MEANS FOR MEASURING THE THERMAL QUANTITIES

MINIMAX APPROXIMATION OF THE RESISTANCE-TEMPERATURE THERMISTOR'S DEPENDENCE

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https://doi.org/10.23939/istcmtm2024.04.020

Abstract. The evidence to support feasibility of using minimax approximation to calculate the parameters of thermistors' thermometric characteristic models has been provided. Minimax approximation ensures the achievement of the minimum possible calibration error, while the least squares method minimizes the sum of squared errors. A rational expression has been proposed to describe the dependence of temperature on thermistor's resistance. The effectiveness of the model in the form of a rational expression is illustrated with actual calibration results. This model provides higher accuracy in reproducing temperature compared to generalized Steinhart-Hart models.

Key words: Thermistor, Thermometric Characteristic, Least Squares Method, Minimax Approximation, Rational Expression, Generalized Steinhart-Hart Model

1. Introduction

Improvements in thermistors manufacturing technology widen their practical use for temperature measurement and monitoring. In addition to the conventional use of thermistors in biomedical devices, automotive electronic systems, and other technical devices [1–5], thermistors have started to be used for high-precision [6–9] and cryogenic temperature measurements [8, 10, 11]. They are now utilized in aircraft manufacturing, space, and other high-tech industries [1]. The expanding applications of thermistors drive the search for more accurate and optimal models of their thermometric characteristics [1, 6, 12].

2. Drawbacks

The practical application of thermistors is limited by noticeable nonlinearity of their thermometric characteristic [13]. To describe the dependence of the temperature on the resistance, the generalized Steinhart-Hart model with parameters calculated by the least squares method is mostly used [6, 12]. In [14], minimax approximation is proposed to calculate the parameters of the thermistor's thermometric characteristic model. The use of minimax approximation to calculate the model parameter values of the thermometric characteristic ensures the minimum possible calibration error within the investigated range [15]. Models calculated using the least squares method may contain particular points where the error value significantly exceeds the average error [16]. The model obtained by the least squares method does not provide an answer regarding the accuracy of calibration results reproduction. Therefore, to substantiate the accuracy of models obtained by the least squares method, their uncertainty has to be calculated [12, 17].

3. Goal

The aim of this study is to provide evidence to support the feasibility of using minimax approximation to calculate the parameter values of the thermistor's thermometric characteristic, and to illustrate the effectiveness of using a rational expression to describe the dependence of temperature on the thermistor's resistance.

4. Application of minimax approximation for calculating the parameters of the thermistor's thermometric characteristic

To confirm the effectiveness of using minimax approximation for calculating the parameters of thermistor thermometric characteristic models, we have used calibration results provided in [6]. This study presents calibration results obtained in May 2014 and February 2015 at the PTB, Germany. We have illustrated the comparison between the results of applying minimax approximation and the least squares method based on a four-parameter generalization of the Steinhart-Hart model [6]

$$T_4(r)=1\Big/\sum_{i=0}^{3} c_i\left(ln(r)\right)^3, \qquad (1)$$

where T – the absolute temperature, $r=R_T/R_s$ – the relative resistance at temperature T, R_T – the resistance at temperature T, $R_s=1001.65\Omega$, and c_i, $i=\overline{0,3}$ – unknown parameters. In [6], the feasibility of using model (1) for the thermometric characteristic of thermistors is indicated. Compared to the Steinhart-Hart model, where $c_2=0$, the authors of [12, 17] claim model (1) is more accurate. The parameter values c_i, $i=\overline{0,3}$ of the model (1) in [12, 17] are calculated using the least squares method.

Based on the calibration results provided in [6], the thermistor's thermometric characteristic model in the form (1) obtained using the least squares method

$$T_{4,14,lsq}(r)=1\Big/\left(\begin{array}{l}6.1510417037_{10}^{\ -8}x^3+2.26274538698_{10}^{\ -6}x^2+\\ \quad +2.56520635435_{10}^{\ -4}x+3.35437275588_{10}^{\ -3}\end{array}\right) \qquad (2)$$

ensures the reproduction of their values with an absolute error of 0.94 mK based on the calibration data obtained in May 2014.

$$T_{4,15,lsq}(r)=1\Big/\left(\begin{array}{l}6.5985545833347_{10}^{\ -8}x^3+2.267036730985_{10}^{\ -6}x^2+\\ \quad +2.565168779395_{10}^{\ -4}x+3.3543781818_{10}^{\ -3}\end{array}\right) \qquad (3)$$

Then, for the calibration data obtained in February 2015 with an error of 1.14 mK , the models (2) and (3) $r=R_T/R_s$, a $x=ln(r)$.

A similar model, calculated using minimax approximation based on the data from May 2014 is in the form:

$$T_{4,14,minm}(r)=1\Big/\left(\begin{array}{l}5.78216036867_{10}^{\ -8}x^3+2.2566107516_{10}^{\ -6}x^2+\\ \quad +2.5652396097358_{10}^{\ -4}x+3.3543783803395_{10}^{\ -3}\end{array}\right) \qquad (4)$$

reproduces with an absolute error of 0.64 mK, while the model based on the calibration data from February 2015

$$T_{4,15,minm}(r)=1\Big/\left(\begin{array}{l}5.95286858_{10}^{\ -8}x^3+2.268338145697_{10}^{\ -6}x^2+\\ \quad +2.5652822930196_{10}^{\ -4}x+3.354376775298_{10}^{\ -3}\end{array}\right) \qquad (5)$$

ensures temperature reproduction with an absolute error of 0.879 mK.

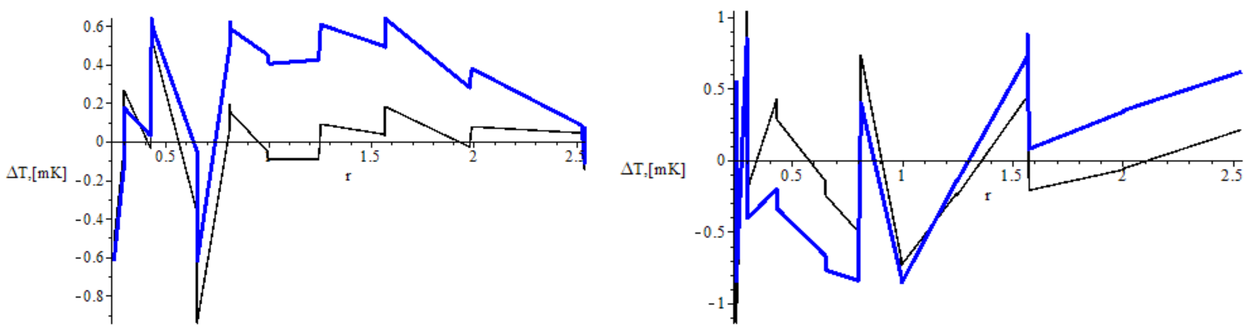

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Fig. 1. The graphs of absolute error: a) models (2) i (4), b) models (3) and (5)

Table 1. The errors in reproducing the calibration results obtained for the models (2) – (5)

Calibration data	Model (2)		Model (4)
May 2014	$r=0.653541$	$\Delta T=0.942$ mK	$\Delta T=0.64$ mK
	Model (3)		Model (5)
February 2015	$r=0.249662$	$\Delta T=1.136$ mK	$\Delta T=0.879$ mK

Error plots of models (2)–(5) are shown in Fig. 1. The error plots for models (4) and (5), with parameters' values calculated using minimax approximation, are distinguished with thicker lines.

From the graphs presented in Fig. 1, it follows that the error of the models the parameters of which were calculated using minimax approximation is uniformly distributed across the entire investigated range of resistance changes. In contrast, for the least squares method, there are particular points where the error in reproducing the calibration results is significantly greater than the average error. The comparison of the errors in reproducing the calibration results of the thermistor models (2) – (5) is presented in Table 1.

Thus, the use of minimax approximation for calculating the parameters of the thermometric characteristic models of the thermistor, compared to the least squares method, ensures higher accuracy in reproducing temperature within the investigated range. The error of the model the parameters of which were calculated using the minimax criterion is equivalent to the accuracy of the reproduction of the calibration results. To substantiate the accuracy of the models constructed using the least squares method, uncertainty is additionally calculated. Therefore, applying minimax approximation for parameter calculation of models based on high-precision data is more appropriate.

5. The application of the rational expression for modeling the thermometric characteristics of thermistors

Based on the results of the thermistor calibration [6], we constructed a model of the thermometric characteristics of the thermistor in the form of a rational expression

$$T(r)=\frac{a_0+a_1\,ln(r)+a_2\,ln(r)^2}{b_0+ln(r)},\qquad (6)$$

where T – the absolute temperature in kelvins, $r=R_T/R_s$ – the relative resistance, and a_0, a_1, a_2 and b_0 are unknown parameters. The values of the parameters of model (6) were calculated using minimax approximation. The method for calculating the parameters of the minimax approximation based on a rational expression is described in [18, 19].

The model of the thermometric characteristics for the calibration data from May 2014 is written as:

$$T_{14,minm}(r)=\frac{4268.786635-28.33192273x-0.7000487623x^2}{14.31912375+x},\qquad (7)$$

where $r=R_T/R_s$, $x=ln(r)$, ensures the reproduction of temperature with an absolute error of 0.622 mK. To confirm the effectiveness of using minimax approximation for calculating the parameters of model (7), we constructed a model similar to (6), where the parameters are calculated using the least squares method.

$$T_{14,lsq}(r)=\frac{4260.2745447-27.68140037x-0.7542877542x^2}{14.290550574+x}.\qquad (8)$$

Model (8) ensures the reproduction of temperature with an absolute error of 0.978 mK.

For the calibration data from February 2015, the model (9) was obtained:

$$T_{15,minm}(r)=\frac{4290.51969658-30.0016619429x-0.589193411759x^2}{14.2907771854+x}\qquad (9)$$

Model (9) ensures the reproduction of temperature with an absolute error of 0.73 mK. The model, based on the calibration data from February 2015, the parameters of which were calculated using the least squares method:

$$T_{15,lsq}(r)=\frac{4260.3340722-27.6845505038x-0.754059735366x^2}{14.2907771854+x}\qquad (10)$$

ensures the reproduction of temperature with an absolute error of 0.96 mK.

The graphs of the absolute error for models (7) – (10) are shown in Fig. 2.

In the graph shown in Fig. 2, the horizontal axis represents the values of $r=R_T/R_s$, while the vertical axis represents the values of the absolute error in temperature reproduction by the models. The graphs of the absolute error for models (7) and (9) are depicted with thicker lines. From the shape of the absolute error graphs for models (7) and (9), it follows that the error in reproducing the temperature values is uniformly distributed across the entire range of investigation. The error of model (7) does not exceed 0.622 mK, while the model (9) exhibits an error of 0.73 mK. The absolute error of the model (8), parameters of which were calculated using the least squares method, is 0.978 mK, and for the model (10), it is 0.96 mK, which is significantly higher than the average error in the investigated range. Therefore, when using the least squares method, the quality of the model is characterized by uncertainty [12], [17]. In contrast, the quality of the model, the parameters of which were calculated using the minimax method, is determined by the approximation error [15].

A comparison of the errors in reproducing the calibration results of the thermistor using models (7) – (10) is presented in Table 2.

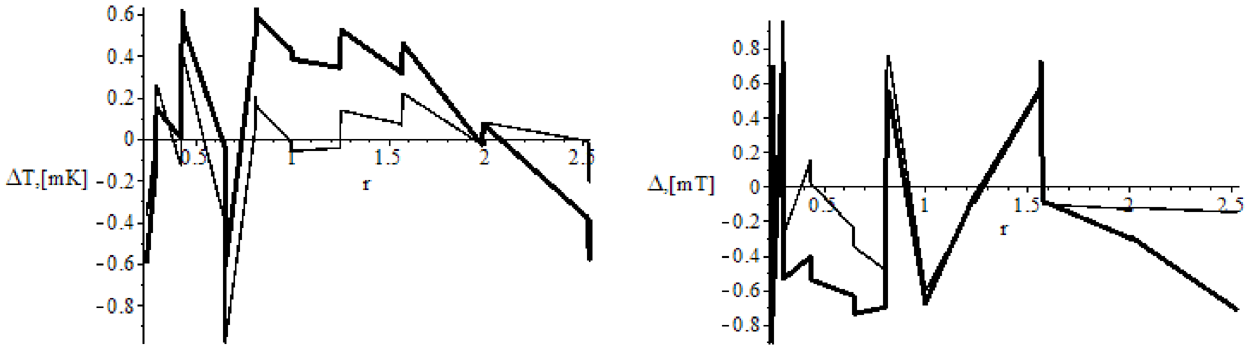

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Fig. 2. The graphs of absolute error: a) models (7) and (8), b) models (9) and (10)

Table 2. The errors in reproducing the calibration results of the thermistor using models (7) – (10) and the models (4) та (5)

Calibration data	Model (8)		Model (7)	Model (4)
May 2014	$r = 0.653541$	$\Delta T = 0.978$ mK	$\Delta T = 0.622$ mK	$\Delta T = 0.64$ mK
	Model (10)		Model (9)	Model (5)
February 2015	$r = 0.2984739$	$\Delta T = 0.96$ mK	$\Delta T = 0.73$ mK	$\Delta T = 0.879$ mK

From the results presented in Table 2, it follows that the models of the thermometric characteristics of the thermistor in the form of the rational expression (6) reproduce the calibration results with higher accuracy than the generalized Steinhart-Hart model (1).

5. Conclusions

The application of minimax approximation for calculating the parameter values of thermistors' thermometric characteristic models ensures the achievement of the smallest possible error in reproducing the calibration results. The error of the model, the parameters of which are calculated using the minimax criterion, is equivalent to the accuracy of reproducing the calibration results. Therefore, compared to the models constructed using the least squares method, there is no need to compute uncertainty.

The use of a rational expression for modeling the thermometric characteristics of thermistors provides higher accuracy in reproducing calibration results compared to the traditionally used generalized Steinhart-Hart model. The results of the parameter calculations for this model also confirm the effectiveness of using minimax approximation.

References

[1] D.R. White, K. Hill, D. del Campo, C. Garcia Izquierdo, Guide on Secondary Thermometry – Thermistor Thermometry, BIPM, Consultative Committee for Thermometry (CCT), 2014.

[2] Hughes-Riley T., Lugoda P., Dias T., Trabi C.L., Morris R.H. A Study of thermistor performance within a textile structure. *Sensors*. 2017. Vol. 17, Iss. 8. Article number 1804. https://doi.org/10.3390/s17081804.

[3] K. Esenowo Jack, E. O. Nwangwu, I. Agwu Etu, E. U. Osuagwu, "A simple thermistor design for industrial temperature measurement," IOSR J. Electr. Electron. Eng., 11, 05, 57–66, 2016. https://doi.org/10.9790/1676-1105035766.

[4] Li Ting, Wang Hongjun, Sun Jianping, Hao Xiaopeng, Wang Chengke, Yu Jiao. Exploration of Calibration Methods for Ocean NTC Thermistor Thermometers. *J. Metrology Science and Technology*, 2021, 65(5): 55-61. https://doi.org/10.12338/j.issn.2096-9015.2020.9027.

[5] Damir Ilić, Josip Butorac, Luka Ferković Temperature measurements by means of NTC resistors and a two-parameter approximation curve, Measurement. Vol. 41, no. 3, April 2008, pp. 294-299. https://doi.org/10.1016/j.measurement.2006.11.007 .

[6] Steffen Rudtsch, Christoph von Rohden, "Calibration and self-validation of thermistors for high-precision

temperature measurements," *Measurement: Journal of the International Measurement Confederation*, 2015, 76, pp. 1–6. https://doi.org/10.1016/j.measurement.2015.07.028 .

[7] Guang Liu, Liang Guo, Chunlong Liu, Qingwen Wu, Evaluation of different calibration equations for NTC thermistor applied to high-precision temperature measurement, *Measurement*, Vol. 120, 2018. pp. 21-27. https://doi.org/10.1016/j.measurement.2018.02.007.

[8] V. F. Mitin et al., "Nanocrystalline SiC Film thermistors for cryogenic applications," *Review of Scientific Instruments*. Vol. 89, no. 2, Feb 2018, pp. 025004-1–025004-5. https://doi.org/10.1063/1.5024505.

[9] D. J. Li, C. G. Xing, "Linearization of high precision human body temperature measurement circuit based on ntc $_{\text{thermistor}}$," *Appl. Mech. Mater*., 713–715, 381–384, 2015. https://doi.org/*10.4028/www.scientific.net/AMM.713-715.381*.

[10] X. Gong, X. Tang, Y. Li, M. Y. Zude Lin and J. Liu, "Investigation the Minimum Measurement Points for Calibration a High Precision NTC Thermistors in Cryogenic Field," *2021 IEEE 16th International Conference on Nano/Micro Engineered and Molecular Systems (NEMS)*, Xiamen, China, 2021, pp. 1088-1091, https://doi.org/10.1109/NEMS51815.2021.9451310.

[11] Lan, Y. , Yang, S. , Chen, G. and Yang, S. (2020) Characteristics of a Type of NTC Thermistors for Cryogenic Applications. *Advances in Materials Physics and Chemistry*, **10**, 167-177. https://doi.org/10.4236/ampc.2020.108012.

[12] Chiachung Chen Evaluation of resistance–temperature calibration equations for NTC thermistors. *Measurement*, Vol. 42. no. 7. 2009, pp. 1103-1111. https://doi.org/10.1016/j.measurement.2009.04.004.

[13] A. Li, H. Wang, L. Dong, Q. Han, M. Wen and X. Wen, "Research on the Adaptability of Thermistor Calibration Equations," in *IEEE Instrumentation & Measurement Magazine*, vol. 26, no. 2, pp. 13-18, April 2023, https://doi.org/10.1109/MIM.2023.10082999.

[14] R. P. Malachivskyi, R. A. Bun, O. V. Shevchuk, "Approximation of Thermometric Characteristics of Thermistors," *Computer Technologies of Printing*, no. 2 (50), 2023. C. 87-97. https://doi.org/10.32403/2411-9210-2023-2-50-87-97, (in Ukrainian).

[15] Remez E. Ya. Fundamentals of the Numerical Methods of Chebyshev Approximation. Naukova Dumka, Kyiv (1969), (in Russian).

[16] V. A. Yatsuk and P. S. Malachivskyy, Methods to Improve Measurement Accuracy, Beskid Bit, Lviv (2008), (in Ukrainian).

[17] White, D. R. Interpolation Errors in Thermistor Calibration Equations. *Int J Thermophys* **38**, 59 (2017). https://doi.org/10.1007/s10765-017-2194-x.

[18] P. S. Malachivskyy, Y. V Pizyur, R. P. Malachivskyi, "Chebyshev approximation by a rational expression for functions of many variables," *Cybernetics and Systems Analysis*, Vol. 56. no. 5, 2020. pp. 811-819. https://doi.org/10.1007/s10559-020-00302-0.

[19] P. S. Malachivskyy, L. S. Melnychok, Y. V Pizyur, "Chebyshev Approximation of Multivariable Functions by a Constrained Rational Expression," Cybernetics and Systems Analysis, Vol. 59. no. 1, 2023. pp. 146–159. https://doi.org/10.1007/s10559-023-00552-8.