Volume 10, Number 2, 2024 120

Synthesis of PI- and PID-Regulators in Control Systems Derived by the Feedback Linearization Method

Andriy Lozynskyy[a,b], Lidiia Kasha[a,*], Stepan Pakizh[a], Roman Sadovyi[a]

[a]*Lviv Polytechnic National University, 12 Stepana Bandery St., Lviv, 79013, Ukraine*
[b]*Casimir Pulaski Radom University, 29 Jatska Malchewskiego St., 26-600, Radom, Poland*

Received: October 17, 2024. Revised: November 11, 2024. Accepted: November 27, 2024.

Abstract

The work proposes a comprehensive approach to the synthesis of the coefficients of PI- and PID-controllers, as well as the coefficients of feedback based on the state variables of the system, using the feedback linearization method for the synthesis of control influences. This approach considers not only the static but also the dynamic characteristics of the system, allowing for higher control accuracy. The feedback linearization method facilitates the transformation of nonlinear systems into linear ones, simplifying their further analysis and controller design. The research shows that the new methodology for synthesizing the coefficients of controllers provides improved system stability, reduces sensitivity to external influences, and decreases the response time of the system to changes in operating conditions. A comparison of the proposed approach with the classical feedback linearization method demonstrates significant advantages in adaptability and accuracy. Specifically, the new methodology accounts for real-time changes in system parameters, which is critically important for complex automated processes. Using a two-mass system as an example, the practical application of this approach for synthesizing a control system is demonstrated, allowing for greater precision in control and reduced energy costs. The results of experimental studies confirm the effectiveness of the proposed methodology, indicating its ability to ensure stable system operation under variable loads and external influences. The analysis showed that the new approach can be utilized not only in traditional automated systems but also in a wide range of applications, such as robotics, industrial automation, and electric drive control systems. This research opens new horizons for the further development of adaptive control methods and can serve as a foundation for future studies in this field.

Keywords: feedback linearization; PID-controller; control system; synthesis; two-mass system.

1. Introduction

The present market competition encourages manufacturers to search for new approaches to improve the efficiency and productivity of technological equipment. At the same time, there is a significant gap between the development of automatic control theory and the practical application of the created methods for synthesising control influences in technical systems. Technological objects are mostly nonlinear in nature. Such a nonlinear system, which is generally described by an n^{th}-order differential equation, is reduced to a system of first-order differential equations:

$$\dot{\bar{x}}(t) = f\left(\bar{x}(t), \bar{\theta}(t)\right) + g\left(\bar{x}(t)\right) \cdot \bar{u}(t) + \bar{\xi}(t), \quad (1)$$

where $\bar{x}(t) = [x_1(t), x_2(t), \ldots x_n(t)]^T$, $x_1(t) = x(t)$, $x_2(t) = \dot{x}(t)$ is vector of system state variables; $\vec{u}(t) \in R^n$ is vector of control influences; $\bar{\xi}(t)$ is vector of external disturbances; $f\left(\bar{x}(t), \bar{\theta}(t)\right)$ and $g\left(\bar{x}(t)\right)$ are nonlinear functions that describe the system; $\bar{\theta}(t)$ is system parameter vector.

[*] Corresponding author. Email address: lidiia.v.kasha@lpnu.ua

This paper should be cited as: A. Lozynskyy, L. Kasha, S. Pakizh, R. Sadovyi. Synthesis of PI- and PID-regulators in control systems derived by the feedback linearization method. *Energy Engineering and Control Systems*, 2024, Vol. 10, No. 2, pp. 120 – 130. https://doi.org/10.23939/jeecs2024.02.120

Improvement of nonlinear system stability, reduction of sensitivity to external influences and decrease of the response time of the system to changes in operating conditions are important tasks that need to be solved.

2. Analysis of publications and research

In view of the complexity of the methods for synthesising nonlinear systems [1]-[4], in the industrial process control systems, traditionally, the control systems synthesised on the basis of linearized models are used. A system model is created by linearizing the system in the area of a certain point in the state space $(t) = x(t) - x_0(t)$, $\Delta u(t) = u(t) - u_{\mathbf{0}}(t)$. The functions $f\left(\bar{x}(t), \bar{\theta}(t)\right)$ and $g\left(\bar{x}(t)\right)$ are expanded in a Taylor series. Given that the operating point region is quite small, the Taylor series expressions with an order higher than the first are neglected. As a result, the system model is obtained in the form of the following equations:

$$\dot{\bar{x}}(t) = \boldsymbol{A} \cdot \bar{x}(t) + \boldsymbol{B} \cdot \bar{u}(t),$$
$$\bar{y}(t) = \boldsymbol{C} \cdot \bar{x}(t), \tag{2}$$

where A, B are the matrix of the system and the matrix of control and disturbing influences, respectively; C is the observability matrix of the system.

When expanded in the Taylor series in the area of a few points, a dynamical system class is contained [5].

The presence of a linear model of the system makes it possible to apply the classical control theory methods to the synthesis of control influences [6]. This advantage is realised by the well-known method of feedback linearization [3],[7]-[9]. By changing to another coordinate basis $z=T(x)$, we obtain the system:

$$\dot{\bar{z}}(t) = \boldsymbol{A} \cdot \bar{z}(t) + \boldsymbol{B} \cdot [\alpha(\bar{x}(t)) + \beta(\bar{x}(t)) \cdot \bar{u}(t)], \tag{3}$$

where A, B are controlled matrices; $\beta(\bar{x}(t)) \neq 0$ for $\bar{x}(t) \in R^n$ and $\beta\left(\bar{x}(t)\right) = L_g L_f^{r-1} T(x)$; $\alpha\left(\bar{x}(t)\right) = L_f^r T(x)$; $L_f T(x) = \frac{\partial T(x)}{\partial x} \cdot f(x)$, $L_g T(x) = \frac{\partial T(x)}{\partial x} \cdot g(x)$ are derivatives of function $T(x)$ with respect to $f(x)$ and $g(x)$; r is an indicator characterising the presence of internal system dynamics.

We synthesize a controlling influence in the form of

$$\bar{u}(t) = \frac{1}{\beta(\bar{x}(t))}\left[-\alpha\left(\bar{x}(t)\right) + \bar{\gamma}(t)\right], \tag{4}$$

and obtain a linear system

$$\dot{\bar{z}}(t) = \boldsymbol{A} \cdot \bar{z}(t) + \boldsymbol{B} \cdot \bar{\gamma}(t). \tag{5}$$

In case of a ‘single input, single output’ system, we will get:

$$z\,(t)^{(r)} = \gamma(t). \text{ (integrator series)} \tag{6}$$

System (6) can be obtained from (2) by applying an approach that is an interpretation of the feedback linearisation method for a linear system [10]. In particular, the application of the feedback linearization method in the case of system (2) gives:

$$\dot{y} = \boldsymbol{C} \cdot \dot{x} = \boldsymbol{C} \cdot \boldsymbol{A} \cdot x + \boldsymbol{C} \cdot \boldsymbol{B} \cdot u,$$

and if $\boldsymbol{C} \cdot \boldsymbol{B} = 0$, we obtain

$$\ddot{y} = \boldsymbol{C} \cdot \boldsymbol{A} \cdot \dot{x} = \boldsymbol{C} \cdot \boldsymbol{A} \cdot \boldsymbol{A} \cdot x + \boldsymbol{C} \cdot \boldsymbol{A} \cdot \boldsymbol{B} \cdot u \;\; etc.$$

It will allow us to obtain an expression for the controlling influence u as:

$$u = -(\boldsymbol{C} \cdot A^{r-1} \cdot \boldsymbol{B})^{-1} \cdot \boldsymbol{C} \cdot A^r \cdot x + (\boldsymbol{C} \cdot A^{r-1} \cdot \boldsymbol{B})^{-1} \cdot \nu\,. \tag{7}$$

After substituting the synthesised control into system (2), we obtain:

$$y\,(t)^{(r)} = \nu. \tag{8}$$

Thus, as it was shown in [10], the control influence u provides compensation for the n-r zeros of the system transfer function.

3. Goal of the paper

The goal of this work is to present the developed methodology for synthesizing the coefficients of controllers for nonlinear technological objects to provide the improved system stability, reduced sensitivity to external influences and decreased response time of the system to changes in operating conditions.

4. Presentation and discussion of the research results

In the case of obtaining a system model as a chain of series-connected integrators, control by the full state vector is traditionally used. In particular, for the case of system (8):

$$\nu = -k_0 \cdot y - k_1 \cdot \dot{y} - \cdots - k_{r-1} \cdot y^{(r-1)},$$

or

$$\nu = -k_0 \cdot \mathbf{C} \cdot x - k_1 \cdot \mathbf{C} \cdot \mathbf{A} \cdot x - \cdots - k_{r-1} \cdot \mathbf{C} \cdot \mathbf{A}^{r-1} \cdot x,$$

which allows implementing the desired root location of the characteristics polynomial.

However, the classical feedback linearization method has two main disadvantages [11]-[13]:

1) Availability of an adequate model of the physical system with correctly determined parameters, which is a rather complex task, both due to the complexity of the technological processes themselves and the presence of uncertainties;

2) Significant impact on the control quality of disturbances.

The second disadvantage can be slightly reduced by implementing a PI- or PID-controller (Fig.1) in the system by using a differential component in the control of the full state vector [14]. In this case, the component of the control influence ν will be formed as:

$$e = U_{contr} - k_1 \cdot y - k_2 \cdot \dot{y} - \cdots - k_{r-1} \cdot y^{(r-1)};$$

$$\nu = k_p \cdot e + k_d \cdot \frac{de}{dt} + k_i \cdot \int e\, dt.$$

For the third-order system, using the transfer through the summer, taking into account (6) and (8), the structure of the control system with the PI-controller will have the form shown in Fig.2.

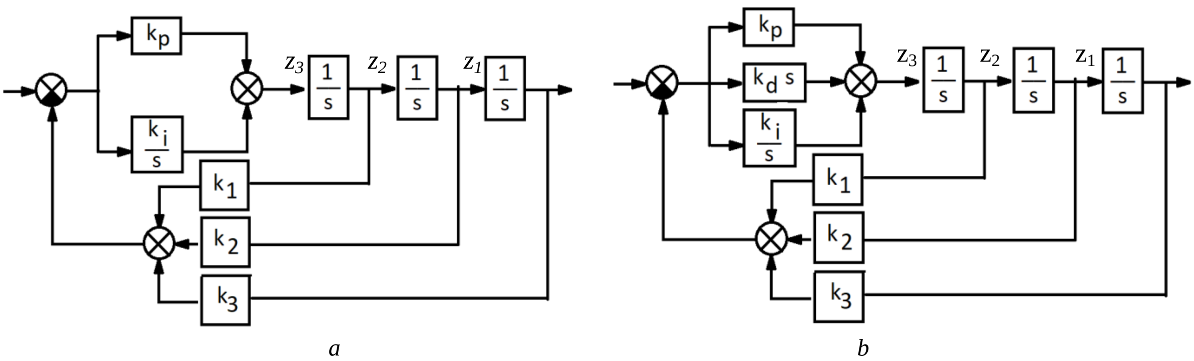

Fig.1. Control system block diagrams based on the full state vector with a PI-controller (*a*) or a PID-controller (*b*).

The transfer function of the closed system will have the form:

$$\frac{Y(s)}{U(s)} = H(s) = \frac{k_p \cdot s + k_i}{s^4 + k_p \cdot k_1 s^3 + (k_p \cdot k_2 + k_i \cdot k_1)s^2 + (k_p \cdot k_3 + k_i \cdot k_2)s + k_i \cdot k_3}. \tag{9}$$

Applying the standard form of the roots placement of a characteristic equation, we obtain:

$$\begin{cases} k_p \cdot k_1 = a_1 \cdot \omega_0; \\ k_p \cdot k_2 + k_i \cdot k_1 = a_2 \cdot {\omega_0}^2; \\ k_p \cdot k_3 + k_i \cdot k_2 = a_3 \cdot {\omega_0}^3; \\ k_i \cdot k_3 = a_4 \cdot {\omega_0}^4. \end{cases} \tag{10}$$

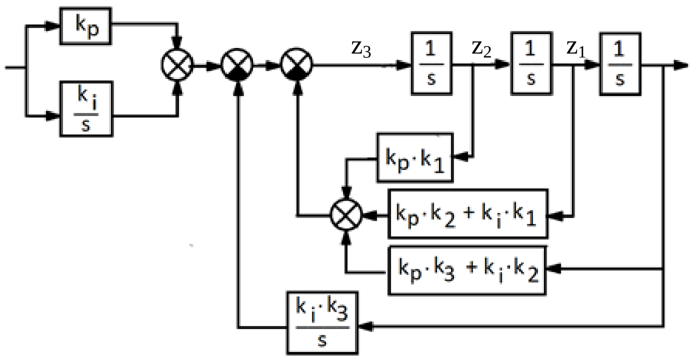

Fig.2. Transformed block diagram of the system with PI-controller.

From the transfer function of the system (9) when s=0, we get:

$$\frac{Y(s=0)}{U(s=0)} = \frac{k_i}{k_i \cdot k_3} = \frac{1}{k_3}.$$

Then $k_3 = U_z / Y_{out}$ and $k_i = \omega_0^4 / k_3 = \omega_0^4 \cdot Y_{out} / U_z$ accordingly. After substitution of k_3 and k_i, the system of equations (10) will take the following form when adjusted to the standard binomial:

$$\begin{cases} k_p \cdot k_1 = 4 \cdot \omega_0; \\ k_p \cdot k_2 + \omega_0^4 \frac{Y_{out}}{U_z} \cdot k_1 = 6 \cdot \omega_0^2; \\ k_p \cdot \frac{U_z}{Y_{out}} + \omega_0^4 \frac{Y_{out}}{U_z} \cdot k_2 = 4 \cdot \omega_0^3. \end{cases} \quad (11)$$

From first equation (11) we obtain $k_1 = \frac{4 \cdot \omega_0}{k_p}$, and from 3rd one $k_2 = \frac{4 \cdot \omega_0^3 - k_p \cdot \frac{U_z}{Y_{out}}}{\omega_0^4 \frac{Y_{out}}{U_z}}$ and by substituting them into the 2nd equation, after some simple transformations, we obtain:

$$k_p^3 - 4 \cdot \omega_0^3 \cdot \frac{Y_{out}}{U_z} \cdot k_p^2 + 6 \cdot \omega_0^6 \left(\frac{Y_{out}}{U_z}\right)^2 \cdot k_p - 4 \cdot \omega_0^9 \left(\frac{Y_{out}}{U_z}\right)^3 = 0. \quad (12)$$

Rewriting equation (12) as

$$\left(k_p - 2 \cdot \omega_0^3 \cdot \frac{Y_{out}}{U_z}\right) \cdot \left(k_p^2 - 2 \cdot \omega_0^3 \cdot \frac{Y_{out}}{U_z} \cdot k_p + 2 \cdot \omega_0^6 \left(\frac{Y_{out}}{U_z}\right)^2\right) = 0,$$

and taking into account that the quadratic equation will have complex-conjugate roots, we get

$$k_p = 2 \cdot \omega_0^3 \cdot \frac{Y_{out}}{U_z}, \qquad k_1 = \frac{2}{\omega_0^2 \cdot \frac{Y_{out}}{U_z}}, \qquad k_2 = \frac{2}{\omega_0 \frac{Y_{out}}{U_z}}.$$

The dynamic characteristics of the system at the above coefficient settings for the case of $\omega_0 = 1$ and $k_3 = U_z / Y_{out} = 1$ are shown in Fig.3.

The obtained results demonstrate an improvement in the control quality compared to the traditional use of the state variable control system. It is important to note that the approach of applying a standard form for adjusting the PI-controller has its limits. For example, in the case of a 2nd order system, the system of equations for determining the controller parameters will be as follows:

$$\begin{cases} k_p \cdot k_1 = 3 \cdot \omega_0; \\ k_p \cdot k_2 + k_i \cdot k_1 = 3 \cdot \omega_0^2; \\ k_i \cdot k_2 = \omega_0^3. \end{cases} \quad (13)$$

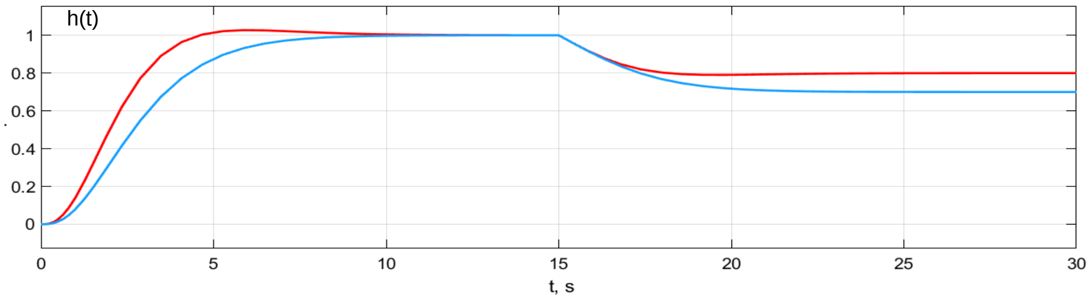

Fig.3. The transient process and response to disturbances at time t = 15 s in the traditional state variable control system (___**blue**) and in the system with an additional PI-controller (___**red**).

From the transfer function of the system at s=0 we obtain $k_2 = U_z / Y_{out}$ and $k_i = \omega_0^3 / k_2 = \omega_0^3 \cdot Y_{out} / U_z$ accordingly. After substitution to (13), the system of equations for determining the controller parameters will become:

$$\begin{cases} k_p \cdot k_1 = 3 \cdot \omega_0; \\ k_p \cdot {U_z}/{Y_{out}} + \omega_0{}^3 \cdot {Y_{out}}/{U_z} \cdot k_1 = 3 \cdot \omega_0{}^2. \end{cases} \quad (14)$$

Determining $k_1 = 3 \cdot \omega_0 / k_p$ from the 1st equation of the system (14) and substituting it into the 2nd equation, we obtain the quadratic equation $k_p{}^2 - 3 \cdot \omega_0{}^2 \cdot {\omega_2}/{M_z} \cdot k_p + 3 \cdot \omega_0{}^4 \left({\omega_2}/{M_z}\right)^2 = 0$ which does not have the real roots. However, if we set the system to the standard form of the minimum root mean square error, the system would look as follows:

$$\begin{cases} k_p \cdot k_1 = 1 \cdot \omega_0; \\ k_p \cdot {U_z}/{Y_{out}} + \omega_0{}^3 \cdot {Y_{out}}/{U_z} \cdot k_1 = 2 \cdot \omega_0{}^2. \end{cases} \quad (15)$$

and obtained quadratic equation $k_p{}^2 - 2 \cdot \omega_0{}^2 \cdot {Y_{out}}/{U_z} \cdot k_p + 1 \cdot \omega_0{}^4 \left({Y_{out}}/{U_z}\right)^2 = 0$, will have a real root $k_p = \omega_0^2 \cdot Y_{out} / U_z$. Then $k_1 = 1 \cdot \omega_0 / k_p = 1 / (\omega_0 \cdot Y_{out} / U_z)$.

In control systems with a PID-controller, the control structure shown in Fig.4 is often used to avoid negative signal response at the controller output when the control signal changes rapidly.

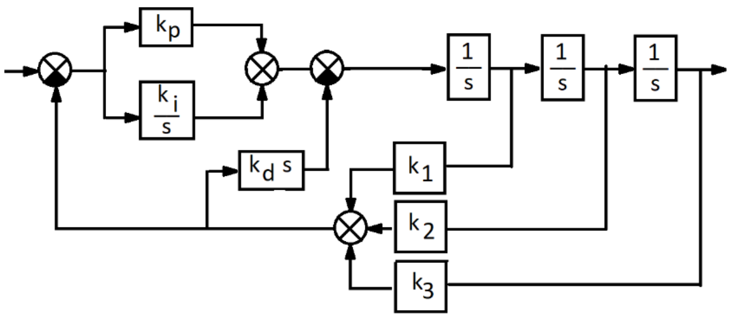

Fig.4. The block diagram of the control system structure with a modified PID-control law.

Then, in the case of the 3rd order system, after moving the PID-controller through the summer, taking into account (6) and (8), the structure of the control system with the modified PID-controller will be shown in Fig.5.

The transfer function of the closed-loop system is given by

$$\frac{Y(s)}{U(s)} = H(s) = \frac{\dfrac{k_p \cdot s + k_i}{1 + k_d \cdot k_1}}{s^4 + \left(\dfrac{k_p \cdot k_1 + k_d \cdot k_2}{1 + k_d \cdot k_1}\right) s^3 + \left(\dfrac{k_p \cdot k_2 + k_d \cdot k_3 + k_i \cdot k_1}{1 + k_d \cdot k_1}\right) s^2 + \left(\dfrac{k_p \cdot k_3 + k_i \cdot k_2}{1 + k_d \cdot k_1}\right) s + \dfrac{k_i \cdot k_3}{1 + k_d \cdot k_1}}. \quad (16)$$

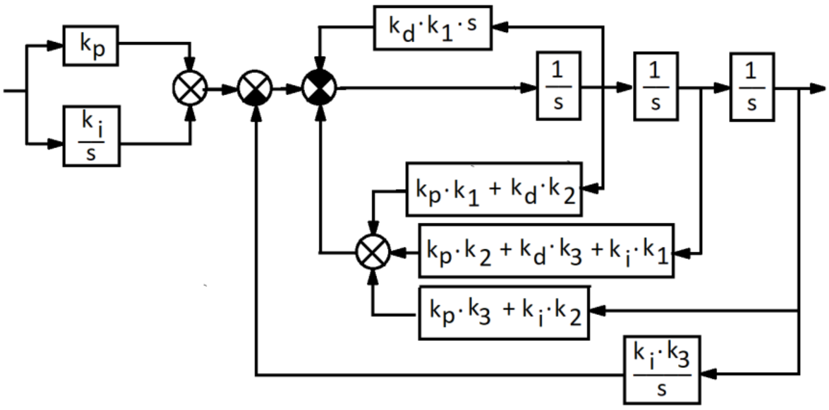

Fig.5. Transformed block diagram of the system with a modified PID-controller.

If we adopt the standard binomial form, we get the system of equations:

$$\begin{cases} \dfrac{k_p \cdot k_1 + k_d \cdot k_2}{1 + k_d \cdot k_1} = 4 \cdot \omega_0; \\ \dfrac{k_p \cdot k_2 + k_d \cdot k_3 + k_i \cdot k_1}{1 + k_d \cdot k_1} = 6 \cdot {\omega_0}^2; \\ \dfrac{k_p \cdot k_3 + k_i \cdot k_2}{1 + k_d \cdot k_1} = 4 \cdot {\omega_0}^3; \\ \dfrac{k_i \cdot k_3}{1 + k_d \cdot k_1} = {\omega_0}^4. \end{cases} \tag{17}$$

System of equations (17), when $1 + k_d \cdot k_1 \neq 0$, can be written as:

$$\begin{cases} k_p \cdot k_1 + k_d \cdot k_2 - k_d \cdot k_1 \cdot 4 \cdot \omega_0 = 4 \cdot \omega_0; \\ k_p \cdot k_2 + k_d \cdot k_3 + k_i \cdot k_1 - k_d \cdot k_1 \cdot 6 \cdot {\omega_0}^2 = 6 \cdot {\omega_0}^2; \\ k_p \cdot k_3 + k_i \cdot k_2 - k_d \cdot k_1 \cdot 4 \cdot {\omega_0}^3 = 4 \cdot {\omega_0}^3; \\ k_i \cdot k_3 - k_d \cdot k_1 \cdot {\omega_0}^4 = {\omega_0}^4. \end{cases} \tag{18}$$

As in the previous case, from the transfer function of the system at s=0, we obtain $k_3 = U_z / Y_{out}$. Setting the value $k_d = a$, we find the values of the control system coefficients. Fig.6 – Fig.9 show the change in the coefficients of the control system for different values of the coefficient k_d and the geometric mean root ω_0.

Fig.10 shows the dependences of the change in the output value and the control signal for the proposed and classical control systems for the case of $\omega_0 = 1$ and $k_3 = U_z / Y_{out} = 1$.

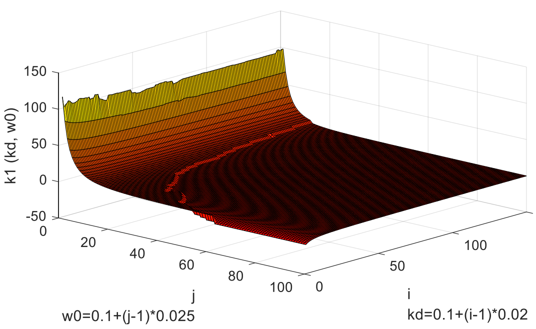

Fig.6. Dependence of the change in the feedback coefficient $k_1(k_d, \omega_0)$.

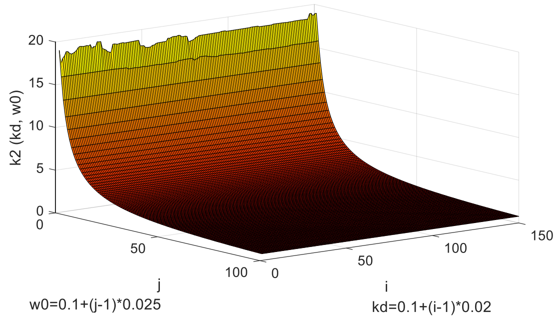

Fig.7. Dependence of the change in the feedback coefficient $k_2(k_d, \omega_0)$.

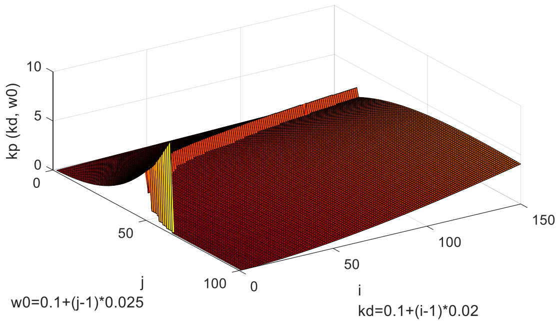

Fig.8. Dependence of change in the gain coefficient of the proportional part of the controller $k_p(k_d\,,\omega_0)$.

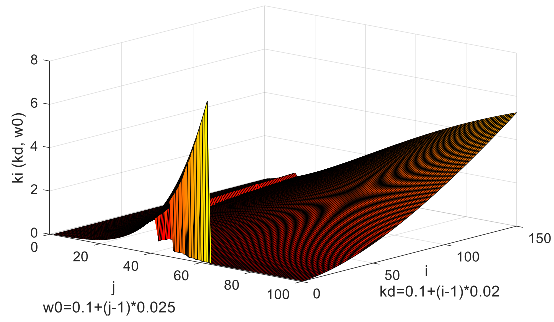

Fig.9. Dependence of change in coefficient of the integral part of the regulator $k_i(k_d\,,\omega_0)$

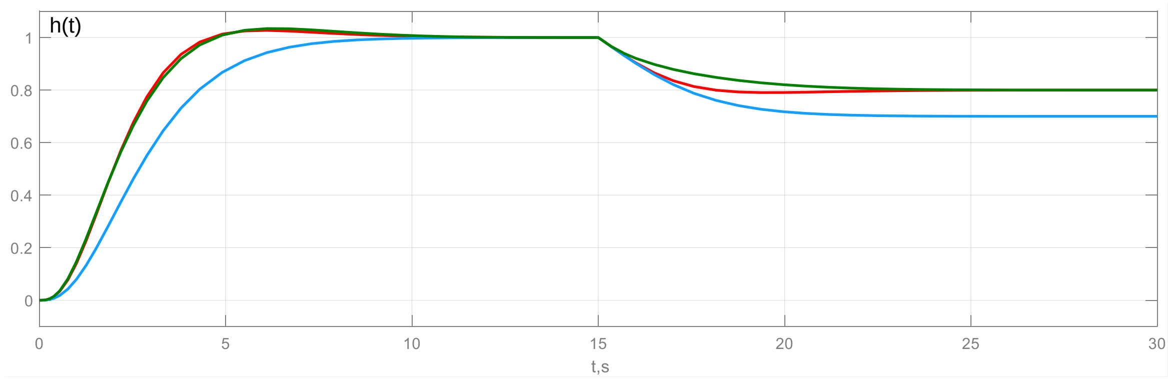

Fig.10. Transient characteristic and disturbance response at time *t* = 15 s in a traditional state variable control system (___**blue**) and in systems with an additional PI-(___**red**) or PID- (___**green**) controller.

The obtained dependencies allow us to assert that the implementation of a differential feedback on the system by state variables during the control synthesis using the binomial form will slightly affect on its dynamic characteristics.

5. The application of the proposed approach for two-mass system

Let's apply the proposed approach to the synthesis of control influence in the two- mass system in Fig.11.

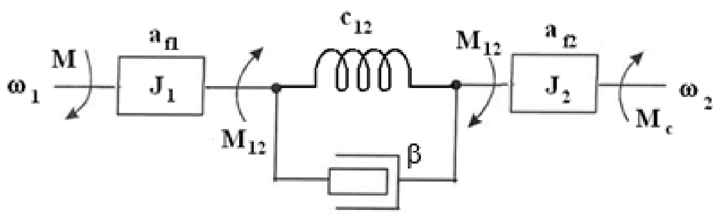

Fig. 11. Block diagram of a two-mass system.

The model of a two-mass system of state variables $\dot{x} = \mathrm{A}\cdot x + \mathrm{B}\cdot u$ in the case of a positioning system can be written as:

$$\begin{bmatrix} \frac{d\omega_1}{dt} \\ \frac{dM_{12}}{dt} \\ \frac{d\omega_2}{dt} \\ \frac{d\psi}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{\beta+a_{f1}}{J_1} & -\frac{1}{J_1} & \frac{\beta}{J_1} & 0 \\ c_{12} & 0 & -c_{12} & 0 \\ \frac{\beta}{J_2} & \frac{1}{J_2} & -\frac{\beta+a_{f2}}{J_2} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ M_{12} \\ \omega_2 \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J_2} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} M \\ M_c \end{bmatrix}. \qquad (19)$$

When applying the feedback linearization method, we set $z_1 = \psi$ and obtain:

$$\dot{z}_1 = z_2 = \omega_2;$$
$$\dot{z}_2 = z_3 = \dot{\omega}_2 = \frac{1}{J_2}\cdot\left(M_{12} + \beta\cdot(\omega_1 - \omega_2) - Mc - a_{f2}\cdot\omega_2\right);$$
$$\dot{z}_3 = z_4 = \ddot{\omega}_2 = \frac{1}{J_2}\cdot\left(\dot{M}_{12} + \beta\cdot(\dot{\omega}_1 - \dot{\omega}_2) - \dot{M}c - a_{f2}\cdot\dot{\omega}_2\right) = \frac{1}{J_2}\cdot\left(c\cdot(\omega_1 - \omega_2) - \dot{M}c\right) +$$
$$+\frac{1}{J_2}\cdot\left(\frac{\beta}{J_1}\cdot\left(M - M_{12} - \beta\cdot(\omega_1 - \omega_2) - a_{f1}\cdot\omega_1\right) - \frac{\beta}{J_2}\cdot\left(M_{12} + \beta\cdot(\omega_1 - \omega_2) - Mc - a_{f2}\cdot\omega_2\right)\right) - \quad (20)$$
$$-\frac{1}{J_2}\cdot\frac{a_{f2}}{J_2}\cdot\left(M_{12} + \beta\cdot(\omega_1 - \omega_2) - Mc - a_{f2}\cdot\omega_2\right).$$

The system has a relative degree of ρ=3, and the expression for the control influence will be as follows:

$$M = \frac{J_1\cdot J_2}{\beta}\left(\nu - \frac{1}{J_2}\cdot\left(c\cdot(\omega_1 - \omega_2) - \dot{M}c\right) - \frac{1}{J_2}\cdot\frac{\beta}{J_1}\cdot\left(-M_{12} - \beta\cdot(\omega_1 - \omega_2) - a_{f1}\cdot\omega_1\right) + \frac{1}{J_2}\left(\frac{\beta}{J_2}\cdot\right.\right.$$
$$\left.\left.\cdot\left(M_{12} + \beta\cdot(\omega_1 - \omega_2) - Mc - a_{f2}\cdot\omega_2\right) + \frac{a_{f2}}{J_2}\cdot\left(M_{12} + \beta\cdot(\omega_1 - \omega_2) - Mc - a_{f2}\cdot\omega_2\right)\right)\right). \quad (21)$$

When $\dot{M}c = 0$ and $\mathrm{Mc} = 0$ we obtain $M = \frac{J_1\cdot J_2}{\beta}\cdot\nu + \frac{J_1\cdot J_2}{\beta}\cdot M_{12}\left(\frac{\beta}{J_1\cdot J_2} + \frac{\beta + a_{f2}}{J_2^2}\right) + \frac{J_1\cdot J_2}{\beta}\cdot(\omega_1 - \omega_2)\cdot$ $\cdot\left(\frac{\beta^2}{J_1\cdot J_2} + \frac{\beta^2 + \beta\cdot a_{f2}}{J_2^2} - \frac{c}{J_2}\right) + \frac{J_1\cdot J_2}{\beta}\cdot\frac{\beta\cdot a_{f1}}{J_1\cdot J_2}\cdot\omega_1 - \frac{J_1\cdot J_2}{\beta}\cdot\frac{a_{f2}\cdot(\beta + a_{f2})}{J_2^2}\cdot\omega_2.$

After substitution the expression for the moment determination, taking into account the assumption, the system of equations (20) will take the below form:

$$\begin{cases}\dot{z}_1 = z_2;\\ \dot{z}_2 = z_3;\\ \dot{z}_3 = \nu.\end{cases} \quad (22)$$

The synthesis of the control influence ν when the PI controller is used and when applying the full state vector control for a third-order system is given above in this article.

For comparison, Fig.12 shows the transient processes in the classical system synthesised by the modal control method and in the system synthesised by the classical feedback linearization method, as well as with the use of an additional PI-controller. The feedback coefficients by state variables in the modal control system are calculated according to the equations given in [15].

In the case when the control is synthesised by the classical feedback linearization method, the feedback coefficients of state variables are equal to $k_1 = 3\cdot\omega_0$, $k_2 = 3\cdot\omega_0^2$, $k_2 = \omega_0^3$, respectively, and the equation for determining the control influence is as follows: $\nu = M_z - k_1\cdot z_3 - k_2\cdot z_2 - k_3\cdot z_1$. At time t = 50 s, we have a displacement disturbance applied to the system.

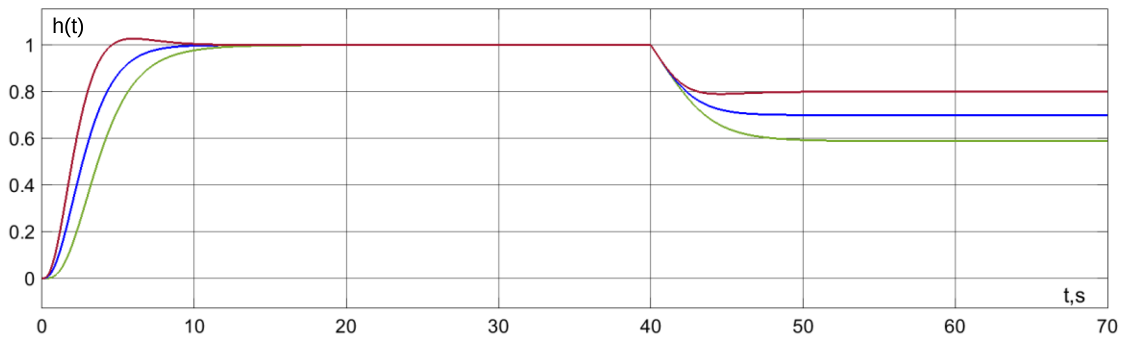

Fig.12. Transient characteristics and response to disturbances at time t = 50 s in case with a traditional modal control system (___**blue**), a classical system synthesised by the feedback linearization method (___**green**), and a system synthesised by the feedback linearization method with an additional PI-controller (___**red**).

The obtained results demonstrate significantly better dynamic and static characteristics in comparison with both the modal control system and the classical system synthesized by the feedback linearization method. As noted in [15], in the system, if internal and external friction are not taken into account in the synthesis of the modal controller, an overshoot occurs. It is caused by the zero of the transfer function and inadequacy of the model used for synthesis of the controller. When taking into account the friction, the overshoot is extremely small, as shown in Fig.12. In the system synthesized by feedback linearization method with a PI-controller, the overshoots are caused by PI-controller which leads to the appearance of a zero in the transfer function of the system (9). It is also worth noting that the system synthesized by the feedback linearization method is adjusted to the standard form of the 3rd order, while the modal controller and the system with an additional PI-controller use the standard form of the 4th order, which affects the system dynamics.

In the case of a traditional two-mass system, its mathematical model will look like:

$$\begin{bmatrix} \frac{d\omega_1}{dt} \\ \frac{dM_{12}}{dt} \\ \frac{d\omega_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{\beta + a_{f1}}{J_1} & -\frac{1}{J_1} & \frac{\beta}{J_1} \\ c_{12} & 0 & -c_{12} \\ \frac{\beta}{J_2} & \frac{1}{J_2} & -\frac{\beta + a_{f2}}{J_2} \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ M_{12} \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J_2} \end{bmatrix} \cdot \begin{bmatrix} M \\ M_c \end{bmatrix}. \tag{23}$$

When applying the feedback linearization method to (23) at $z_1 = \omega_2$, we obtain

$$\begin{aligned} \dot{z}_1 = z_2 = \dot{\omega}_2 = 1/J_2 \cdot \left(M_{12} + \beta \cdot (\omega_1 - \omega_2) - Mc - a_{f2} \cdot \omega_2\right); \\ \dot{z}_2 = z_3 = \ddot{\omega}_2 = 1/J_2 \cdot \left(\dot{M}_{12} + \beta \cdot (\dot{\omega}_1 - \dot{\omega}_2) - \dot{M}c - a_{f2} \cdot \dot{\omega}_2\right). \end{aligned} \tag{24}$$

The system has a relative degree of $\rho = 2$, and the equation for the control influence M will be similar to the case of a positional two-mass system. Adjusting the controller coefficients using the standard binomial form, as described above, is impossible. In our research, we will use the standard form: $s^3 + \omega_0 \cdot s^2 + 2 \cdot \omega_0^2 \cdot s + \omega_0^3$. Then, in the case of synthesis of a PID-controller, we obtain the following system of equations:

$$\begin{cases} \frac{k_p \cdot k_1 + k_d \cdot k_2}{1 + k_d \cdot k_1} = 1 \cdot \omega_0; \\ \frac{k_p \cdot k_2 + k_i \cdot k_1}{1 + k_d \cdot k_1} = 2 \cdot {\omega_0}^2; \\ \frac{k_i \cdot k_2}{1 + k_d \cdot k_1} = {\omega_0}^3. \end{cases} \tag{25}$$

From the transfer function of the system at s=0, we obtain $k_2 = M_z / \omega_2$. At the differential regulator coefficient $k_d = 0.85$, the values of the control system coefficients for $\omega_0 = 1$ are equal to $k_1 = 1.6001$; $k_p = 0.9437$; $k_i = 2.23601$ respectively.

For comparison, in Fig.13 are shown transient processes in a classical system synthesised by the classical feedback linearisation method and with the use of an additional PI-controller or PID-controller. The synthesis of the control input v using a PI-controller and control by the full state vector for a 2nd order system is given above in this article.

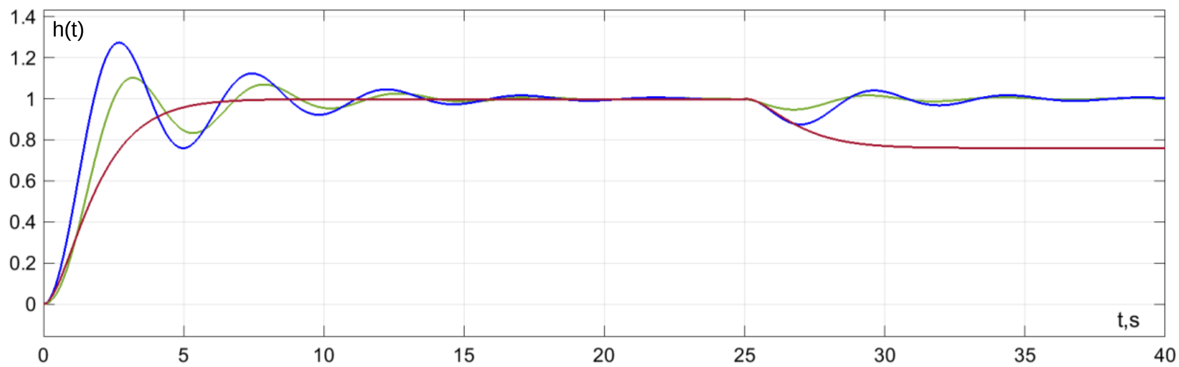

Fig.13. Transient process and response to disturbances at time t = 25 s in the system synthesised by the feedback linearization method when adjusting to the binomial form (___**red**) with an additional PID- controller (___**green**) and PI-controller (___**blue**) when adjusting the system to the form $s^3 + \omega_0 \cdot s^2 + 2 \cdot \omega_0^2 \cdot s + \omega_0^3$.

In the case when the control is synthesised by the classical feedback linearization method, the feedback coefficients for the state variables are $k_1 = 2 \cdot \omega_0$, $k_2 = \omega_0^2$, respectively, and the control influence is $\nu = M_z - k_1 \cdot z_2 - k_2 \cdot z_1$. At time t = 25 s, a disturbance acts on the system.

The obtained results allow us to confirm that the application of a PI- or PID-controller to the control structure provides an astatic control system by disturbance. At the same time, the introduction of a differential component, when tuned to the standard form of the minimum mean square error, significantly improves the system's dynamic characteristics, in comparison with the control synthesised on the basis of the binomial form.

6. Conclusion

The proposed approach to the complex adjustment of the coefficients of PI- or PID-controllers and feedback coefficients on state variables in control systems based on the full state vector improves the static and dynamic properties of the system, which is confirmed by the results shown in Fig.3 and Fig.10.

Depending on the chosen standard form for adjusting the control system coefficients, effect of the differential component of the control influence on the system characteristics changes, as shown by the results in Fig.10 and Fig.13.

The application of the proposed approach for the synthesis of the control influence in a two-mass system has demonstrated its effectiveness in comparison with traditional modal control. The presence of internal dynamics of the system influences the nature of transients in systems synthesised using the feedback linearization method.

References

[1] Hassan K. Khalil, (2022) Nonlinear Systems. *Prentice Hall*, New York, 2002, 768 p.

[2] Isidori, A. (1995) Nonlinear control systems, *Springer-Verlag*, pp. 550. https://doi.org/10.1007/978-1-84628-615-5

[3] Ortega, R., Garcı´a-Canseco, E. (2004) Interconnection and damping assignment passivity-based control: A survey, *European J. of Control*, Vol. 10(5), pp. 432-450. https://doi.org/10.3166/ejc.10.432-450

[4] Shchur, I., Rusek, A., & Biletskyi, Y. (2014). Energy-shaping optimal load control of PMSG in a stand-alone wind turbine as a port-controlled Hamiltonian system. *Przegląd Elektrotechniczny* (Electrical Review), 5, pp. 50-55.

[5] Lozynskyy, A., & Demkiv, L. (2016). Application of dynamic systems family for synthesis of fuzzy control for electromechanical systems. *Advances in Electrical and Electronic Engineering*, 14(5), pp. 543-550. https://doi.org/10.15598/aeee.v14i5.1717

[6] Lozynskyi, O. Yu., Lozynskyi, A. O., Paranchuk, Ya. S., Marushchak, Ya. Yu., & Tsapa, V. B. (2016). Methods for synthesizing optimal linear systems. *Lviv Polytechnic Publishing House*, 392 p. (in Ukrainian)

[7] Lascu, C., Jafarzadeh, S., Fadali, M. S., & Blaabjerg, F. (2017). Direct torque control with feedback linearization for induction motor drives. *IEEE Transactions on Power Electronics*, 32(3), pp. 2072-2080. https://doi.org/10.1109/TPEL.2016.2564943

[8] Choi, Y.-S., Choi, H. H., & Jung, J.-W. (2016). Feedback linearization direct torque control with reduced torque and flux ripples for IPMSM drives. *IEEE Transactions on Power Electronics*, 31(5), pp. pp. 3728-3737. https://doi.org/10.1109/TPEL.2015.2460249

[9] Baltag, A., Livint, G., Belehuz, L., & Baciu, A. G. (2023). Application of feedback linearization method to wind turbines with PMSG for extracting maximum power from wind energy. *In 2023 10th International Conference on Modern Power Systems (MPS*), Cluj-Napoca, Romania, pp. 01-06. https://doi.org/10.1109/MPS58874.2023.10187595

[10] Lozynskyy, A., Lozynskyy, O., Marushchak, Ya., & Kasha, L. (2020). Synthesis of combine feedback control of electromechanical system by feedback linearization method. *In IEEE Problems of Automated Electrodrive. Theory and Practice (PAEP)* Kremenchuk, Ukraine, pp. 1-6. https://doi.org/10.1109/PAEP49887.2020.9240776

[11] Accetta, A., et al. (2019). Robust control for high performance induction motor drives based on partial state-feedback linearization. *IEEE Transactions on Industry Applications*, 55(1), pp. 490-503. https://doi.org/10.1109/TIA.2018.2869112

[12] Gastaldini, C. C., Vieira, R. P., Azzolin, R. Z., & Grndling, H. A. (2013). Feedback linearization control with sliding mode speed observer for three-phase induction machines. *In 2013 Brazilian Power Electronics Conference*, Gramado, Brazil, pp. 345-349. https://doi.org/10.1109/COBEP.2013.6785138

[13] Morawiec, M., Strankowski, P., Lewicki, A., Guziński, J., & Wilczyński, F. (2020). Feedback control of multiphase induction machines with backstepping technique. *IEEE Transactions on Industrial Electronics*, 67(6), pp. 4305-4314. https://doi.org/10.1109/TIE.2019.2931236

[14] Boukas, T. K., & Habetler, T. G. (2004). High-performance induction motor speed control using exact feedback linearization with state and state derivative feedback. *IEEE Transactions on Power Electronics*, 19(4), pp. 1022-1028. https://doi.org/10.1109/TPEL.2004.830042

[15] Marushchak, Y. Yu., Lozynskyi, A. O., & Kushnir, A. P. (2011). Dynamics of two-mass systems for stabilization of modes in electric arc furnaces. *Lviv Polytechnic Publishing House*, 224 p. (in Ukrainian)

Синтез ПІ- та ПІД-регуляторів у системах керування, отриманих методом лінеаризації зворотним зв’язком

Андрій Лозинський[a,b], Лідія Каша[a], Степан Пакіж[a], Роман Садовий[a]

[a]*Національний університет «Львівська політехніка», вул. Степана Бандери, 12, Львів, 79013, Україна*
[b]*Радомський університет ім. Казіміра Пуласкєго, вул. Яцка Мальчевскєго, 29, Радом, 26-600, Польща*

Анотація

В роботі запропоновано комплексний підхід до синтезу коефіцієнтів ПІ- та ПІД-регуляторів, а також коефіцієнтів зворотних зв’язків за змінними стану системи при застосуванні методу лінеаризації зворотним зв’язком для синтезу керуючих впливів. Цей підхід враховує не лише статичні, але й динамічні характеристики системи, що дозволяє досягти більш високої точності в управлінні. Метод лінеаризації зворотним зв’язком забезпечує перетворення нелінійних систем у лінійні, що спрощує їх подальший аналіз та проектування контролерів. Дослідження показує, що нова методологія синтезу коефіцієнтів регуляторів забезпечує покращену стабільність системи, знижує чутливість до зовнішніх впливів та зменшує час реагування системи на зміни в умовах експлуатації. Порівняння запропонованого підходу з класичним методом лінеаризації зворотним зв’язком демонструє значні переваги у адаптивності та точності. Зокрема, нова методологія дозволяє враховувати зміни в параметрах системи в реальному часі, що є критично важливим для складних автоматизованих процесів. На прикладі двомасової системи продемонстровано практичне застосування цього підходу для синтезу системи керування, що надає можливість досягти більшої точності в управлінні та зменшити енергетичні витрати. Результати експериментальних досліджень підтверджують ефективність запропонованої методології, вказуючи на її здатність забезпечувати стабільну роботу системи в умовах змінних навантажень і зовнішніх впливів. Аналіз показав, що новий підхід може бути використаний не лише в традиційних автоматизованих системах, але й в широкому спектрі застосувань, таких як робототехніка, промислова автоматизація та системи управління електричними приводами. Дане дослідження відкриває нові горизонти для подальшого розвитку адаптивних методів управління та може слугувати основою для майбутніх досліджень у цій галузі.

Ключові слова: лінеаризація зворотним зв’язком; ПІД-регулятор; система керування; синтез; двомасова система.