{Bi}, \quad g_2 = g_3 = \Lambda_2 \left(\mu_n^2 + \mu_m^2\right), \quad g_4 = \Lambda_3 \left(\mu_n^2 + \mu_m^2\right) + \mathrm{Bi} + \Lambda_1,$$
$$\mathrm{Bi} = \frac{\alpha_z h}{\lambda_0}, \quad \mu_n = \frac{h\pi n}{a}, \quad \mu_m = \frac{h\pi m}{b}, \quad \tau' = \frac{\lambda_0}{c_v^0 h^2}\tau, \quad C^* = C_1 C_3 - C_2^2,$$
$$\{\Lambda_1, \Lambda_2, \Lambda_3\} = \frac{1}{2h\lambda_0} \{A^\lambda, B^\lambda, D^\lambda\}, \quad \{C_1, C_2, C_3\} = \frac{1}{2hc_v^0} \{A^c, B^c, D^c\},$$

λ_0 and c_v^0 are some characteristic coefficients of thermal conductivity and heat capacity, respectively; $-p_1$ and $-p_2$ are the roots of the quadratic equation

$$C^* p^2 + [C_1 g_4 + C_3 g_1 - C_2(g_3 + g_2)]\, p + g_1 g_4 - g_2 g_3 = 0,$$

$$Q_{mn} = \frac{4}{ab} \int_0^a \int_0^b t_c(x, y) \sin\frac{\pi n}{a} x\, \sin\frac{\pi m}{b} y\, dx\, dy, \tag{6}$$

$$Z_j(\tau') = \int_0^{\tau'} t^+(v)\, e^{-p_j(\tau' - v)} dv, \quad (j = 1, 2). \tag{7}$$

3. Determination of the stress-strain state

To investigate the stress-strain state of the plate caused by the temperature field (5), we use the basic thermoelastic equations of the first-order shear theory [7], written for isotropic plates in terms of generalized displacements u_i, w, γ_i $(i = 1, 2)$.

Physical equations for stresses at any point of the plate take the form:

$$\sigma_{11} = \frac{E(z)}{1-\nu^2}\left[\partial_1 u_1 + \nu\,\partial_2 u_2 + z\left(\partial_1\gamma_1 + \nu\partial_2\gamma_2\right) - (1+\nu)\,\alpha_t(z)\,t\right],$$
$$\sigma_{22} = \frac{E(z)}{1-\nu^2}\left[\partial_2 u_2 + \nu\,\partial_1 u_1 + z\left(\partial_2\gamma_2 + \nu\partial_1\gamma_1\right) - (1+\nu)\,\alpha_t(z)\,t\right],$$
$$\sigma_{12} = \frac{E(z)}{2(1+\nu)}\left(\partial_2 u_1 + \partial_1 u_2\right), \quad \sigma_{13} = \frac{E(z)}{2(1+\nu)}\left(\gamma_1 + \partial_1 w\right), \quad \sigma_{23} = \frac{E(z)}{2(1+\nu)}\left(\gamma_2 + \partial_2 w\right), \tag{8}$$

where $E(z)$ and $\alpha^t(z)$ are the modulus of elasticity and the coefficient of thermal linear expansion, which depend on the coordinate z; ν is the Poisson's ratio, which is assumed to be constant; u_i, w are the displacements of the points of the average surface; γ_i are the angles of rotation of the normal. Physical equations for forces and moments in the middle surface of the plate can be written as:

$$N_1 = A\left(\partial_1 u_1 + \nu\partial_2 u_2\right) + B\left(\partial_1\gamma_1 + \nu\partial_2\gamma_2\right) - A^t T_1 - B^t T_2/h,$$
$$N_2 = A\left(\partial_2 u_2 + \nu\partial_1 u_1\right) + B\left(\partial_2\gamma_2 + \nu\partial_1\gamma_1\right) - A^t T_1 - B^t T_2/h,$$
$$M_1 = B\left(\partial_1 u_1 + \nu\partial_2 u_2\right) + D\left(\partial_1\gamma_1 + \nu\partial_2\gamma_2\right) - B^t T_1 - D^t T_2/h,$$
$$M_2 = B\left(\partial_2 u_2 + \nu\partial_1 u_1\right) + D\left(\partial_2\gamma_2 + \nu\partial_1\gamma_1\right) - B^t T_1 - D^t T_2/h,$$

$$N_{12} = \frac{1-\nu}{2}\big(A(\partial_1 u_2 + \partial_2 u_1) + B(\partial_1\gamma_2 + \partial_2\gamma_1)\big),$$
$$M_{12} = \frac{1-\nu}{2}\big(B(\partial_1 u_2 + \partial_2 u_1) + D(\partial_1\gamma_2 + \partial_2\gamma_1)\big),$$
$$Q_1 = k'A\frac{1-\nu}{2}\big(\gamma_1 + \partial_1 w\big), \quad Q_2 = k'A\frac{1-\nu}{2}\big(\gamma_2 + \partial_2 w\big). \tag{9}$$

Here

$$\{A, B, D\} = \frac{1}{1-\nu^2}\int_{-h}^{h} E(z)\{1, z, z^2\}dz, \quad \{A^t, B^t, D^t\} = \frac{1}{1-\nu}\int_{-h}^{h} E(z)\,\alpha_t(z)\{1, z, z^2\}dz;$$

N_i, N_{12}, Q_i are normal, shear, and shear forces, respectively; M_i, M_{12} are bending and torsional moments; k' is the shear coefficient [21].

The equilibrium equations take the form:

$$A\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)u_1 + A\frac{1+\nu}{2}\partial_{12}^2 u_2 + B\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)\gamma_1 + B\frac{1+\nu}{2}\partial_{12}^2\gamma_2 = A^t\partial_1 T_1 + B^t\partial_1 T_2/h,$$
$$A\frac{1+\nu}{2}\partial_{12}^2 u_1 + A\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)u_2 + B\frac{1+\nu}{2}\partial_{12}^2\gamma_1 + B\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)\gamma_2 = A^t\partial_2 T_1 + B^t\partial_2 T_2/h,$$
$$\left[\frac{1-\nu}{2}Ak'\left(\partial_1^2 + \partial_2^2\right)\right]w + \frac{1-\nu}{2}Ak'\partial_1\gamma_1 + \frac{1-\nu}{2}Ak'\partial_2\gamma_2 = 0,$$
$$B\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)u_1 + B\frac{1+\nu}{2}\partial_{12}^2 u_2 - \frac{1-\nu}{2}Ak'\partial_1 w + \left[D\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right) - \frac{1-\nu}{2}k'A\right]\gamma_1$$
$$+ D\frac{1+\nu}{2}\partial_{12}^2\gamma_2 = B^t\partial_1 T_1 + D^t\partial_1 T_2/h,$$
$$B\frac{1+\nu}{2}\partial_{12}^2 u_1 + B\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)u_2 - \frac{1-\nu}{2}k'A\partial_2 w + D\frac{1+\nu}{2}\partial_{12}^2\gamma_1$$
$$+ \left[D\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right) - \frac{1-\nu}{2}k'A\right]\gamma_2 = B^t\partial_2 T_1 + D^t\partial_2 T_2/h. \tag{10}$$

Boundary conditions at the edges have the following form:

$$\begin{aligned} x = 0, a: \quad & u_2 = w = \gamma_2 = 0, \quad N_1 = M_1 = 0, \\ y = 0, b: \quad & u_1 = w = \gamma_1 = 0, \quad N_2 = M_2 = 0. \end{aligned} \tag{11}$$

The system of differential equations (10) together with the boundary conditions (11) constitutes the boundary value problem of temperature stresses for isotropic inhomogeneous rectangular plates in generalized displacements. This problem has been solved by the finite double Fourier transform method in the x and y coordinates. From the known generalized displacements, the temperature stresses (8) and the force-moments (9) were found.

4. Results and discussion

As an example, consider a three-layer shell of irregular symmetrical structure. Let the physical and mechanical characteristics of these layers be equal to $q^{(1)} = q^{(3)} = \{c_{ij}^{(1)}, \beta_{ii}^{(1)}, \lambda_{ii}^{(1)}, c_v^{(1)}\}$, $q^{(2)} = \{c_{ij}^{(2)}, \beta_{ii}^{(2)}, \lambda_{ii}^{(2)}, c_v^{(2)}\}$, and their thicknesses are $h_1 = h_3$ and h_2, $(2h_1 + h_2 = h)$. Then the expressions for the integral characteristics $A^q = \{A, A^t, A^\lambda, A^c\}$, $B^q = \{B, B^t, B^\lambda, B^c\}$ and $D^q = \{D, D^t, D^\lambda, D^c\}$ through the characteristics of the layers are determined by the formulas [3]

$$A^q = 2h\left[2q^{(2)} - \frac{h_1}{h}\big(q^{(2)} - q^{(1)}\big)\right], \quad B^q = 0, \quad D^q = \frac{2h^3}{3}\left\{q^{(1)} + \big(q^{(2)} - q^{(1)}\big)\left(1 - \frac{h_1}{h}\right)^3\right\}. \tag{12}$$

Numerical studies were performed for the following ambient temperature distribution functions:

$$t_c(x, y) = t^* N(x)\, N(y), \tag{13}$$

$$t^+(\tau) = 1 - e^{-\beta^* \tau}, \tag{14}$$

where

$$N(x) = S_-\big(x - (x_0 - a_0)\big) - S_+\big(x - (x_0 + a_0)\big), \quad N(y) = S_-\big(y - (y_0 - b_0)\big) - S_+\big(y - (y_0 + b_0)\big);$$

$S_\pm(x)$ are asymmetric unit functions; $2a_0$ and $2b_0$ are the dimensions of the heating region; (x_0, y_0) are the coordinates of the center of this region; β^* is a parameter characterizing the rate of increase in the temperature of the medium to a given surface distribution; t^*, $\beta^* = \text{const}$.

From the relations (6) and (13) the expressions for the Fourier coefficients Q_{nm} are obtain

$$Q_{nm} = \frac{16t^*}{mn\pi^2} \sin\frac{\pi n a_0}{a} \sin\frac{\pi n x_0}{a} \sin\frac{\pi m b_0}{b} \sin\frac{\pi m y_0}{b}.$$

In the case of uniform heating over the entire surface of the plate ($x_0 = \frac{a}{2}$, $y_0 = \frac{b}{2}$, $a_0 = \frac{a}{2}$, $b_0 = \frac{b}{2}$) we obtain $Q_{nm} = \frac{4t^*}{mn\pi^2}\,(1 - \cos\pi n)\,(1 - \cos\pi m)$.

The time function is calculated using the formulas (7) and (14)

$$Z_j(\tau') = \frac{1}{p_j}\big(1 - e^{-p_j \tau'}\big) + \frac{1}{\beta^* - p_j}\big(e^{-\beta^* \tau'} - e^{-p_j \tau'}\big).$$

The materials of the shell layers were metal (Ti-6Al-4V) and ceramics (ZrO_2) with the following physical and mechanical characteristics [13]:

— **metal:** $\nu = 0.321$, $E_m = 66.2\,\text{GPa}$, $\alpha_m^t = 10.3 \cdot 10^{-6}\,1/\text{K}$, $\lambda_m = 18.1\,\text{W/mK}$, $c = 808.3\,\text{J/(kg K)}$;
— **ceramics:** $\nu = 0.333$, $E_c = 117\,\text{GPa}$, $\alpha_c^t = 7.11 \cdot 10^{-6}\,1/\text{K}$, $\lambda_c = 2.036\,\text{W/mK}$, $c = 615.6\,\text{J/(kg K)}$.

The values of the other parameters are as follows: $h/a = 0.025$, $a/b = 1$, $k' = 5/6$, $\tau' = 3$, $E_0 = 10^2\,\text{GPa}$, $\alpha_0^t = 10^{-5}\,1/\text{K}$, $\lambda_0 = 1\,\text{W/mK}$, $c_0 = 6 \cdot 10^2\,\text{J/(kg K)}$.

The dimensionless temperature $t' = \frac{t}{t^*}$, temperature characteristics $T_i' = \frac{T_i}{t^*}$, radial deflections $w' = \frac{w}{ht^*\alpha_0^t}$, normal forces $N_1' = \frac{N_1}{t^* h E_0 \alpha_0^t}$, bending moments $M_1' = \frac{M_1}{t^* h^2 E_0 \alpha_0^t}$ and normal stresses $\sigma_1' = \frac{\sigma_{11}}{t^* E_0 \alpha_0^t}$ were calculated in the center of a square three-layer plate metal/ceramic/metal (M/K/M) with a uniform distribution of ambient temperature over the surface.

Figure 1 shows the values of these quantities for $\text{Bi} = 1$, $\beta^* = 0.5$, and different values of the ratio of the thickness of the middle layer to the front layer h_2/h_1. For M/K/M plates, as the ratio h_2/h_1

increases, the role of ceramics increases, and the stresses on the surface $z = h$ increase, since the elastic modulus for ceramics is greater than for metal.

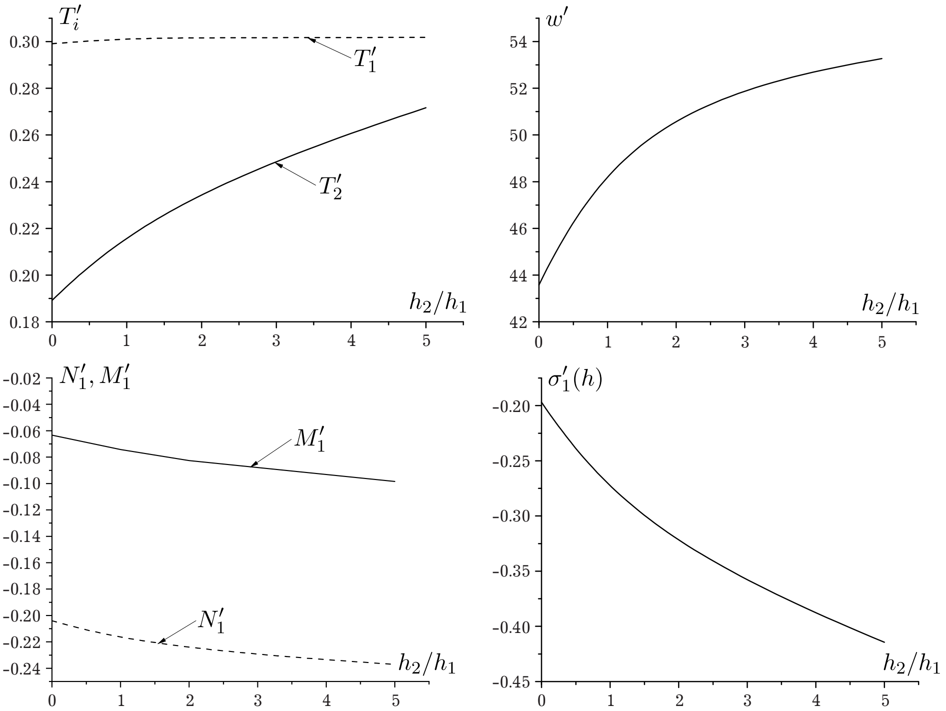

Fig. 1. Temperature characteristics T_i', radial deflections w', normal forces N_1', bending moments M_1' and normal stresses σ_1' depending on the ratio of the thickness of the middle layer to the front layer h_2/h_1 have been calculated at $\mathrm{Bi} = 1$, $\beta^* = 0.5$.

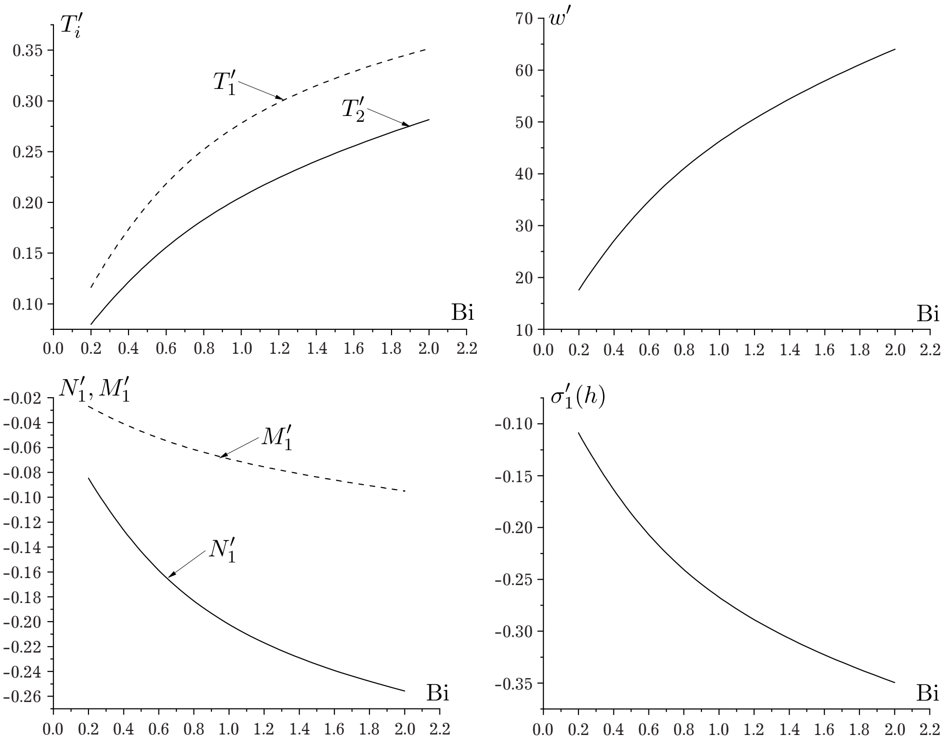

Fig. 2. Temperature characteristics T_i', radial deflections w', normal forces N_1', bending moments M_1' and normal stresses σ_1' depending on Bi have been calculated at $h_2/h_1 = 1$, $\beta^* = 0.5$.

Figure 2 illustrates the dependence of the temperature and the stress-strain state of the plate on the heat transfer between the front surfaces of the plate and the environment. The calculations were performed for $h_2/h_1 = 1$, $\beta^* = 0.5$, and different values of the heat transfer coefficient Bi. It can be seen that with an increase in heat transfer, the temperature and, consequently, the plate stress increases.

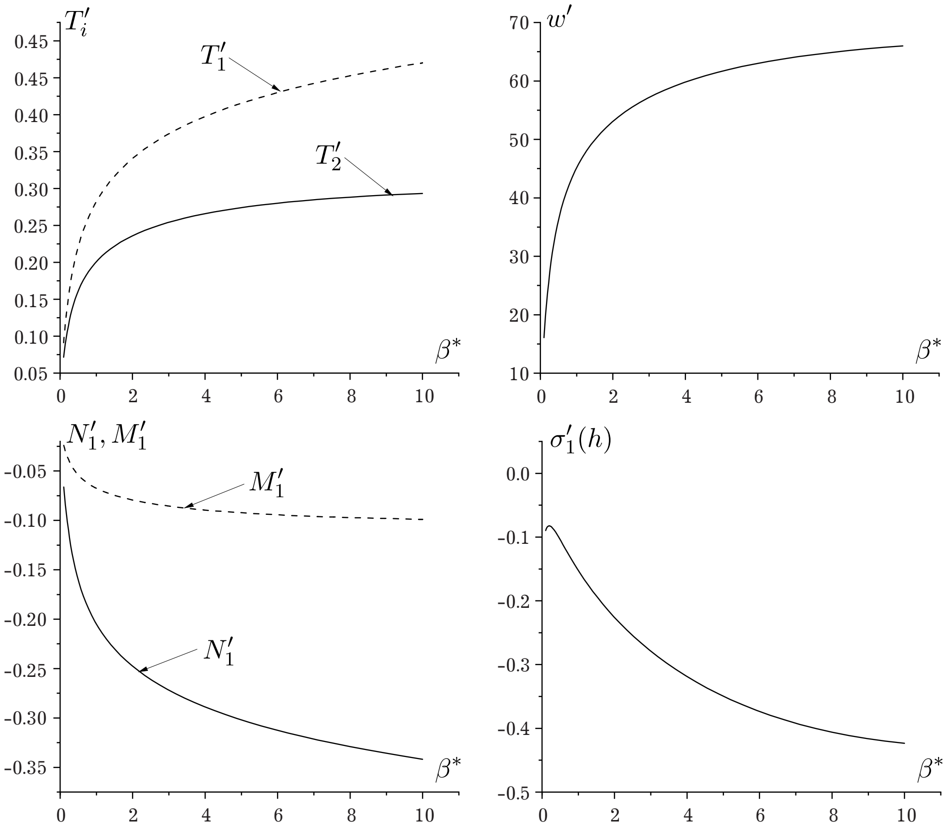

Fig. 3. Temperature characteristics T_i', radial deflections w', normal forces N_1', bending moments M_1' and normal stresses σ_1' depending on β^* have been calculated at $h_2/h_1 = 1$, $\mathrm{Bi} = 1$.

Figure 3 shows the values of the temperature and stress-strain state of the plate for $h_2/h_1 = 1$, $\mathrm{Bi} = 1$, and different values of the coefficient β^*, which characterize the speed of reaching the steady-state temperature field and stress-strain state by the characteristics of the temperature field.

5. Conclusions

Based on the equations of the first-order linear shear theory with five degrees of freedom, a methodology for solving the problems of thermal conductivity and thermoelasticity for a layered plate of irregular structure with layers made of ceramics and metal is developed. The plate is heated by the environment by heat transfer through the side surfaces according to Newton's law. The effect of the heat transfer coefficient, time parameter, and the order of layers of different materials and their thicknesses on the components of the stress-strain state and temperature field was investigated. It follows from the numerical analysis that by changing the order of layers and their thicknesses, it is possible to influence the magnitude of stresses in a multilayer structure.

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Термонапружений стан тришарової прямокутної пластини за умов нестаціонарного конвективного нагрівання

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Розглядається прямокутна ізотропна пластина шаруватої нерегулярної структури. Вона конвективно нестаціонарно нагрівається зовнішнім середовищем. Для визначення її термонапруженого стану записано вихідні співвідношення нестаціонарної задачі теплопровідності та термопружності з використанням п'ятимодальної математичної моделі зсувної теорії термопружності. З використанням методів інтегральних перетворень Фур'є і Лапласа знайдено загальні розв'язки нестаціонарної задачі теплопровідності та квазістатичної задачі термопружності для шарнірно опертої на краях розглядуваної пластини. Числовий аналіз температурного поля, радіальних прогинів, нормальних зусиль, згинних моментів і нормальних напружень залежно від геометричних параметрів та критерію Біо виконано для тришарової пластини. Матеріали її шарів виготовлені з кераміки і металу. Проаналізовано температуру і механічні параметри для структури шарів пластини — “метал–кераміка–метал”.

Ключові слова: *тришарова пластина; конвективний теплообмін; нестаціонарний нагрів; температура; термонапружений стан.*