COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 14, No. 1, 2024

MATHEMATICAL MODEL OF TRANSIENT PROCESSES OF DC MOTOR WITH PERMANENT-MAGNET EXCITATION

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https://doi/org/10.23939/jcpee2024.01.012

Abstract: Permanet magnet DC motors (PM DCM) have an advantage over electromagnetically excited commutator motors due to their better energy performance. The application of analytical methods to study PM DCM requires significant simplifications, and the use of field methods is quite labour-intensive. In our opinion, the use of electric and magnetic circuit methods makes it possible to achieve the accuracy of calculating the PM DCM that is sufficient for engineering needs. The purpose of the article is to develop a mathematical model of transient processes in PM DCM based on the use of the theory of electric and magnetic circuits. The article proposes an equivalent scheme for the magnetic circuit of PM DCM and a system of equations describing it. There are given equations for transient processes in PM DCM and an algorithm for their solution, which involves the integration of the solution of the system of equations of the magnetic state at each step. The proposed mathematical model of transient processes in PM DCM can be used to analyze these processes, as well as in design.

Key words: direct current motor, permanent-magnet excitation, an equivelent scheme of the magnetic circuit, equation of the magnetic state, transient processes.

1. Introduction

PM DCM have an advantage over electromagnetically excited commutator motors due to the absence of losses in the excitation winding, which leads to increased efficiency and better use of active materials. The use of permanent-magnet excitation makes it possible to simplify the design and reduce the cost of manufacturing DC micromotors. PM DCM are widely used in the automotive industry, toys, electrical household appliances, portable power tools, etc.

The fundamental work [1] considers the use of permanent magnets in electric machines, including PM DCM. The ratios for calculating the overall dimensions of a disk-type magnetoelectric DC motor are given in [2]. For a built-in DC torque motor with permanent-magnet excitation, methods of increasing the maximum torque are proposed in [3]. Methods for reducing the impact of the armature reaction at the initial moment of starting the PM DCM of the automobile starter using the finite element method are considered in [4]. The influence of the angle of brush displacement from the neutral on the characteristics of the PM DCM using the finite element method is studied in [5].

Analytical research methods [2, 3] that require adopting assumptions for simplification, are the least accurate, but allow obtaining formulas suitable for engineering design. The use of field methods [1, 4, 5] makes it possible to achieve the highest accuracy, but it is quite labour-intensive. In our opinion, accuracy sufficient for engineering practice can be achieved using methods of electric and magnetic circuits. Such an approach is proposed in [6] to study the magnetic state of a shaded-pole induction motor using a branched equivalent scheme of the magnetic circuit. The general approach to the analysis of the PM DCM based on the equivalent scheme of the magnetic circuit is presented in [7], and the branched equivalent scheme is detailed in [8]. [9] presents an experimental verification of the adequacy of the calculation of the PM DCM magnetic circuit.

2. Equivalent scheme of a magnetic circuit and equation of magnetic state

For the design of the PM DCM with a radially magnetized permanent magnet in the form of a parallelepiped, [9] provides an equivalent scheme of a magnetic circuit with concentrated parameters (Fig. 1). In this scheme, the sections of the air gap correspond to permanent magnetic resistances, and ferromagnetic sections correspond to nonlinear magnetic resistances, represented by the dependence of the magnetizing force on the magnetic flux $F\,[\Phi]$.

The active zone of the armature under the magnet within the pole division is divided by radial planes into m sections; for the scheme shown in Fig. 1, $m = 5$.

The system of nonlinear equations of the magnetic state, composed by the method of contour fluxes, corresponds to the equivelent scheme of the magnetic circuit of the PM DCM. In this system, the primary unknowns are the contour magnetic fluxes, and the secondary ones are the magnetic fluxes in the branches of the circuit.

If, for nonlinear magnetic resistances, the ratio between contour fluxes and fluxes in branches is taken into account, then the system of equations of the magnetic state will have the form (1). This system contains (*m*–1) equations of armature, two equations of the stator circuits, and one equation containing the elements of the stator and armature, i.e. a total of (*m*+2) equations.

$$\begin{aligned}
&R_{\delta 1}(\Phi_{cr1}-\Phi_{cs1})+R_{\delta 2}(\Phi_{cr1}-\Phi_{cr2})-\\
&-F_{z1}[\Phi_{\delta 1}]+F_{z2}[\Phi_{\delta 2}]-F_{ar1}[\Phi_{ar1}]=T_2-T_1;\\
&R_{\delta 2}(\Phi_{cr2}-\Phi_{cr1})+R_{\delta 3}(\Phi_{cr2}-\Phi_{cr3})-F_{z2}[\Phi_{\delta 2}]+\\
&+F_{z3}[\Phi_{\delta 3}]-F_{ar2}[\Phi_{ar2}]=T_3-T_2;\\
&R_{\delta 3}(\Phi_{cr3}-\Phi_{cr2})+R_{\delta 4}(\Phi_{cr3}-\Phi_{cr4})-\\
&-F_{z3}[\Phi_{\delta 3}]+F_{z4}[\Phi_{\delta 4}]-F_{ar3}[\Phi_{ar3}]=T_4-T_3;\\
&R_{\delta(m-1)}\times\left(\Phi_{cr(m-1)}-\Phi_{cr(m-2)}\right)+\\
&+R_{\delta m}\times\left(\Phi_{cr(m-1)}+\Phi_{cs1}\right)-\\
&-F_{z(m-1)}\left[\Phi_{\delta(m-1)}\right]+F_{zm}[\Phi_{\delta m}]-\\
&-F_{ar(m-1)}\left[F_{ar(m-1)}\right]=T_m-T_{(m-1)}.\\
&R_{\sigma s}(\Phi_{cs1}-\Phi_{cs2})+R_{\sigma s}(\Phi_{cs1}+\Phi_{cs3})+\\
&+R_{\delta 1}(\Phi_{cs1}-\Phi_{cr1})+R_{\delta m}\left(\Phi_{cs1}+\Phi_{cr(m-1)}\right)+\\
&+F_{z1}[\Phi_{\delta 1}]-F_{zm}[\Phi_{\delta m}]-F_{ar}[F_{ar}]+F_{s1}[\Phi_{s1}]=\\
&=T_1+T_m;\\
&R_{\sigma s}(\Phi_{cs2}-\Phi_{cs1})+F_{s2}[\Phi_{s2}]+R_m(\Phi_{cs2}-\Phi_{cs3})=F_m;\\
&R_{\sigma s}(\Phi_{cs3}+\Phi_{cs1})+F_{s3}[\Phi_{s3}]+R_m(\Phi_{cs3}-\Phi_{cs2})=\\
&=-F_m.
\end{aligned}\qquad(1)$$

When solving the system of equations of the magnetic state, it is necessary to know the ratio between the fluxes in the branches and the contour fluxes.

Let's create column vectors of contour fluxes in the form of

$\vec{\Phi}_{cr}=\left(\Phi_{cr1},\Phi_{cr2},\ldots,\Phi_{cr(m-1)}\right)_*$ – a column vector of rotor's contour fluxes;

$\vec{\Phi}_{cs}=\left(\Phi_{cs1},\Phi_{cs2},\Phi_{cs3}\right)_*$ – a column vector of stator's contour fluxes;

$$\vec{\Phi}_1=\left(\Phi_{cp1},\Phi_{cp2},\ldots,\Phi_{cp(m-1)},\Phi_{cs1},\Phi_{cs2},\Phi_{cs3}\right)_*$$

$$\vec{\Phi}_1=\left(\Phi_{cr1},\Phi_{cr2},\ldots,\Phi_{cr(m-1)},\Phi_{cs1},\Phi_{cs2},\Phi_{cs3}\right)_*$$

– a column vector of contour fluxes, or primary unknowns of size (*m* + 2).

Let's create column vectors of fluxes in branches and give the matrices that make it possible to go from contour fluxes to fluxes in branches

$\vec{\Phi}_\delta=\left(\Phi_{\delta 1},\Phi_{\delta 2},\ldots,\Phi_{\delta m}\right)_*$ is a column vector of fluxes in air gap sections;

$$\vec{\Phi}_\delta=c_\delta\vec{\Phi}_1,\qquad(2)$$

where $c_\delta=\begin{matrix}-1&0&0&0&1&0&0\\1&-1&0&0&0&0&0\\0&1&-1&0&0&0&0\\0&0&0&\text{O}&0&0&0\\0&0&0&1&1&0&0\end{matrix}$ – the matrix of connections with a size of $m\times(m+2)$;

$\vec{\Phi}_s=\left(\Phi_{s1},\Phi_{s2},\Phi_{s3}\right)_*$ is a column vector of fluxes in the sections of the stator yoke;

$$\vec{\Phi}_s=c_s\vec{\Phi}_{cs};\ c_s=\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix};$$

$$\vec{\Phi}_\sigma=\left(\Phi_{\sigma 1},\Phi_{\sigma 2}\right)_*;\ \vec{\Phi}_\sigma=c_\sigma\vec{\Phi}_{cs};\ c_\sigma=\begin{matrix}-1&1&0\\-1&0&-1\end{matrix};$$

$\Phi_m=\Phi_{cs2}-\Phi_{cs3}$;

$\vec{\Phi}_{ar}=\left(\Phi_{ar},\Phi_{ar1},\Phi_{ar2},\ldots,\Phi_{ar(m-1)}\right)_*$ is a column vector of fluxes in the sections of the rotor yoke, which contains m elements;

$$\vec{\Phi}_{ar}=c_{as}\vec{\Phi}_{cs}+c_{ar}\vec{\Phi}_{cr*}$$

where

$c_{as}=\begin{matrix}-1&0&0\\0&0&0\\0&0&0\\\vdots&\vdots&\vdots\\0&0&0\end{matrix}$ – a matrix of connections with dimension $m\times 3$;

$c_{ar}=\begin{matrix}0&0&0&0\\-1&0&0&0\\0&-1&0&0\\0&0&\text{O}&0\\0&0&0&-1\end{matrix}$ –a matrix of connections with dimension $m\times(m-1)$.

We will present the column vector of fluxes in branches as

$$\vec{\Phi}_2 = \left(\vec{\Phi}_\delta, \vec{\Phi}_s, \vec{\Phi}_{ar}, \vec{\Phi}_\sigma, \Phi_m\right)_*$$

Let us write the relationship between the fluxes in the branches and the contour vector equation

$$\vec{\Phi}_2 = f\,[\vec{\Phi}_1]$$

Let us create column vectors of the differences between the magnetizing forces of the armature sections and the magnetizing forces of the magnet with a size (m+2) elements

$\overrightarrow{\Delta T_a} =$

$= \left(T_1 - T_2, T_2 - T_3, \ldots, T_{(m-1)} - T_m, -T_1 - T_m, 0, 0\right)_*$;

$\vec{F}_m = \left(0, 0, \ldots, -F_m, F_m\right)_*$.

Let us write the magnetizing force of the *i*-th armature branch of with coordinate η_i in the form [10]

$$T_i = T_{a\max}\left(2\,\eta_i / \tau\right); \qquad T_{a\max} = \left(N / 4p\right)\cdot\left(i_a / 2_a\right), \tag{3a,b}$$

where η_i is the angle between the *i*-th branch of the armature and the pole axis; τ is the pole division; i_a represents the motor armature current; *N* is the total number of conductors of the armature winding; 2*a* stands for the number of parallel branches; *p* is the number of pairs of poles.

Let us transform (3 a, b) into the form

$$T_i = \frac{N}{2p}\frac{1}{2a}\frac{1}{\tau}\,i_a\eta = \kappa_T i_a \eta_i, \tag{4}$$

where $k_T = \frac{N}{2p}\frac{1}{2a}\frac{1}{\tau}$ – constant coefficient for this PM DCM.

In the (m – 1) equations of the armature circuits and in the equation with the elements of the stator and armature circuits, there are differences in the magnetizing forces of the armature branches, which will be presented in the form

$$\Delta T = T_i - T_{(i+1)} = k_t i_a\left(\eta_i - \eta_{(i+1)}\right) \tag{5}$$

Let us create a column vector of armature's branch coordinates

$$\vec{\eta} = \left(\eta_1, \eta_2, \ldots, \eta_m, 0, 0\right)_*$$

Then the column vector of the differences in the magnetizing forces of the armature sections will be written in the form

$$\overrightarrow{\Delta T_a} = k_t i_a c_t\,\vec{\eta}$$

where $c_t = \begin{matrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{O} & \mathbf{O} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$ is a square matrix of size (m+2).

Therefore, the nonlinear system of equations of the magnetic circuit of the PM DCM, written according to Kirchhoff's laws, in vector form will have the form

$$f\left[\vec{\Phi}_1, \vec{\Phi}_2\right] + \overrightarrow{\Delta T_a} + \vec{F}_m = 0, \quad \vec{\Phi}_2 = f\,[\vec{\Phi}_1]. \tag{6a, b}$$

Let us apply Newton's iterative method to solve this system. By substituting the linear vector equation (6b) into (6a), secondary unknowns $\vec{\Phi}_2$ can be excluded, but then we will obtain a cumbersome and inconvenient system to solve.

We will use the algorithm given in [11] to solve system (6a,b). The linear vector equation generated by system (6a, b) of nonlinear vector equations at the ***j***-th iteration will have the form

$$A^{(j-1)}\cdot\Delta\vec{\Phi}_1^{(j)} = -\vec{H}_1^{(j-1)}, \tag{7}$$

where $\Delta\vec{\Phi}_1^{(j)}$ is the correction of the root at the ***j***-th iteration;

$\vec{H}_1^{(j-1)} = f\left[\vec{\Phi}_1^{(j-1)}, \Phi_2^{(j-1)}\right] + \overrightarrow{\Delta T_a} + \vec{F}_m$ is the value of the discrepancy calculated for the (j–1)th approximation of the unknowns $\vec{\Phi}_1$ and $\vec{\Phi}_2$

$$A^{(j-1)} = m_1^{(j-1)} + m_2^{(j-1)}\cdot m_{12}^{(j-1)}, \tag{8}$$

– the value of the Jacobi matrix calculated for the (***j–1***)th approximation of the vectors $\vec{\Phi}_1$ and $\vec{\Phi}_2$

$$m_1^{(j-1)} = \frac{df\left[\vec{\Phi}_1^{(j-1)}, \Phi_2^{(j-1)}\right]}{d\vec{\Phi}_1};$$

$$m_2^{(j-1)} = \frac{df\left[\vec{\Phi}_1^{(j-1)}, \Phi_2^{(j-1)}\right]}{d\vec{\Phi}_2}; \quad m_{12}^{(j-1)} = \frac{d\vec{\Phi}_2\left[\vec{\Phi}_1^{(j-1)}\right]}{d\vec{\Phi}_1}.$$

is the value of the derivatives calculated for the (***j–1***)-th approximation of the vectors $\vec{\Phi}_1$ and $\vec{\Phi}_2$.

To obtain the (***j–1***)-th approximation of the root $\vec{\Phi}_1$, we use the formula

$$\vec{\Phi}_1^{(j)} = \vec{\Phi}_1^{(j-1)} + \Delta\vec{\Phi}_1^{(j)}. \tag{9}$$

3. Equations of transient processes and algorithm for their solution

To calculate the transient processes in PM DCM, it is necessary to integrate numerically the system of differential equations containing the equation of voltage balance and the equation of armature motion. We will use the explicit method of numerical integration, which involves solving the system of equations of the magnetic state (5 a, b) at each step of integration, i.e. finding the fluxes in the branches of the equivalent scheme. Based on these data, we determine the working magnetic flux of the air gap, the electromotive force of the armature winding, the electromagnetic moment and the differential inductance of the armature winding – the quantities that are included in the equations of the voltage and moment balance.

Let us write the voltage equation of the PM DCM in the form

$$\frac{d\Psi_a}{dt} + i_a\, r_a + e_a - u_a = 0\,, \qquad (10)$$

where Ψ_a is the total flux coupling of the armature winding; r_a is the total resistance of the armature circuit; e_a is the electromotive force of the armature winding; u_a is the applied voltage.

The electromotive force of the armature winding is determined by the well-known formula [10]

$$e_a = c_M \cdot \omega \cdot \Phi_\delta\,, \qquad (11)$$

where ω is the angular frequency of rotation of the armature; Φ_δ is the total flux in the air gap; $c_M = (p\,N)/(2\,\pi\,a)$ is the coefficient constant for a given PM DCM.

The flux Φ_δ is equal to the sum of the fluxes of individual sections of the air gap

$$\Phi_\delta = l_\delta \int_0^\pi B_\eta d_\eta = l_\delta \sum_{i=1}^{m} B_\eta\, \Delta\eta_i = \sum_{i=1}^{m} \Phi_{\delta i}\,, \qquad (12)$$

where l_a is the active length of the armature steel; B_η is the magnetic induction of the area of the air gap with the coordinate η; $\Delta\eta_i$ is an arc that corresponds to the ***i***-th section of the air gap.

Let us present the total flux coupling of the armature winding as the sum of the flux coupling caused by the field in the air gap $\Psi_{a\delta}$ and the flux coupling of the scattering fields $\Psi_{a\sigma}$

$$\Psi_a = \Psi_{a\delta} + \Psi_{a\sigma}\,, \qquad (13)$$

where $\Psi_{a\sigma} = L_{a\sigma} i_a$; $L_{a\sigma} = \lambda_{a\sigma}\, l_a$; $L_{a\sigma}$ is the leakage inductance of the armature winding, $\lambda_{a\sigma}$ is the specific leakage inductance of the armature winding, which is determined according to known formulas [10].

Differentiating (13) with respect to time *t*, we obtain

$$\frac{d\Psi_a}{dt} = \frac{d\Psi_{a\delta}}{dt} + L_{a\sigma}\frac{di_a}{dt}\,. \qquad (14)$$

Let us transform equation (14) into the form

$$\begin{aligned} &\frac{d\Psi_{a\delta}}{dt} = \frac{\partial\Psi_{a\delta}}{\partial i_a}\frac{di_a}{dt} = L_{a\delta}\frac{di_a}{dt}; \\ &\frac{d\Psi_a}{dt} = L_{a\delta}\frac{di_a}{dt} + L_{a\sigma}\frac{di_a}{dt} = \left(L_{a\delta} + L_{a\sigma}\right)\frac{di_a}{dt}. \end{aligned} \qquad (15)$$

Taking (15) into account, let us transform equation (10) into a form convenient for numerical integration

$$\frac{di_a}{dt} = \frac{1}{\left(L_{a\delta} + L_{a\sigma}\right)}\left(u_a - i_a\, r_a - e_a\right). \qquad (16)$$

Let us write the flux coupling of the armature winding, due to the working flux, in the form

$$\Psi_{a\delta} = \frac{N}{2a}\Phi_\delta\,. \qquad (17)$$

Taking into account (2), we obtain

$$\Phi_\delta = \sum_{i=1}^{m}\Phi_{\delta i} = c_\Psi \cdot \vec{\Phi}_\delta = c_\Psi\left(c_\delta \cdot \vec{\Phi}_1\right), \qquad (18)$$

where $c_\Psi = (1,\ 1,...1)$ is a row vector consisting of *m* single elements.

From (17) and (18) we write

$$L_{a\delta} = \frac{d\Psi_{a\delta}}{di} = \frac{N}{2a} c_\Psi \left(c_\delta \cdot \frac{d\vec{\Phi}_1}{di}\right). \qquad (19)$$

Therefore, to determine the differential inductance $L_{a\delta}$ at each step of integration, it is necessary to find the derivative $d\vec{\Phi}_1 / di$. Differentiating system (6 a, b) with respect to the current i_a gives

$$\frac{d\vec{f}\left[\vec{\Phi}_1, \vec{\Phi}_2\right]}{d\vec{\Phi}_1} \cdot \frac{d\vec{\Phi}_1}{di_a} + \frac{d\vec{f}\left[\vec{\Phi}_1, \vec{\Phi}_2\right]}{d\vec{\Phi}_2} \cdot \frac{d\vec{\Phi}_2}{di_a} + \kappa_T \cdot c_T \cdot \vec{\eta} = 0 \qquad (20a)$$

$$\frac{d\vec{\Phi}_2}{di_a} - \frac{d\vec{\Phi}_2}{d\vec{\Phi}_1} \cdot \frac{d\vec{\Phi}_1}{di_a} = 0\,. \qquad (20b)$$

After transformation (20 a,b) we obtain

$$A \cdot \frac{d\vec{\Phi}_1}{di_a} = -\kappa_T \cdot c_T \cdot \vec{\eta}\,, \qquad (21)$$

where A is the value of the Jacobi matrix according to (8).

Let us solve (21) with respect to the derivative

$$\frac{d\vec{\Phi}_1}{di_a} = \left(A^{-1}\right)\cdot\left(-\kappa_т \cdot c_т \cdot \vec{\eta}\right). \qquad (22)$$

Substituting (22) into (19), we obtain

$$L_{a\delta} = \frac{N}{2a} c_\Psi \left[c_\delta \left(A^{-1}\right)\cdot\left(-\kappa_t \cdot c_t \cdot \vec{\eta}\right)\right], \qquad (23)$$

Let us present the equation of moments of PM DCM in the form

$$J\frac{d\omega}{dt} = M_{em} - M_0, \qquad (24)$$

where M_{em} is the electromagnetic moment of the motor; M_0 is the resistance moment of the mechanism on the shaft; J is the moment of inertia of the armature.

Let us determine the electromagnetic moment of the motor due to the fluxes in the areas of the air gap

$$M_{em} = 2p\frac{D_a}{2}\int_0^{\pi} B_\eta l_\delta i_a d\eta = 2p\frac{D_a}{2}\sum_{i=1}^{m} B_\eta\, l_\delta\, i_a \Delta\eta_i = 2p\frac{D_a}{2}\, i_a \sum_{i=1}^{m} \Phi_{\delta i}., \qquad (25)$$

Taking into account (18), we obtain

$$M_{em} = 2p\frac{D_a}{2}\, i_a\, c_\Psi \left(c_\delta \cdot \vec{\Phi}_1\right). \qquad (26)$$

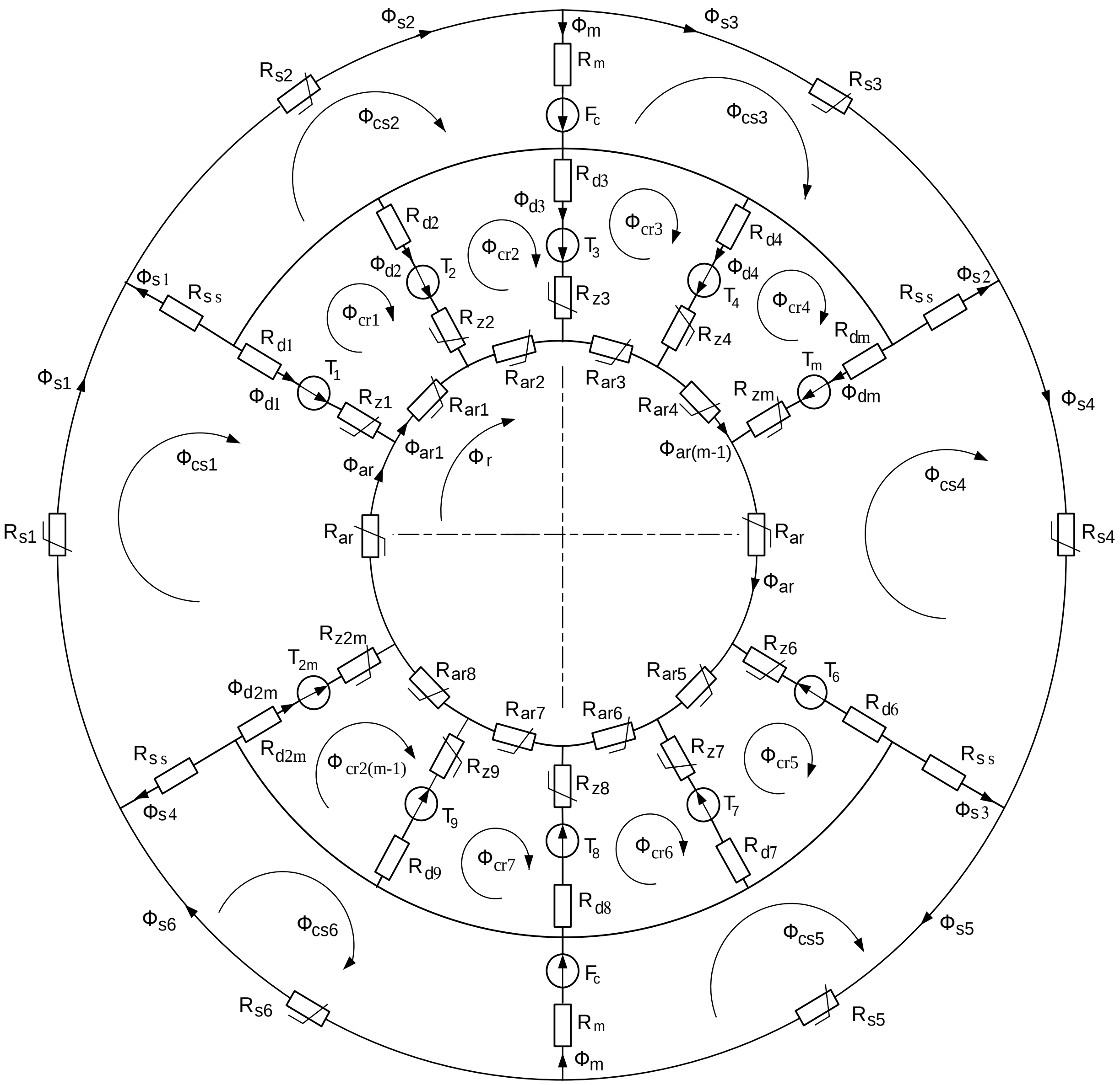

Fig. 1. An equivalent scheme of the PM DCM *magnetic circuit.*

4. Conclusions

The complete system of equations of transient processes of the PM DCM consists of the system of equations of the magnetic state and equations of balance of voltages and moments. The system of equations of the magnetic state corresponding to the equivalent scheme and the algorithm for its solution are presented. The mathematical model of the PM DCM is based on a branched equivalent scheme of the magnetic circuit. The equations of balance of voltages and moments of the PM DCM are presented. The algorithm for finding the required quantities (magnetic flux of the air gap, electromotive force of the armature winding, electromagnetic moment and differential inductance of the armature winding) at each step of integration is given based on the results of the previous solution of the system of equations of the magnetic state. The proposed mathematical model of transient processes of the PM DCM can be used to analyze these processes, as well as in design.

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МАТЕМАТИЧНА МОДЕЛЬ ПЕРЕХІДНИХ ПРОЦЕСІВ ДВИГУНА ПОСТІЙНОГО СТРУМУ ЗІ ЗБУДЖЕННЯМ ВІД ПОСТІЙНИХ МАГНІТІВ

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Двигуни постійного струму зі збудженням від постійних магнітів (ДПС ПМ) внаслідок кращих енергетичних показників мають перевагу над колекторними двигунами з електромагнітним збудженням. Застосування для дослідження ДПС ПМ аналітичних методів потребує істотних спрощень, а використання польових методів є достатньо трудомістким. На наш погляд, застосування методів електричних та магнітних кіл дає змогу досягти достатньої для інженерних потреб точності розрахунку ДПС ПМ. Метою статті є розроблення математичної моделі перехідних процесів ДПС ПМ на основі використання теорії електричних та магнітних кіл. В статті запропонована заступна схема магнітного кола ДПС ПМ та система рівнянь, яка її описує. Наведено рівняння перехідних процесів ДПС ПМ та алгоритм їх розв’язання, який передбачає на кожному кроці інтегрування розв’язання системи рівнянь магнітного стану. Запропонована математична модель перехідних процесів ДПС ПМ може бути використана для аналізу цих процесів, а також під час проєктування.

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Received: 28.05.2024, Accepted: 25.09.2024

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