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# CALCULATION OF THE STRENGTH OF THE ELEMENTS OF THE FREE-WHEEL BALL CLUTCH OF A BICYCLE

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**Abstract.** The work describes the structure and operational principles of a bicycle's free-wheel ball clutch. It also provides a comprehensive methodology for calculating the clutch's working components' transverse forces, bending moments, and movements. The calculations are illustrated for two scenarios of half-coupling loading: two concentrated opposite forces and a uniform load. The provided materials and data enable designers to make informed decisions regarding the selection of materials for manufacturing and the primary geometric parameters of the free-wheel ball clutch.

**Keywords:** coupling, clutch, free-wheel clutch, stress, half-coupling, half-coupling connection.

## Introduction

Couplings are responsible parts of machines and mechanisms that significantly affect the load's level and nature on the mechanical drive's kinematic chains. Also, they often perform the functions of protective devices of the responsible mechanical drives against overloads, as well as the functions of regulators of the speed of movement and the direction of energy transmission.

## Problem Statement

Couplings are used to connect shafts that have a common axis of rotation. Couplings also allow you to connect shafts in case of radial or angular displacement of shaft axes. Free-wheel couplings are used for automatic connection and disconnection of shafts without stopping the engine and also allow the torque to be transmitted in only one direction.

## Review of Modern Information Sources on the Subject of the Paper

Known studies of free-wheel couplings mainly concern roller and ratchet mechanisms. The research data are given in the works of scientists V. S. Polyakov, M. M. Ivanov, S. G. Nagornyak, and O. A. Rakhovsky and others [8]. In recent years, Lviv Polytechnic National University has developed some designs of free-running ball couplings, which are protected by patents for inventions and utility models. These designs were studied in the works of V. V. Malashchenko, O. I. Sorokivskyi, V. V. Malashchenko, I. E. Kravets, and A. O. Borys and others [1–5, 9–11].

## Objectives and Problems of Research

This work aims to develop a methodology for calculating the strength of a bicycle freewheel ball clutch. This technique will allow for determining the developed ball clutch's main geometric and kinematic parameters during its design.

**Main Material Presentation**

The general view of the freewheel ball clutch of a bicycle is shown in Fig. 1, *a*. The ball coupling consists of a driven half-coupling one and a driving half-coupling 4, between which the balls 5 of the bearing are installed. The sprockets 2 of the chain transmission are installed on the outer surface of the driving half-coupling 4. The main parts of the coupling are fastened with the help of nut 3. Fig. 1 shows the side surface of the driven half-coupling 1 with six grooves on the inner end surface. Slots are made on the outer surface of the driving half-coupling 4 for installing sprockets 2. Fig. 1, *c* shows the side surface of the driving half-coupling 4 with grooves 7 on the outer end surface. Balls 8 are installed between the driver 4 and driven 1 half-couplings in grooves 6 and 7. The number of grooves 6 of the driver half-coupling 4 corresponds to the number of balls 8 in them.

The freewheel ball clutch works as follows. The clutch is set in motion using a chain transmission, which transmits the load to the sprockets 2. Next, the sprockets 2 transmit the torque through the splines to the driving half-coupling 4. During the rotation of the driving half-coupling 4 clockwise, balls 5 roll in the grooves 6 opposite to the direction of movement. When reaching the peripheral ends of groove 6, ball 5 falls into grooves 7 of the driven half-coupling 1 and wedge between grooves 6 and 7. The driven half-coupling 1 begins to rotate with the driver 4 and transmit torque. Since the driven half-coupling is fixed to the axis of the bicycle wheel, the torque is further transmitted to the wheel.

In the case of a change in the direction of rotation of the sprockets 2, as well as in the case when the angular speed of rotation of the driven half-coupling 1 is greater than the angular speed of the driving half-coupling 4, the balls 5 are pushed out of engagement by the side surfaces of the grooves 6 and 7. The ball 5 moves to the peripheral ends of groove 6 and opens the kinematic connection between half-couplings 1 and 4. The coupling goes into free-running mode, and torque is not transmitted.

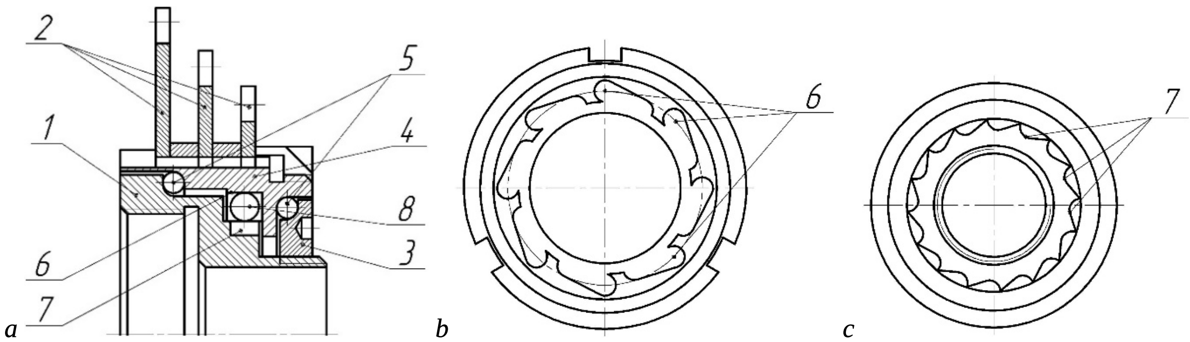


**Fig. 1.** The design of the free-wheel ball clutch of a bicycle

During the bicycle freewheel ball clutch operation, all the load is transmitted using one or more balls 8, which are engaged in the grooves 6 and 7 of the driving 4 and driven 1 half-coupling. The ball 8 presses on the grooves 7 of the driven half-coupling 1 to transmit the torque, made on a cylindrical surface. The most loaded element of the ball joint is the driven half-coupling in the area where grooves 7 are made. To simplify the calculation of the coupling strength, we assume that the loaded part of the driven half-coupling has the shape of a ring. We will use the well-known classical theory, which allows us to calculate the strength of a semi-coupling as a circular ring [6]. The calculation diagram of the load of the driven half-coupling is shown in Fig. 2.

We assume that the section's axis is in the ring's plane (the working area of the half-coupling). External forces also act in this plane. A statically indeterminate ring is closed when the load is applied at a right angle. In the case of calculating a thin ring, the dependencies established in the theory of rectilinear rods [6] can be considered valid. A statically determined system can be obtained by cutting the ring in a certain section $q = 0$ (Fig. 2). We ignore the effect of regular and shear forces on the deformation.

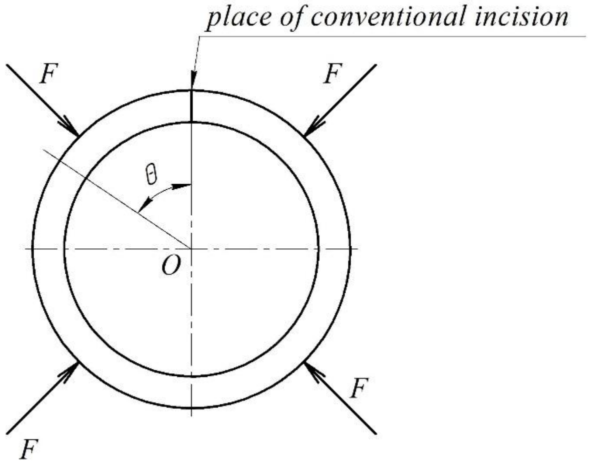


**Fig. 2.** The calculation diagram of the forces acting on the driven half-coupling

If the external loads have an axis of symmetry, then it is advisable to cut this axis (Fig. 2). For a symmetrically loaded ring (half-coupling), the bending moment in the section is q [6]

$$M(\theta) = M_F(\theta) - \frac{2\cos\theta}{\pi}\int_0^{\pi} M_F(\theta)\cos\theta d\theta - \frac{1}{\pi}\int_0^{\pi} M_F(\theta)d\theta, \tag{1}$$

where $M_F$ – bending moment in section q; q – the running angle of the section.

For example, consider a more straightforward case when two oppositely placed balls are engaged. Then, consider the semi-coupling as a ring loaded by two concentrated forces (Fig. 3).

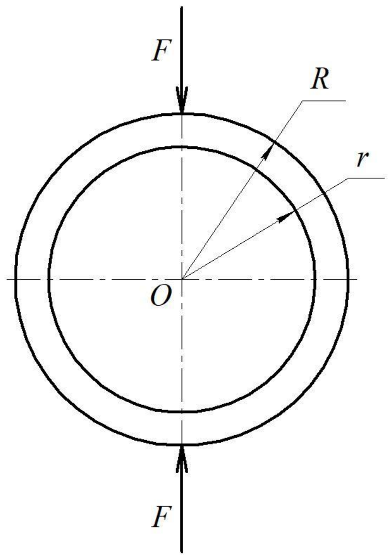


**Fig. 3.** The loading of the half-coupling by two concentrated forces

According to this theory, the cut is made along the axis of symmetry, thus dividing the load equally along the edges of the section, and we get [6]

$$M_F(\theta) = -\frac{F}{2}\sin\theta. \tag{2}$$

We determine the components of equation (1)

$$\int_0^{\pi} M_F(\theta)\sin\theta d\theta = -\frac{FR}{2}\int_0^{\pi}\sin\theta d\theta = -FR;\ \int_0^{\pi} M_F(\theta)\cos\theta d\theta = 0. \tag{3}$$

From equation (1), we obtain the dependence for determining the bending moment in the section [6]

$$M(\theta) = FR\left(\frac{1}{\pi} - \frac{\sin\theta}{2}\right). \tag{4}$$

In the case when the plane $\theta = 0$, we have the value of the bending moment

$$M(0) = \frac{FR}{\pi}. \tag{5}$$

In the case when the plane $\theta = \pi/2$, we have the value of the bending moment

$$M\left(\frac{\pi}{2}\right) = FR\left(\frac{1}{\pi} - \frac{1}{2}\right). \tag{6}$$

In the case when all the balls are engaged, the load on the half-coupling is distributed more evenly. Consider the example of loading a half-coupling with eight concentrated equal radial forces, which are applied uniformly around the circle (Fig. 4). That is, the freewheel ball clutch contains eight balls that are engaged and transmit torque from the driving to the driven half-coupling.

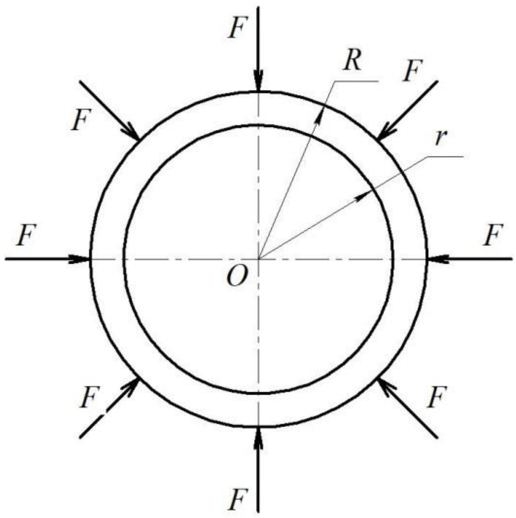


**Fig. 4.** The loading of the half-coupling by eight concentrated forces

For this case, we determine the bending moments and normal transverse forces in the cross-section of the half-coupling in the case $0 \le \theta \le \alpha$ [6]:

$$M(\theta) = FR\left(\frac{1}{\alpha} - \frac{\cos\left({}^{\alpha}/_{2} - \theta\right)}{2\sin\left(\frac{\alpha}{2}\right)}\right); \; N(\theta) = -\frac{F}{2\sin\left(\frac{\alpha}{2}\right)}\cos\left(\frac{\alpha}{2} - \theta\right), \tag{7}$$

where $\alpha = \frac{2\pi}{n}$ – the force application angle in radians; *n* – the number of concentrated forces.

The displacement of the point of force application relative to the center of the half-coupling is determined by the formula

$$w = \frac{FR^2}{EJ} \cdot \frac{1}{2\sin^2\left({}^{\alpha}/_{2}\right)}\left(\frac{\alpha}{4} + \frac{1}{4}\sin\alpha \cdot \frac{2\sin^2\left({}^{\alpha}/_{2}\right)}{\alpha}\right) + \frac{FR}{EA_1} \cdot \frac{1}{2\sin^2\left({}^{\alpha}/_{2}\right)}\left(\frac{\alpha}{4} + \frac{\sin\alpha}{4}\right), \tag{8}$$

where $E$ – is the modulus of elasticity of the half-coupling material; $A_1$ – cross-sectional area; $J$ – the moment of inertia of the section.

In the case when there will be a larger number of balls in the engagement of the ball coupling, and the angle between the concentrated forces will be a<20°, the movement of the point of force application relative to the center of the ring will be

$$w = \frac{FR^2}{EJ} \cdot \frac{\alpha^2}{720°} + \frac{FR}{EA_1} \cdot \frac{1}{\alpha}. \tag{9}$$

According to the formulas given above, we will calculate the free-wheel ball clutch of a bicycle, which has the following geometric dimensions: ball radius of 5 mm; the outer radius of the working area of the driven semi-coupling $\boldsymbol{R}$ = 24 mm; the inner radius of the driven half-coupling $\boldsymbol{r}$ = 11 mm; width of the working area $\boldsymbol{h}$ = 10 mm; modulus of elasticity of the steel material $\boldsymbol{E} = 2.1 \cdot 10^5$ MPa. When two oppositely placed balls are engaged (Fig. 3), we take the force acting from one ball on the driven half-coupling equal to $\boldsymbol{F}$ = 1000 N.

For this case of a symmetrically loaded driven half-coupling, according to formula (2), the load is distributed evenly along the edges of the section

$$M_F(\text{q}) = -\frac{1000}{2} sin\text{q} = -500\, N \cdot m.$$

According to equation (4), we determine that the bending moment in the section q = 45° is equal

$$M(45°) = 1000 \cdot 0.24 \cdot \left(\frac{1}{\pi} - \frac{\sin\ 45°}{2}\right) = -0.85\, N \cdot m.$$

From equation (5), we determine the bending moment in the section for the plane q = 0°

$$M(0°) = \frac{1000 \cdot 0.24}{3.14} = 7.64\, N \cdot m.$$

From equation (6), we determine the bending moment in the section for the plane q = p/2

$$M\left(\frac{\pi}{2}\right) = 1000 \cdot 0.24 \cdot \left(\frac{1}{3.14} - \frac{1}{2}\right) = -4.36\, N \cdot m.$$

The diagram of the bending moments of the driving half-coupling's working section is shown in Fig. 5.

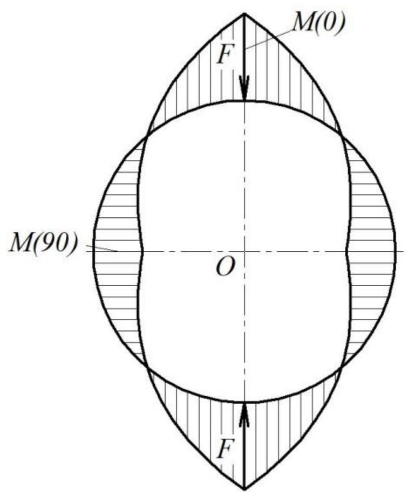


**Fig. 5.** Diagram of bending moments in the case of half-coupling loading by two concentrated forces

Consider the case when all the balls are engaged, and the load on the half-coupling is distributed evenly (Fig. 4). We assume that eight engaged balls transmit the torque. We think the load is distributed evenly between the balls, and the force transmitted by one ball to the driven half-coupling equals $\boldsymbol{F}$ = 250 N.

According to formula (7), we calculate the bending moments in the section of the half-coupling in the section 0£q£a:

$$M(0°) = 250 \cdot 24\left(\frac{1}{0{,}785} - \frac{cos\left(0.785/_2 - 0\right)}{2sin\left(\frac{0.785}{2}\right)}\right) = 2.1\ N \cdot m;$$

$$M(10°) = 250 \cdot 24\left(\frac{1}{0.785} - \frac{cos\left(0.785/_2 - 0.175\right)}{2sin\left(\frac{0.785}{2}\right)}\right) = 1.78\ N \cdot m;$$

$$M(20°) = 250 \cdot 24\left(\frac{1}{0.785} - \frac{cos\left(0.785/_2 - 0.349\right)}{2sin\left(\frac{0.785}{2}\right)}\right) = 1.65\ N \cdot m;$$

$$M(30°) = 250 \cdot 24\left(\frac{1}{0.785} - \frac{cos\left(0.785/_2 - 0.524\right)}{2sin\left(\frac{0.785}{2}\right)}\right) = 1.69\ N \cdot m;$$

$$M(45°) = 250 \cdot 24\left(\frac{1}{0.785} - \frac{cos\left(0.785/_2 - 0.785\right)}{2sin\left(\frac{0.785}{2}\right)}\right) = 2.1\ N \cdot m;$$

where $\alpha = \frac{2 \cdot 3.14}{8} = 0.785\ rad$ – angle of force application in radians; $n$ – the number of concentrated forces, $n = 8$.

According to formula (7), we calculate the normal forces in the half-coupling section in the case of 0£q£a:

$$N(0°) = -\frac{250}{2sin\left(\frac{0.785}{2}\right)} cos\left(\frac{0.785}{2} - 0\right) = -301.78\ N;$$

$$N(10°) = -\frac{250}{2sin\left(\frac{0.785}{2}\right)} cos\left(\frac{0.785}{2} - 0.175\right) = -318.91\ N;$$

$$N(20°) = -\frac{250}{2sin\left(\frac{0.785}{2}\right)} cos\left(\frac{0.785}{2} - 0.349\right) = -326.34\ N;$$

$$N(30°) = -\frac{250}{2sin\left(\frac{0.785}{2}\right)} cos\left(\frac{0.785}{2} - 0.524\right) = -323.85\ N;$$

$$N(45°) = -\frac{250}{2sin\left(\frac{0.785}{2}\right)} cos\left(\frac{0.785}{2} - 0.785\right) = -301.78\ N.$$

We calculate the cross-sectional area using the formula

$$A_1 = \pi(R^2 - r^2) = 3.14 \cdot (24^2 - 11^2) = 1429\ mm^2 = 0.00143\ m^2.$$

The formula calculates the moment of inertia of the section

$$J_y = \frac{\pi(D^3 - d^3)}{32D} = \frac{\pi(R^3 - r^3)}{4R} = \frac{3.14(24^3 - 11^3)}{4 \cdot 24} = 409\ mm^3.$$

According to formula (8), we calculate the displacement of the point of force application relative to the center of the half-coupling

$$w = \frac{250 \cdot 0.24^2}{2.1 \cdot 10^5 \cdot 409 \cdot 10^{-6}} \cdot \frac{1}{2 \cdot sin^2\left(0.7855/_2\right)} \left( \frac{0.785}{4} + \frac{1}{4} \sin \ 0.785 \cdot \frac{2\, sin^2\left(0.785/_2\right)}{0.785} \right) +$$

$$+ \frac{250 \cdot 0.24}{2.1 \cdot 10^5 \cdot 1429 \cdot 10^{-6}} \cdot \frac{1}{2\, sin^2\left(0.785/_2\right)} \left( \frac{0.785}{4} + \frac{sin\, 0.785}{4} \right) = 0.02697\, mm,$$

where $E$ – the modulus of elasticity of the half-coupling material, for steel $E = 2.1 \cdot 10^5$ MPa.

The obtained maximum deflections of the working section of the driven half-coupling in the case of a uniform load are about 0.003 mm, which allows them to be neglected during design.

The diagram of the bending moments of the driving half-coupling's working section, when all the balls are engaged and the load is distributed evenly, is shown in Fig. 6.

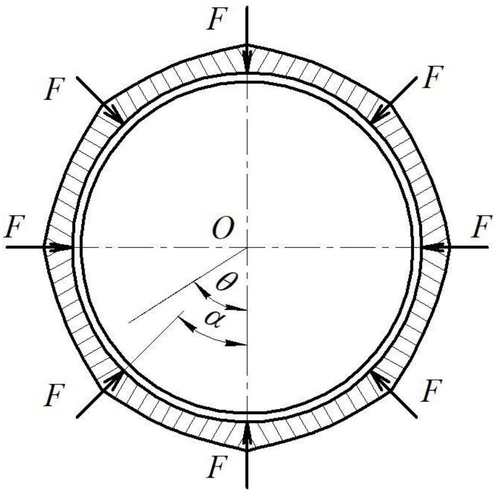


**Fig. 6.** Diagram of bending moments in the case of loading of the driven half-coupling under the action of eight concentrated forces

Comparing the obtained calculation results, it is noticeable that in the case of eight balls in engagement, the curve of bending moments is more evenly distributed along the working surface of the driven half-coupling compared to the case with two concentrated forces. Also, the value of the bending moments in the sections of the half-coupling in the case of a uniform load is 3.6 times lower than in the case of engagement of only two balls, 7.64 N·m versus 2.1 N·m. When two balls are in contact, the most enormous bending moments occur in the places of action of forces, and there are also areas with minimum moments and moments of the opposite direction.

The resulting dependencies (2)–(6) are essential for solving the applied problem of determining the main force characteristics of free-wheel ball clutches in the case of simultaneous engagement of two balls. This case is the most unfavorable for the clutch operation and allows you to assess the critical loads during the torque transfer from the driving to the driven link.

During the free-wheel ball clutch design, the most appropriate material for manufacturing half couplings is selected. In the case of obtaining significant deflections of the working surface of half-couplings, the outer diameter of the driven half-coupling is increased; that is, the rigidity is increased.

Dependencies (7)–(9) make it possible to estimate the strength of the elements of the ball joint in the case of its uniform load when all balls are engaged.

The results confirm the practical value of the proposed methodology for calculating the strength of half-couplings of a new free-running ball coupling. The case when two balls are in contact can be considered extreme, and the values of transverse moments and deflections can be determined only to identify their maximum probable values. In reality, in practice, several balls will be engaged more often than not. Therefore, the working stresses in the cross-sections and the deflections of the half-couplings when they are uniformly loaded will be much smaller.

It should be noted that the actual stresses and deflections of half-couplings will always be in the interval between their minimum and maximum values. Also, in addition to the number of working balls, the number and accuracy of manufacturing the half-coupling grooves can affect the load distribution in the free-running ball coupling. Increasing the accuracy of manufacturing grooves will affect the increase in the number of balls that are simultaneously engaged.

## Conclusions

The article proposes a methodology for calculating the strength of one of the structures of free-wheel ball clutches, which allows the selection of the main elements of these couplings for use in drives of machines and mechanisms. According to this method, it is possible to calculate forces in the contact zone of parts, bending moments, and axial deformations of the main elements of ball couplings and evaluate the strength of their elements.

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