

# Effect of nanoparticle shape on natural convection in hybrid nanofluid inside square cavity

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A numerical study of natural convection in a square cavity with  $Al_2O_3$ -Cu/water hybrid nanofluid, focusing on the effects of nanoparticle shape, is conducted. The governing partial differential equations and corresponding boundary conditions are transformed into nondimensional forms and solved using the finite element method. The flow and heat transfer characteristics are graphically illustrated and explained for different nanoparticle volume fractions and shapes, with corresponding average Nusselt numbers. It has been observed that a variety of nanoparticle shapes effect, as the empirical nanoparticle shape factor m increases, the total surface area of the nanoparticle increases. This causes more heat can be dissipated and in turn, produces a higher heat transfer rate.

**Keywords:** natural convection flow;  $Al_2O_3$ -Cu/water hybrid nanofluid; square cavity; finite element method; nanoparticle shape.

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## 1. Introduction

Natural convection in closed cavities is one of the most important subjects of fluid flow and heat transfer. This subject has received much attention due to many energy-related applications, such as heat transfer in buildings, nanofluid heat transfer enhancement and control, solar energy collectors, semiconductor manufacturing, and nuclear energy management. The motion of the fluid in natural convection occurs due to the buoyancy forces imposed on the fluid when its density of the heat transfer surface is described as a result of thermal expansion of the fluid in a non-uniform temperature distribution. Most authors considered the flow and heat transfer inside a square or rectangular cavity because the numerical solution is not complicated, and the geometry is very simple to simulate.

Nanofluids are used in various heat transfer applications. Different nano sized particles such as  $Al_2O_3$ , Cu, CuO,  $SO_2$ , TiO<sub>2</sub> are added in the base fluid to enhance the thermal characteristics of the base flow. Many studies have modeled and simulated the natural convection of a suspension of nanoparticles in a base fluid in cavity. Hashim et al. [1] and Alsabery et al. [2] investigated the flow and heat transfer of nanofluids in a wavy wall cavity. Tahmasebi et al. [3] and Ghalambaz et al. [4] and studied the effect of the presence of a layer of porous media and a layer of a solid wall on the flow and natural convection heat transfer of nanofluids. The influence of thermal radiation on entropy generation within a closed cavity filled with  $Al_2O_3$ -water was investigated in the numerical study of Zhang et al. [5]. Sedeghi et al. [6] reviewed and summarized the findings of the published literature on enhancement of natural convection in various enclosures filled using nanofluid. Nazir et al. [7] studied the effect of radiation and heat generation on natural convection flow of nanofluid inside the triangular cavity.

Hybrid nanofluids have been the subject of many scientific works in past decades. The hybrid nanofluids is a mixture of two kinds of solid nanoparticles in a base fluid to improve the thermophysical

properties of single type of solid nanoparticle. The Al<sub>2</sub>O<sub>3</sub>-Cu/water hybrid nanofluid also has a higher heat transfer rate than the Al<sub>2</sub>O<sub>3</sub>/water nanofluid, as reported by Olatundun and Makinde [8]. Revnic et al. [9] investigated the MHD natural convection effects inside the discrete heating wavy cavity filled with hybrid-nanofluid with particles. Recently, several articles have reported the enhancement of energy transport processes through hybrid nanofluids [10–15]. It was found that the heat transfer rate of the hybrid nanofluid is superior to the Cu/water nanofluid. Besides that, it was observed that the highest increment of Nusselt number was achieved in the use of blade-shaped nanoparticles, compared to the platelet, cylindrical, brick, and spherical shapes.

Based on the previous works, the present study the effects of nanoparticles shape on natural convection in a square cavity with  $Al_2O_3$ -Cu/water hybrid nanofluid. To our best knowledge this problem has not been analyzed yet. The proposed problem aims to provide beneficial information on the efficiency of hybrid nanofluid and shapes of nanoparticles in the flow containing the assumed conditions, which may present in the manufacturing or engineering processes.

## 2. Problem formulation

In this study, a square cavity was used to assess how its thermal system performs, its reactivity to heat transfer, and its thermal conductivity on natural convection. The Rayleigh number had a substantial

influence on the flow profile and heat transmission within the cavity, as well as the thickness of the thermal barrier layer, in a square form. The enclosure is a square with the hot wall EFGJ, the top wall BC is cooled with a constant temperature  $T_c$  and the hot wall EFGJ is heated with a constant temperature  $T_h$ , where  $T_h > T_c$  as shown in Figure 1. The walls AB, CD, DE, and JA are thermally insulated. The enclosure is filled with Cu-Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O hybrid nanofluid.

The governing equations of Navier–Stokes and energy examine steady, two-dimensional viscous incompressible flow are written. The following dimensional system of equations for hybrid nanofluid:



 $T_c$ 

C

B

Fig. 1. Physical model.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial y} + \frac{1}{2} \frac{\partial p}{\partial y} + \frac{u_{brf}}{\partial y} \left( \frac{\partial^2 u}{\partial y} - \frac{\partial^2 u}{\partial y} \right)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),\tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g\left(T - T_c\right),\tag{3}$$

$$\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{4}$$

The boundary conditions: at all surfaces:

$$U = V = 0, (5)$$

at 
$$AB$$
,  $CD$ ,  $EF$  and  $GJ$ :

u

$$\frac{\partial U}{\partial Y} = 0,\tag{6}$$

at BC, DE, FG, and JA:

$$\frac{\partial V}{\partial X} = 0. \tag{7}$$

The thermal properties of base fluid and empirical correlation for hybrid nanofluid is given in Table 1 [4]. The thermophysical properties of water and solid hybrid nanoparticles Alumina  $(Al_2O_3)$ 

and Copper (Cu) are stated in Table 2 and the nanoparticle empirical shape factor is mentioned in Table 3 where m is the shape factor.

Properties	Hybrid Nanofluid Correlations					
Density	$\rho_{hnf} = \varphi_{\text{Al}_2\text{O}_3}\rho_{Al_2O_3} + \varphi_{Cu}\rho_{Cu} + (1-\varphi)\rho_f,$					
	where $\varphi_{hnf} = \varphi_{Al_2O_3} + \varphi_{Cu}$					
Heat capacity	$\left(\rho C_p\right)_{hnf} = \varphi_{\text{Al}_2\text{O}_3} \left(\rho C_p\right)_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} \left(\rho C_p\right)_{Cu} + \left(1 - \varphi\right) \left(\rho C_p\right)_f$					
Dynamic Viscocity	$\mu_{hnf} = \mu_f \left( 1 - \varphi_{\text{Al}_2\text{O}_3} - \varphi_{\text{Cu}} \right)^{-2.5}$					
Thermal Diffusivity	$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}$					
Thermal Conductivity	$\frac{k_{hnf}}{\varphi} = \left[ \left( \frac{\varphi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} k_{\text{Cu}}}{\varphi} \right) + (m-1)k_f - (m-1) \varphi \left( k_f - \left( \frac{\varphi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} k_{\text{Cu}}}{\varphi} \right) \right) \right]$					
Therman Conductivity	$k_f = \left[ \left( \frac{\varphi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} k_{\text{Cu}}}{\varphi} \right) + (m-1)k_f - \varphi \left( k_f - \left( \frac{\varphi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} k_{\text{Cu}}}{\varphi} \right) \right) \right]$					
Electrical Conductivity	$\frac{\sigma_{hnf}}{\varphi} = \frac{\left[\left(\frac{\varphi_{\text{Al}_2\text{O}_3}\sigma_{\text{Al}_2\text{O}_3}+\varphi_{\text{Cu}}\sigma_{\text{Cu}}}{\varphi}\right) + 2\sigma_f + 2\left(\varphi_{\text{Al}_2\text{O}_3}\sigma_{\text{Al}_2\text{O}_3}+\varphi_{\text{Cu}}\sigma_{\text{Cu}}\right) - 2\varphi\sigma_f\right]}{\left[\left(\frac{\varphi_{\text{Al}_2\text{O}_3}\sigma_{\text{Al}_2\text{O}_3}+\varphi_{\text{Cu}}\sigma_{\text{Cu}}}{\varphi}\right) + 2\sigma_f + $					
Electrical Conductivity	$-\frac{1}{\sigma_f} = \left[ \left( \frac{\varphi_{Al_2O_3} \sigma_{Al_2O_3} + \varphi_{Cu} \sigma_{Cu}}{\varphi} \right) + 2\sigma_f - \left( \varphi_{Al_2O_3} \sigma_{Al_2O_3} + \varphi_{Cu} \sigma_{Cu} \right) + \varphi\sigma_f \right]$					
Thermal Expansion	$(\rho\beta)_{hnf} = \varphi_{\text{Al}_2\text{O}_3} (\rho\beta)_{\text{Al}_2\text{O}_3} + \varphi_{\text{Cu}} (\rho\beta)_{\text{Cu}} + (1-\varphi) (\rho\beta)_f$					

 Table 1. Thermophysical properties correlations.

Table 2.	Thermophysical	properties.
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Properties	Water $(H_2O)$	Alumina $(Al_2O_3)$	Copper (Cu)
$ ho~({ m kg/m^3})$	997.1	3970	8933
$C_p~({ m J/kgK})$	4179	765	385
$k  ({ m W/mK})$	0.613	40	400
$ ho~({ m S/m})$	$5.5 imes10^{-6}$	$35  imes 10^6$	$59.6  imes 10^6$
$\beta~(1/{ m K})$	$21  imes 10^{-5}$	$0.85  imes 10^{-5}$	$1.67  imes 10^{-5}$
$\Pr$	6.2		
$\alpha$	$1.47\times 10^{-7}$		

Table 3. Nanoparticle empirical shape.

Shape	Shape Factor
Sphere	3
Cube/brick	3.7
Cylindrical	4.9

Here,  $\mu_{hnf}$ ,  $\rho_{nhf}$ ,  $k_{hnf}$ , and  $(\rho C_p)_{hnf}$  are the dynamic viscosity, density, thermal conductivity, and heat capacity of the hybrid nanofluid, respectively,  $\varphi_{hnf}$  is the nanoparticle volume fraction ( $\varphi = 0$  corresponds to a regular heat transfer fluid,  $\varphi_{Al_2O_3}$  for Al<sub>2</sub>O<sub>3</sub> nanoparticle and  $\varphi_{Cu}$  for Cu nanoparticle), the suffix f is for water, and  $C_p$  is the heat capacity at

constant pressure.

Considering the following parameters;

$$(X,Y) = \frac{(x,y)}{L}, \quad (U,V) = \frac{(uL,vL)}{\alpha_{hnf}}, \quad P = \frac{pL^2}{\rho_j \alpha_{hnf}^2}, \quad \theta = \frac{(T-T_c)}{(T_h - T_c)}.$$
 (8)

The governing equations (1)-(4) are reduced to the following set of dimensionless equation:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\frac{\nu_{hnf}}{\nu_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right),\tag{9}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\frac{\nu_{hnf}}{\nu_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f} \Pr\operatorname{Ra}\theta,\tag{10}$$

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \frac{\alpha_{hnf}}{\alpha_f} \left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right),\tag{11}$$

where  $\Pr = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f}$  is the Prandtl number and  $\operatorname{Ra} = \frac{g(\rho\beta)_f (T_h - T_c)(\rho C_p)_f L^3}{\mu_f k_f}$  is the Rayleigh number. The corresponding non-dimensional boundary conditions are:

at BC:

$$\theta = 0, \tag{12}$$

at EF, FG and GJ:

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ьt	tect	ot	nanoparticle	sha	ape on	natural	convection	ın	hybric	i nanoti	IIId	insic	le square	cavity
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$$\theta = 1, \tag{13}$$

at 
$$AB$$
 and  $CD$ :

$$\frac{\partial \theta}{\partial X} = 0,\tag{14}$$

at DE and JA:

$$\frac{\partial \theta}{\partial Y} = 0.$$

## 3. Computational procedure

The Galerkin weighted residual technique accompanying with the finite element method do apply to investigate the control equations of Eqs. (9)–(11). The governing Eqs. (9)–(11) along with the associated boundary condition (13)–(15) is solved using the Galerkin weighted residual along with finite element method. Alsabery et al. [16] has been thoroughly examined and approved this method. Figure 2 displays the problem's finite element mesh. Five different finite element meshes have been considered and the value of Nu<sub>avg</sub> are carried out and presented in Table 4. Therefore, all the simulations have been carried out at mesh size extra fine with 22428 elements grid system. The iteration is reported until the normalized residual of the governing equations less than  $10^{-6}$ .



Fig. 2. Example of mesh generation.

#### Table 4. Comparison for different type of mesh size.

Predefined	Domain	Boundary		
mesh size	elements	elements	$Nu_{avg}$	CPU time (s)
Normal	2163	159	15.445	14
Fine	3466	198	15.607	14
Finer	8680	402	15.935	20
Extra fine	22428	772	16.109	49
Extremely fine	30254	772	16.106	81

## 4. Results and discussion

The steady-state results presented in this work are generated for different pertinent dimensionless groups: Rayleigh number  $(10^3 \leq \text{Ra} \leq 10^5)$ , W = 0.2, H = 0.4. The default parameters are assigned values unless otherwise stated. Figures 3 to 5 show contours of stream function (streamlines) and temperature (isotherms) with Pr = 6.2,  $\varphi_{hnf} = 0.02$ , W = 0.2, D = 0.4 and H = 0.4 for different values of the Rayleigh number and nanoparticle shape. For low Ra value, the streamlines are characterized by two symmetrical counter-rotating vortexes occupying the entire cavity body. The corresponding isotherms are mostly parallel to the vertical walls except along the top surface of the heater. The contribution of convection is noticeable at high Ra as evident by the departure of the isotherms from the vertical pattern. As Ra increases to  $Ra = 10^5$ , convection mechanism becomes more pronounced and consequently the central vortex moves upward (Figure 5a). Figures 3b to 5bshows that the isotherms are horizontal inside the very thin boundary layers. This can be attributed to high convection current within the cavity which also causes a reduction in the temperature gradients in the centre of the cavity. Figure 6 presents the variations of  $Nu_{avg}$  for different Rayleigh Numbers with different nanoparticle shapes at H = 0.4 and  $\varphi_{hnf} = 0.02$ . From Figure 6, the value of Nu<sub>avg</sub> increases as Ra increases with sphere (m = 3) being the lowest and cylindrical (m = 4.9) being the highest. As the empirical nanoparticle shape factor increase, the total surface area of the nanoparticle increases. This cause more heat can be dissipated and in turns, produces a higher heat transfer rate.

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Fig. 3. (a) Streamlines and (b) isotherms for Ra = 10000.



Fig. 4. (a) Streamlines and (b) isotherms for Ra = 50000.



Fig. 5. (a) Streamlines and (b) isotherms for Ra = 100 000.



Fig. 6. Variations of the average Nusselt number with Rayleigh number and nanoparticle shape.

Figures 7–9 show the streamlines and isotherms at Ra =  $10^4$  for the Cu-Al<sub>2</sub>O<sub>3</sub> hybrid nanofluid with  $\varphi_{hnf} = 0.02$ ,  $\varphi_{hnf} = 0.035$  and  $\varphi_{hnf} = 0.05$ , respectively. We found that symmetrical behaviour in both the streamlines and the contour maps of the isotherms due to the temperature distribution imposed at the bottom wall and the boundary conditions on vertical walls. The flow is mostly formed of two counter-rotating circulation cells, as shown by the Ra and the value of the solid volume fraction. When the solid volume fraction of nanoparticles increases, the fluid travels slower in the container. The presence of nanoparticles has a significant impact on the heat transfer rate through the enclosure. The heat transmission in this scenario is greatest at  $\varphi_{hnf} = 0.05$  and is increased by the presence of nanoparticles, which have a substantially higher thermal conductivity than water. Figure 10 presents the variations of Nu<sub>avg</sub> for different solid volume fraction and nanoparticle shapes at H = 0.4 and Ra =  $10^4$ . It is found that cylindrical (m = 4.9) nanoparticle shape produces the highest heat transfer rate while sphere (m = 3) is the lowest. The unusual behavior of Nu<sub>avg</sub> for lower  $\varphi_{hnf}$  is suspected due to the shape of enclosure, the heating and cooling position and the aspect ratio of the cold wall or hot wall.





Fig. 7. (a) Streamlines and (b) isotherms for  $\varphi_{hnf} = 0.02$ .



Fig. 8. (a) Streamlines and (b) isotherms for  $\varphi_{hnf} = 0.035$ .



Fig. 9. (a) Streamlines and (b) isotherms for  $\varphi_{hnf} = 0.05$ .



Fig. 10. Variations of the average Nusselt number with nanoparticle volume fraction and nanoparticle shape.

# 5. Conclusions

In this paper, the effect of nanoparticle shape on natural convection using hybrid nanofluid inside a square cavity was investigated. The average Nusselt number was calculated by deriving numerical solutions for various factors from associated formulation research. The numerical results are presented both graphically and in tabular form for specific parameter values. Hence, the findings of this study can be summarized as follows:

- With the increase in Ra, both Nu<sub>avq</sub> and heat transfer increase.
- As the nanoparticle shape factor increases, the overall surface area grows, allowing for greater heat dispersion.
- As the solid volume fraction increases, the values of Nu<sub>avg</sub> increase significantly.
- As a result, the cylindrical nanoparticle shape enhances heat transfer the most, compared to cube and spherical shapes.

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## 1.

# Вплив форми наночастинок на природну конвекцію в гібридному нанофлюїді всередині квадратної порожнини

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Чисельно досліджено природну конвекцію в квадратній порожнині з гібридним нанофлюїдом  $Al_2O_3$ -Cu/вода з врахуванням впливу форми наночастинок. Основні диференціальні рівняння в частинних похідних з відповідними граничними умовами перетворено в безрозмірні, а потім розв'язано за допомогою методу скінченних елементів. Характеристики потоку та теплопередачі були графічно проілюстровані та пояснені для різних параметрів об'ємних часток наночастинок та параметрів форми наночастинок із середніми числами Нуссельта. Помічено, що вплив різних форм наночастинок полягає в тому, що зі збільшенням емпіричного коефіцієнта форми наночастинок m збільшується загальна площа поверхні наночастинки. Це призводить до того, що більше тепла може розсіюватися, і, у свою чергу, створюється більша швидкість теплопередачі.

Ключові слова: природний конвекційний потік; гібридний нанофлюїд Al<sub>2</sub>O<sub>3</sub>-Cu/вода; квадратна порожнина; метод скінченних елементів; форма наночастинок.