INFORMATION AND MEASUREMENT TECHNOLOGIES IN MECHATRONICS AND ROBOTICS

ADAPTIVE MODELING OF UNDERWATER ROBOT FLUID DYNAMICS BASED ON FORCE MEASUREMENT DEVICE

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Abstract. This work describes the development and testing of a data-driven hydrodynamic model for quadruped robots designed for adaptive and intelligent interaction in dynamic environments. To effectively manage and interpret sensor data, we employ Gaussian Process Regression (GPR) to model the underlying uncertainties in fluid-structure interactions, allowing for more precise predictions in complex and varying environments. The probabilistic nature of GPR enables quadruped robots to handle noisy data and provide robust, uncertainty-aware decision-making strategies.

We train and evaluate the model using real-time sensor data, which includes ambient environmental factors and the robot's internal states. A key focus of our study is the robot's adaptive response to different hydrodynamic conditions, such as varying speeds and fluid dynamics. The results demonstrate that the GPR-based model efficiently learns and adapts to these dynamic conditions, leading to accurate force prediction and enhanced autonomous performance in a range of real-world scenarios.

Key words: Force measuring device, Data-Driven Hydrodynamic Model, Online Learning, Underwater Robot

1. Introduction

Quadruped robots have garnered attention for applications like subsurface inspections and planetary exploration, but their use in aquatic environments is limited due to challenges in hydrodynamic modeling. Accurately predicting underwater forces is complex because of fluid-structure interactions (FSI) and changing conditions such as leg shape or movement patterns. Traditional models, like those based on Navier-Stokes equations, are precise but computationally heavy and unsuitable for real-time applications [1, 2].

Fluid-structure interaction models, while detailed, also require significant resources, making them impractical for real-world scenarios. These methods also struggle to account for the dynamic nature of changing robot structures during motion.

In contrast, Gaussian Process Regression (GPR) offers a data-driven, probabilistic approach that captures uncertainties in fluid dynamics. GPR models not only predict forces accurately but also adapt to varying conditions and noise in real-time data. This makes GPR a promising tool for underwater robots, offering robust predictions in unpredictable environments.

2. Drawbacks

Traditional hydrodynamic models, though accurate, are computationally demanding and unsuitable for real-time applications. Fluid-structure interaction methods also require extensive resources and struggle to adapt to changing robot structures during movement [3].

While GPR models are more flexible, they have their limitations. As the dataset grows, GPR becomes computationally expensive, which can limit its real-time use. The model's performance also heavily depends on selecting the right hyperparameters and kernel functions, making it sensitive to noisy or sparse data.

3. Goal

This paper aims to develop a novel approach for modeling the hydrodynamics of quadruped robots with swimming capabilities using GPR. The key objectives of this study are:

– Unique Dataset Collection: To gather a comprehensive dataset through controlled towing experiments, capturing a wide range of motion scenarios. This dataset is crucial for training and validating the hydrodynamic models.

– Development of a GPR-Based Hydrodynamic Model: To design an innovative GPR framework that leverages its probabilistic nature to capture uncertainties in fluid-structure interactions, allowing for more precise force predictions. This model will be optimized to handle varying leg configurations and dynamic environments.

– Achieving Robust Prediction under Varying Conditions: To demonstrate the model's ability to accurately predict force states across different speeds, fluid dynamics, and environmental conditions. GPR's uncertainty quantification will be critical in enhancing the model's adaptability and robustness in dynamic, real-time underwater environments.

The ultimate goal of this study is to significantly improve hydrodynamic modeling for quadruped robots, particularly in underwater applications, by providing a more adaptive, accurate, and robust prediction framework that can handle complex and variable conditions [4].

4. GPR-based Model

GPR is a non-parametric, probabilistic model used to learn the dynamics of a system by capturing the relationships between input and output data. GPR is particularly effective for modeling continuous-time changes in forces as a function of sensor inputs while providing uncertainty estimates. The model defines a distribution over functions and uses training data to update this distribution. The core of GPR lies in predicting a continuous function based on a set of observed data points [5,6,7,8].

Given a dataset

$$\mathcal{D} = \{(\mathrm{x_i}, \mathrm{y_i})\}_{\mathrm{i=1}}^{\mathrm{n}}$$

where $\mathrm{x_i}$ represents the input (e.g., sensor data), and y_i represents the output (e.g., hydrodynamic force), GPR models the relationship between inputs and outputs as a multivariate Gaussian distribution:

$$y(x) \sim \mathcal{GP}\big(m(x), k(x, x')\big)$$

where $m(x)$ is the mean function (typically assumed to be zero), and $k(x, x')$ is the covariance (kernel) function that defines the similarity between points x and x'.

The covariance function $k(x, x')$ plays a crucial role in determining the behavior of the model. A common choice is the Radial Basis Function (RBF) kernel:

$$k(x, x') = \sigma_f^2 \exp\left(-\frac{\left\|x - x'\right\|^2}{2l^2}\right) \quad (1)$$

where σ_f^2 is the signal variance, and l is the length scale, which controls how quickly the function can vary.

The GPR prediction for a new test point x_* is given by the conditional distribution:

$$\mathrm{p}(\mathrm{y}_*|\mathrm{x}_*, X, y) = \mathcal{N}\big(\mu(\mathrm{x}_*), \sigma^2(\mathrm{x}_*)\big) \quad (2)$$

where the mean and variance of the prediction are:

$$\mu(\mathrm{x}_*) = k\big(\mathrm{x}_*, \mathrm{X}\big)[\mathrm{K}(\mathrm{X}, \mathrm{X}) + \sigma_\mathrm{n}^2 \mathrm{I}]^{-1}\mathrm{y} \quad (3)$$

$$\sigma^2(\mathrm{x}_*) = \mathrm{k}(x_*, \mathrm{x}_*) - \mathrm{k}(x_*, \mathrm{X})[\mathrm{K}(\mathrm{X}, \mathrm{X}) + \sigma_\mathrm{n}^2 \mathrm{I}]^{-1}\mathrm{k}(X, x_*)$$

Here, $K(X, X)$ is the covariance matrix of the training inputs, σ_n^2 is the noise variance, and $\mathrm{k}(x_*, X)$represents the covariance between the test point x and the training points.

The advantage of GPR is its ability to provide not only a mean prediction but also a measure of uncertainty for each prediction. This is critical in dynamic environments, such as underwater robotics, where sensor noise and environmental variability can significantly affect the accuracy of force predictions.

In this study, we optimize the hyperparameters of the kernel function (e.g., σ_f^2 , l and σ_n^2) using a maximum likelihood estimation approach, and the model is trained on real-time sensor data collected during robot movements. GPR's ability to model the underlying uncertainties in fluid-structure interactions makes it highly suitable for robust predictions in varying underwater environments.

Middle: The Gaussian Process Regression (GPR) model processes the kinematic information and maps it to a latent vector h, incorporating uncertainty estimates to account for variations in the environment and the robot's movements. Right: The prediction of the hydrodynamic force trajectory $\mathrm{F_b}$ is generated by the GPR model based on the initial condition $\mathrm{F_0}$, providing a probabilistic force prediction across the time steps $[t_1, t_2, \dots, t_L]$.

5. Model Architecture

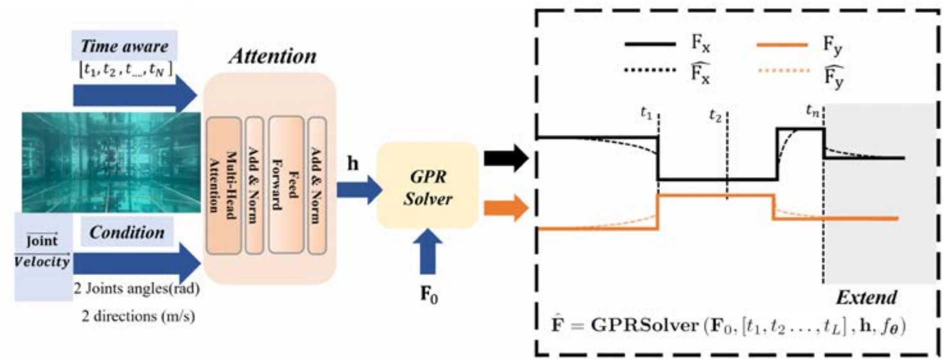

Fig. 1. Overview. Left: The robot's motion under different conditions is represented in time steps $[t_1, t_2, \dots, t_N]$, capturing key kinematic data.

The proposed model architecture aims to predict the hydrodynamic forces acting on a quadruped robot using GPR based on a sequence of observational data. The key components of the model are GPR and the covariance (kernel) function. The steps involved in the model architecture are as follows:

- **Input Data:** The input consists of a sequence of kinematic observations $\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}$, each representing motion parameters sampled at uniform time intervals. These parameters include two joint angles and two linear velocities, providing a comprehensive description of the robot's movement.

- **Covariance (Kernel) Function:** The GPR model uses a kernel function to learn the relationship between the inputs and the hydrodynamic forces. The covariance function $\boldsymbol{k}(\boldsymbol{x}, \boldsymbol{x}')$ defines the similarity between data points $\boldsymbol{x}$ and $\boldsymbol{x}'$, crucial for predicting forces. The most used kernel in this study is the Radial Basis Function (RBF) kernel, given by:

$$k(x, x') = \sigma_f^2 \exp\left(-\frac{\left\|x - x'\right\|^2}{2l^2}\right) \tag{4}$$

where σ_f^2 is the signal variance and l is the length scale. This kernel allows the GPR model to capture both smooth and rapidly changing dynamics, which are critical in underwater environments.

Prediction Framework: The GPR model predicts the forces $\mathbf{F}(\mathbf{t})$ acting on the robot at any time $\boldsymbol{t}$, based on the input sequence of observations. Given a test input $\mathbf{x}_*$, the model provides a Gaussian distribution over possible values of the force, with mean $\boldsymbol{\mu}(\mathbf{x}_*)$ and variance $\boldsymbol{\sigma^2}(\boldsymbol{x}_*)$.

- **Model Training and Hyperparameter Optimization:** The GPR model's hyperparameters, inclu-ding the signal variance $\boldsymbol{\sigma_f^2}$, length scale $\boldsymbol{l}$, and noise variance $\boldsymbol{\sigma_n^2}$, are optimized using maximum likelihood estimation (MLE). The training data consists of real-time sensor measurements from the robot's movements in water, and the hyperparameter tuning ensures that the model accurately captures the underlying dynamics.

- **Prediction:** The output of the GPR model is a set of predicted force vectors $\mathbf{F_1}, \mathbf{F_2}, \ldots, \mathbf{F_L} \in \mathbf{R^2}$, where $\mathbf{L}$ is the prediction length, spanning various time intervals. The model primarily predicts forces in the $\mathbf{x}$ and $\mathbf{y}$ directions, as forces in the $\mathbf{z}$-axis (due to gravity and buoyancy) are assumed to remain constant.

The GPR-based architecture provides robust and adaptive force predictions by leveraging the uncertainty quantification inherent in Gaussian Processes. Unlike traditional machine learning models that offer point estimates, the GPR model offers a distribution of possible outcomes, making it well-suited for underwater environments where sensor noise and dynamic fluid conditions can introduce significant uncertainty.

6. Learning Objective

The learning objective of the GPR model is to minimize the prediction error between the predicted force vectors and the ground truth measurements while accounting for uncertainty. The process involves the following steps:

- **Dataset Preparation:**

The dataset consists of sequences of observation data along with corresponding force measurements. The data is divided into training, validation, and test sets. Formally, the dataset can be represented as:

$$\mathcal{D} \coloneqq \left\{\left(F_0^j, x_0^j\right), \left(F_1^j, x_1^j\right), \ldots, \left(F_L^j, x_L^j\right)\right\}_{j=1}^{M} \tag{5}$$

where $\mathrm{F_i^j}$ is the predicted force at time step i for trajectory j, $\mathrm{x_i^j}$ is the corresponding input observation, L is the total number of time steps per trajectory, and M is the number of trajectories in the dataset.

- **Loss Function:**

In GPR, the model outputs a mean prediction along with a variance estimate for the force vector at each time step. The loss function takes into account both the prediction error and the uncertainty estimate. The negative log marginal likelihood (NLML) is commonly used as the objective function to optimize GPR models, which maximizes the likelihood of the observed data given the predicted mean and covariance.
The NLML loss function is formulated as:

$$\mathcal{L}(\theta) = \frac{1}{2}\mathrm{y}^\top \mathrm{K}_\theta^{-1}\mathrm{y} + \frac{1}{2}\log|\mathrm{K}_\theta| + \frac{\mathrm{n}}{2}\log(2\pi) \tag{6}$$

where $\mathbf{y}$ represents the observed forces, K_θ is the covariance matrix parameterized by θ (which includes the signal variance σ_f^2, length scale l, and noise variance σ_n^2, and n is the number of data points. Minimizing this function allows the model to fit the data while properly accounting for uncertainty.

- **Backpropagation and Gradient Calculation:**

Unlike traditional neural networks, GPR uses analytical gradients for hyperparameter optimization. The gradient of the negative log marginal likelihood with respect to the hyperparameters θ is computed to update the kernel's parameters:

$$\frac{\partial \mathcal{L}(\theta)}{\partial \theta} = \frac{1}{2}\boldsymbol{y}^\top K_\theta^{-1}\frac{\partial K_\theta}{\partial \theta}K_\theta^{-1}\boldsymbol{y} - \frac{1}{2}Tr\left(K_\theta^{-1}\frac{\partial K_\theta}{\partial \theta}\right) \tag{7}$$

This gradient allows for backpropagation through the kernel's hyperparameters, ensuring that the model can adjust the signal variance, length scale, and noise variance to improve predictions.

- **Optimization:**

Hyperparameter optimization is typically carried out using gradient-based optimization algorithms such as the Adam optimizer or L-BFGS. These methods update the hyperparameters of the covariance function by minimizing the NLML loss. During training, the model adjusts the kernel parameters iteratively to maximize the likelihood of the observed data while minimizing prediction uncertainty.

The overall objective of the GPR-based model is not only to minimize the prediction error but also to ensure that the uncertainty estimates are well-calibrated, which is particularly crucial in dynamic environments like underwater robotics.

7. Experiments

7.1. Setup:

To train and evaluate our GPR-based hydrodynamic force prediction model, we conducted a series of controlled towing experiments. These experiments were designed to provide a rich dataset for training, validation, and testing, ensuring the accurate modeling of the robot's interactions with the surrounding fluid. The key components of the experimental setup are described below **[9,10,11]**:

- **Pool Environment:**

All experiments were conducted in a controlled pool environment, ensuring repeatable and consistent hydrodynamic conditions. The water temperature and depth were maintained constant throughout the experiments to avoid external variability in fluid dynamics.

- **Towing Mechanism:**

A specialized towing mechanism was employed, capable of pulling the robot at varying speeds and directions. The towing speeds ranged from 0.2 m/s to 0.5 m/s, with increments of 0.1 m/s. The robot was towed in three primary directions: along the x-axis, y-axis, and diagonally at 45 degrees (xy). These variations allowed us to capture diverse motion scenarios, which are critical for training the GPR model.

- **Force Sensors:**

High-precision force sensors were installed on the robot to capture hydrodynamic forces acting on the robot in real-time. The sensors recorded force data along the x, y, and z axes. Since the GPR model is focused on predicting forces in the x and y directions (with z-axis forces assumed constant), this detailed sensor data provides a comprehensive training dataset.

- **Robot Configuration:**

To simulate different locomotion scenarios, the quadruped robot's limb configurations were varied by adjusting joint angles. The robot was tested under a range of movement patterns, providing sufficient input diversity for the GPR model to learn fluid-structure interactions effectively.

Fig. 2. Robot configuration

The dataset derived from these experiments contains a sequence of kinematic observations (joint angles and velocities) paired with corresponding force measurements. This comprehensive dataset is critical for training the GPR model to predict hydrodynamic forces while quantifying the uncertainty in the predictions.

The dataset was collected during the towing experiments described. These experiments were specifically designed to measure the forces acting on the quadruped robot across 192 distinct towing speeds and joint configurations.

Table 1. Input and output formats of the Datasets

Expr.	**Input**	**Output**
Expr1	Condition x: [batch, 100, 4] and Initial: F_0 [batch, 2]	[batch, 100, 2]
Expr2	Condition x: [batch, 50, 4] and Initial: F_0 [batch, 2]	[batch, 50, 2]
Expr3	Condition x: [batch, 50, 4] and Initial: F_0 [batch, 2]	[batch, 50, 2]

Note: batch refers to the batch size during the training stage.

7.2 Dataset

From **Table 1**, the dataset is augmented in the following ways:

- **Expr1:** This experiment extends the time series to 100-time steps, representing sequential data of the quadruped robot maintaining a constant attitude angle. The Gaussian Process Regression (GPR) model is tasked with predicting the forces in two axial directions (x and y) over these 100-time steps. The focus is on testing the GPR's ability to model the hydrodynamic forces under static conditions.

- **Expr2:** This experiment highlights the comparative predictive capabilities of GPR models under variable conditions for online learning. The temporal length of each condition is reduced to 10-time steps, randomly selected from the dataset. Additionally, a condition variable is concatenated to the input data, varying across five distinct scenarios. This setup allows the GPR model to generalize across different environmental conditions, making it more versatile in predicting forces in dynamic contexts.

- **Expr3:** This experiment addresses the increased complexity of dynamic conditions. Random perturbations are introduced at each time step, with magnitudes equal to 10% of the standard deviation of the respective force values. Similar to Expr2, the trajectories are resampled across 192 distinct conditions to test how well the GPR model adapts to noisy and dynamically changing scenarios. This experiment is key to understanding the robustness of GPR in environments with variable and unpredictable conditions.

7.3 Model Prediction Performance

In the following experiments, we evaluate the performance of different Gaussian Process Regression (GPR) models in predicting dynamic hydrodynamic forces on a quadruped robot. We compare the model's predictions with the ground truth using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Several kernel configurations, including the Radial Basis Function (RBF) and Matern kernels, are tested, along with the addition of different noise levels and varying training data conditions.

Table 2. Performance on Different Conditions

Models	**MAE-S**	**RMSE-S**	**MAE-C**	**RMSE-C**	**MAE-N**	**RMSE-N**
GPR-RBF	9.8e-3	4.0e-3	4.1e-3	5.0e-3	5.3	6.8
GPR-Matern	8.3e-3	3.9e-3	3.5e-3	2.8e-3	4.0	5.3
GPR-RBF (Noisy)	5.4e-3	6.2e-4	2.8e-3	1.5e-3	2.8	4.9
GPR-Matern (Noisy)	**3.7e-4***	**5.8e-4***	**2.4e-4***	**5.1e-5***	**2.0***	**4.0***

Note: The suffix -S indicates static conditions in Expr1, -C denotes conditions that change over time in Expr2, and -N represents noisy and changing conditions in Expr3. The * symbol is used to highlight the best-performing models.

8. Analysis

From Table 2, it is evident that GPR models with different kernel choices perform well under various dynamic conditions. The Matern kernel generally outperforms the RBF kernel, especially when the data includes variability and noise. This is likely due to the Matern kernel's ability to model rougher functions and better capture the underlying complexities of the hydrodynamic forces.

In scenarios with noisy conditions (as in Expr3), the GPR-Matern model demonstrates significantly better performance, with errors as low as 4.0, making it highly suitable for deployment in real-world underwater environments where sensor noise is prevalent.

Additionally, GPR's uncertainty quantification provides a distribution of possible outcomes, which proves advantageous in noisy environments. The GPR-Matern (Noisy) model providing the most accurate and robust predictions.

9. Limitations

Despite the strengths of GPR, the model's reliance on a large amount of training data and its sensitivity to kernel selection can limit real-time applicability. Furthermore, GPR's computational complexity may increase as the dataset grows. Future work will focus on reducing this complexity and testing the model in real-world aquatic environments to validate its robustness and performance.

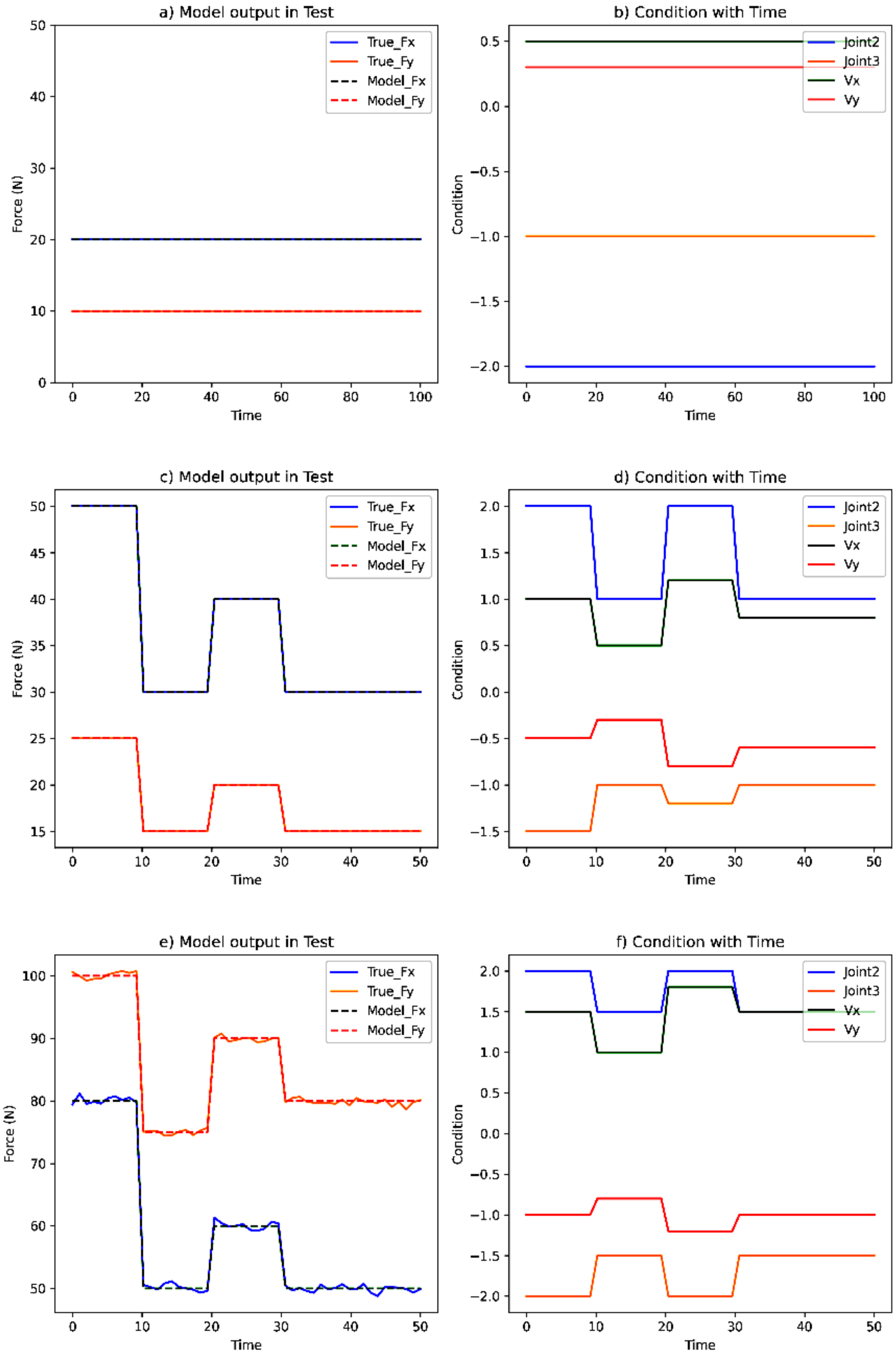

Fig. 3. Model Prediction Performance under the different conditions
with the time sequence length. (a) and (b) present the static force and conditions over time in expr1. (c) and (d) illustrate the change in force and conditions over time in expr2. (e) and (f) depict the noisy orce dynamics and conditions over time in expr3. In (a), (c), and (e), he dot lines mean that the model prediction trajectories

10. Conclusions

This paper introduces a data-driven approach to learning complex hydrodynamic models for underwater quadruped robots using Gaussian Process Regression (GPR). We employed GPR with various kernel functions, such as the Radial Basis Function (RBF) and Matern kernels, to predict dynamic forces based on kinematic trajectories, providing a probabilistic framework for estimating hydrodynamic forces. By leveraging the model's uncertainty quantification, we enhance the robot's adaptability to changing and noisy environments.

The proposed GPR model offers a scalable solution for underwater robotics, balancing computational efficiency and prediction accuracy. Our experiments showed that the GPR model, particularly with the Matern kernel, performs well across varying conditions, including static, dynamic, and noisy environments, with precise predictions of hydrodynamic forces.

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