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DETERMINING THE INFLUENCE OF CONTINUOUS SECTION SHAPE AND DIMENSIONS ON STRESSES OVER A WIDE RANGE OF VIBRATION FREQUENCY

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Abstract. *Problem statement.* Considering the prospects of using discrete-continuous vibration machines, there is a need to study the properties of continuous sections, which are elastic plates, in a wide range of frequencies. *Purpose.* It consists of studying the frequency response of continuous sections of various shapes and sizes from the harmonic disturbance force and identifying the frequency ranges in which the maximum stresses will be observed. *Methodology.* A schematic diagram of a discrete-continuous oscillating system is given. A set of necessary parameters is selected, which the studied samples of continuous sections should possess. Rectangular, diamond-shaped, X-shaped, parabolic convex and parabolic convex plates of different thicknesses corresponding to the given parameters were designed. Control point arrays were selected to investigate stresses in the plates. For each of the plates, a linear dynamic analysis was carried out with the help of simulation modeling. The dependence of the maximum stresses at the control points of the investigated plates on the frequency of the harmonic disturbance was determined. *Findings (results) and originality (novelty).* A linear dynamic analysis of continuous sections of various shapes and thicknesses was performed for the first time. The results obtained from the conducted research, in general, indicate the presence of stress amplifications occurring in plates of various shapes and sizes at certain frequencies of harmonic disturbance. In all investigated types of plates, stress amplification was found at the first and third natural frequencies of oscillations. It was determined that the largest stresses occur in the parabolic convex plate and the smallest – in the parabolic convex. *Practical value.* Frequency ranges have been established in which, with harmonic disturbance of continuous sections, it is possible to obtain significant dynamic amplification of oscillations of discrete-continuous vibrating machines with an electromagnetic drive. Recommendations for choosing the optimal geometric parameters of the plates as continuous sections of vibration machines, in which they could be operated for an infinite number of load cycles, are described. *Scopes of further investigations.* The reaction of continuous sections with various shapes and dimensions to the simultaneous interaction of several forces with different frequencies and amplitudes of disturbance requires further research.

Keywords: vibrating machine, oscillating system, continuous section, electromagnetic drive, linear dynamic analysis.

Introduction

The widespread introduction of energy-efficient technologies in various industries has contributed to the spread of both new types of vibration technological equipment and the improvement of parameters and modes of operation of existing structures. A new class of vibration machines was created, which uses the dynamic potential of continuous sections – structures endowed with both inertial and stiffening parameters. Due to the use of inter-resonance modes of operation, such technological equipment is much more energy efficient than classic two-mass vibration machines. At the same time, for the most part, one of the resonances is formed by the oscillating system as a whole, and the second is determined by the first natural frequency of oscillations of the continuous section. Considering the multifrequency of the system, due to the presence of an infinite number of natural frequencies and forms of oscillations of the continuous section in the form of a plate, there is a potential for further improvement of the energy efficiency of this type of vibration equipment. There is a possibility of the presence of frequency ranges where the dynamic amplification of oscillations of the continuous section and the working body of the oscillating system would acquire maximum values.

Problem Statement

Studying the influence of geometric dimensions and shapes of continuous sections on stress in a wide range of frequencies would allow us to improve both the energy efficiency indicators of this vibrating machine class and its operational characteristics.

Therefore, in this article, the authors will try to form ideas for further improvement of the operating modes of discrete-continuous vibration machines with an electromagnetic drive by conducting a linear dynamic analysis of continuous sections of various shapes and sizes.

Analysis of Modern Information Sources on the Subject of the Article

Improvement of methods of calculation and design of highly efficient inter-resonance vibration technological equipment has been carried out for a long time [1, 2]. For the most part, three-mass discrete systems were considered, the ultralight reactive mass of which had a rapid-like characteristic in the interresonance zone of oscillations. The relatively recently created class of discrete-continuous inter-resonance vibration machines has some advantages [3], particularly the ability to use the dynamic potential of a continuous section in the form of a rod or plate. However, in these studies, to achieve the necessary operating modes of the equipment, only the first natural frequency of oscillations of the continuous section was used.

I. Nazarenko and O. Dedov [4, 5] investigated the operation of vibration machines in various frequencies. In particular, work for compaction of concrete mixtures in the frequency range from 100 to 300 Hz is considered in work [4]. These studies describe the impact of higher harmonics in dynamic systems due to their complex movement in the process of interaction with the technological load. However, one-mass vibrating machines with an inertial drive were considered.

Multi-frequency vibro-impact technological equipment is studied in works [6, 7]. The influence of the stiffness coefficient of intermediate supports on the frequency of natural oscillations of a rod system containing a flat spring is considered [6]. In this paper, an analysis of the dynamic strength and service life of a flat spring was also carried out to further apply the obtained results in the creation of resonant vibroimpact systems. The elastic element in the form of a flat plate in the considered work has another rigid fixation in the oscillating mass. The dynamic analysis of the rod two-mass vibration shock system with intermediate supports was carried out in [7]. Based on the dynamic analysis results, a stress check was carried out, taking into account the conditions of bending and contact stiffness.

In the article [8], using the approximate Rayleigh – Ritz method, the first natural frequency of oscillations of the continuous section (plate) of the inter-resonance vibrating table with an electromagnetic drive is established. The basic function of the deflections of the surface of the continuous section is presented as a product of the transverse deflections along two coordinates with hinged fastening at four points. For this purpose, the kinetic and potential energies of the continuous section were presented in integral form.

Considering the promising development of discrete-continuous vibration machines, it is expedient to study the strength of continuous sections using linear dynamic analysis (harmonic disturbance) over a wide range of frequencies.

Statement of Purpose and Tasks of Research

Study of the frequency response of continuous sections of various shapes and sizes from a harmonic disturbance force. Identification of frequency ranges in which maximum stresses will be observed. Determination of the stresses that will occur at the control points of the continuous sections of the vibrating machine.

The Main Material Presentation

The continuous section is the third oscillating mass of the vibrating table with an electromagnetic drive (Fig. 1). This equipment is intended for the compaction of concrete mixtures used for the formation of concrete and reinforced concrete products. Another example of the use of a vibrating table is the possibility of its use in foundry production for the compaction of molding mixtures.

A vibrating table with an electromagnetic drive consists of the first 1, the second 2, and the third 3 oscillating masses. The first 1 and the second 2 oscillating masses are connected by means of an elastic node 4. The third oscillating mass 3, which is a continuous section in the form of an elastic plate, is fixed in the second discrete oscillating mass 2 with the hinged supports 5. The movable structure of the vibrating table is attached to a fixed base 6 through the system of vibration isolators 7.

The continuous section in the vibrating table plays the role of the third (reactive) mass, which transmits disturbances from the electromagnetic drive to the entire vibrating system. For the effective operation of the vibrating table, it is necessary to correctly select the parameters of the continuous section (type of fastening, geometric dimensions, material, natural frequencies of oscillations).

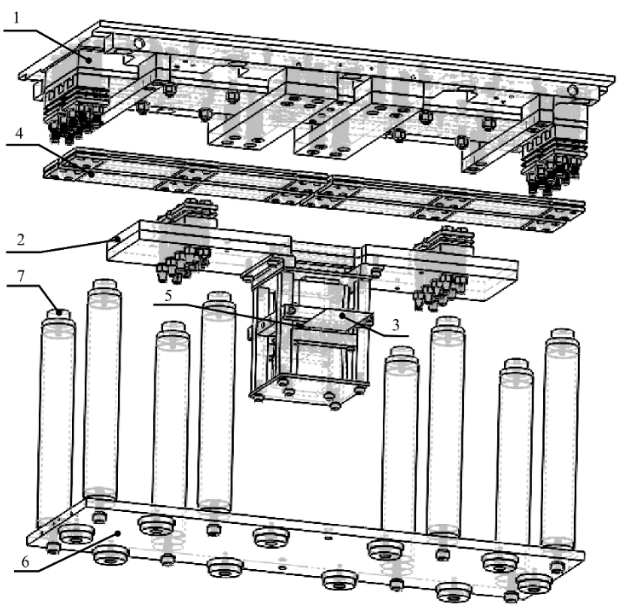

Fig. 1. Exploded view of the vibration table: 1 – first (active) mass; 2 – second (intermediate) mass; 3 – third mass (continuous section); 4 – system of elastic units connecting active and intermediate masses; 5 – hinged supports; 6 – fixed base; 7 – system of vibration isolators

One of the most important parameters of a continuous section during the synthesis of a discrete-continuous system is its natural oscillation frequency ω_{in}. The rational selection of the natural frequencies of oscillations of the continuous section in combination with the establishment of a specific mode of its disturbance allows for obtaining high energy efficiency in the operation of this type of discrete-continuous vibration machine. The use of resonant modes of operation common in vibration technology [1] can have a negative effect on the longevity of the functioning of continuous sections, which are thin metal plates. Therefore, in the case of discrete-continuous vibration machines, the interresonance mode of oscillations is used [2]. It consists of disturbing the oscillating system at the frequency of forced oscillations between two resonant peaks, one of which is caused by the first natural frequency of the continuous section. However, the continuous section, being a flexible body with infinite degrees of freedom [2], has many of its natural forms and frequencies of oscillations. A change in the geometric dimensions and the shape of the plate significantly affects the value of the natural frequencies of oscillations. Taking into account that the drive of the vibrating machine works from the power grid at a constant frequency, we take as the necessary parameter the first natural frequency of oscillations of the continuous section, which should be $\omega = 49 \pm 0{,}2$ Hz.

Also, an important parameter of the continuous section in the case of discrete-continuous vibration machines is the dimensions for its attachment in the intermediate mass. The scheme of fixing continuous sections is shown in Fig. 2.

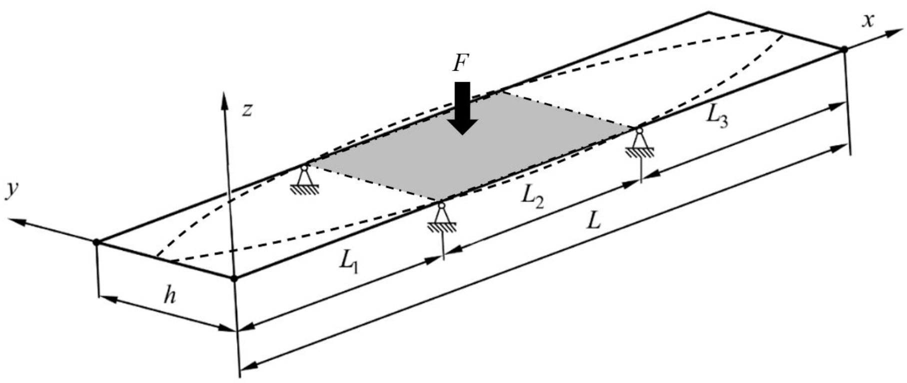

Fig. 2. Schematic diagram of fixing plates

From Fig. 2, it can be seen that the mutual location of the points of hinge fastening has clearly defined geometric parameters of the width h and length of the middle section L_2. The force of plate disturbance by an electromagnet F is applied uniformly to the area marked in gray. These parameters should be constant for all samples of the investigated plates. The lengths of the ends of the plate L_1 and L_3 must be equal to each other.

In general, the totality of all the necessary parameters that the investigated plate samples should have been presented in the form:

$$\begin{cases} \omega_{1n} = 49 \pm 0{,}2 \text{ Hz}, \\ L_1 = L_3, \\ L_2 = 176 \text{ mm}, \\ h = 87{,}5 \text{ mm}, \\ F = 30 \text{ N}, \\ b = 2;\ 2{,}5;\ 3;\ 3{,}5;\ 4 \text{ mm}, \\ z = 0{,}03, \end{cases} \tag{1}$$

where b – thickness of the plates; z – damping coefficient, which is selected taking into account the fastening conditions, the geometric dimensions and material of the plates, as well as the frequency range of the harmonic disturbance [9, 10].

For research, we choose the following types of plates: rectangular, diamond-shaped, X-shaped, parabolic convex, and parabolic convex. The material of the plates is AISI 1020 carbon steel. Control points $P_1 ... P_i$ are defined on each plate, for which the dependence of the maximum von Mises stress on the frequency of the harmonic disturbance by force F will be displayed.
Thus, the general conditions for conducting research by using a simulation modeling have been formed.

Rectangular plate. The simplest and most common type of continuous section, for which, taking into account the symmetry of fastening, 5 control points are established for studying stresses (Fig. 3): at the apex of the rectangle (P_1), in the middle of the side of the wing of the plate (P_2), between the hinged supports along the width (P_3), at the point of intersection of the diagonals of the rectangle (P_4) and between the hinged supports along the length (P_5).

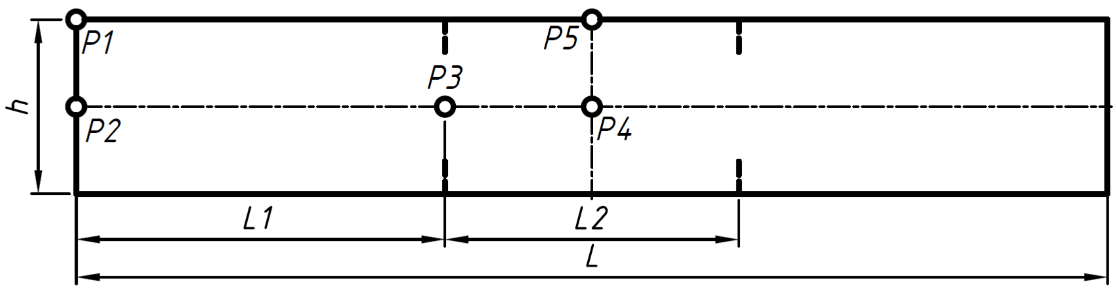

Fig. 3. Calculation scheme of the rectangular plate

Samples of rectangular plates with thicknesses determined in the system (1) were designed. Parameters of samples of rectangular plates are presented in the Table. 1.

Table 1

Geometric dimensions and natural frequencies of oscillations of rectangular plates

No	Thick-ness b, mm	Width h, mm	Length L_1, mm	Length L_2, mm	Length L, mm	Natural frequencies of oscillations, Hz				
						ω_{n1}	ω_{n2}	ω_{n3}	ω_{n4}	ω_{n5}
1	2	87.5	129.5	176	435	48.93	71.40	235.27	300.07	307.82
2	2.5	87.5	149.5	176	475	49.00	69.16	271.94	321.53	327.10
3	3	87.5	167.5	176	511	49.07	67.68	302.06	341.23	345.88
4	3.5	87.5	184.5	176	545	48.93	66.31	332.32	358.57	362.65
5	4	87.5	200	176	576	48.96	65.43	337.31	375.39	379.10

As can be seen from the Table 1, in rectangular plates with increasing thickness, when the first natural frequencies of oscillation are close in value, the values of the third (ω_{n3}), fourth (ω_{n4}), and fifth (ω_{n5}) natural frequencies increase. The value of the second (ω_{n2}) natural frequency of oscillation decreases.

Graphs of the dependence of stresses at the control points P_1 - P_5 of rectangular plates on the frequency of disturbance in the range $0 - 1000$ Hz are shown in Fig. 4.

From Fig. 4, it can be seen that the greatest stresses occur when the plates are disturbed by a harmonic force at the first and third natural frequencies of oscillation. At the same time, it is possible to observe a greater flatness of resonance peaks at the third natural frequency. It is obvious that the value of the maximum stresses decreases when the thickness of the plate increases. However, it is visible that the stress values that occur at the eighth natural frequency of plate oscillations (in the range of 500–600 Hz) almost do not change with the increase in the thickness of the continuous section.

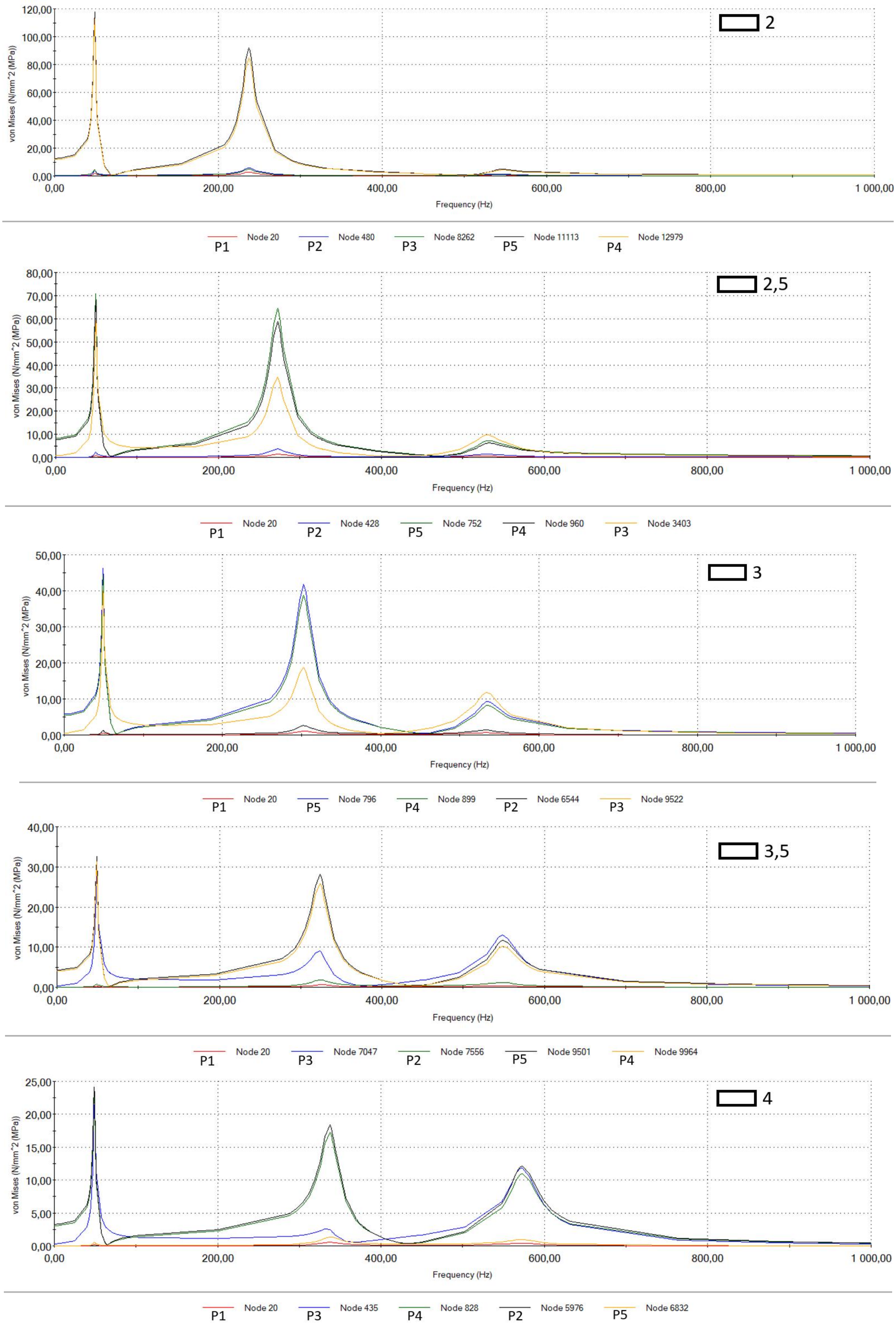

Fig. 4. Graphs of the dependence of stresses at control points P_1 - P_5 of rectangular plates on the frequency of disturbance (the thickness of the plates is shown in the upper right corner)

Diamond-shaped plate. Continuous section type with variable width. In the diamond-shaped plate, the stresses are investigated at four control points: at the end of the plate (P_1), between the hinge supports along the width (P_2), in the middle of the plate (P_3), and between the hinge supports along the length of the plate (P_4).

The narrowing of the ends in a diamond-shaped plate causes an increase in length compared to a rectangular plate at the same values of the first natural oscillation frequency. With the same thicknesses of the diamond-shaped plates, it is possible to more effectively disturb the working body of the discrete-continuous inter-resonance vibration machine [11]. The calculation scheme of the diamond-shaped plate is presented in Fig. 5.

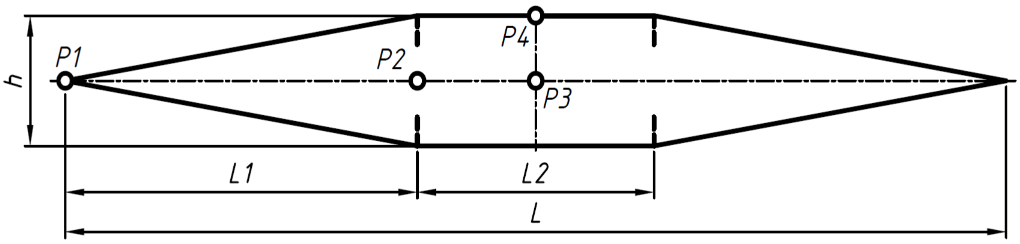

Fig. 5. Calculation scheme of the diamond-shaped plate

The parameters of the diamond-shaped plate samples designed concerning conditions (1) are presented in the Table. 2.

Table 2

Geometric dimensions and natural frequencies of oscillations of diamond-shaped plates

No	Thick-ness *b*, mm	Width *h*, mm	Length L_1, mm	Length L_2, mm	Length *L*, mm	Natural frequencies of oscillations, Hz				
						ω_{n1}	ω_{n2}	ω_{n3}	ω_{n4}	ω_{n5}
1	2	87.5	198	176	572	49.07	67.36	200.92	306.78	357.89
2	2.5	87.5	228	176	632	49.02	65.17	220.33	295.31	370.10
3	3	87.5	255	176	686	48.97	63.70	230.44	286.82	394.65
4	3.5	87.5	279.5	176	735	49.01	62.73	235.16	281.21	425.37
5	4	87.5	302.5	176	781	49.00	61.90	236.71	276.44	456.31

In diamond-shaped plates with increasing thickness, when the first natural frequencies of oscillation are close in value, the values of the third (ω_{n3}) and fifth (ω_{n5}) natural frequencies increase. The value of the second (ω_{n2}) and fourth (ω_{n4}) natural oscillation frequencies decreases.

Graphs of the dependence of stresses at the control points P_1 - P_4 of diamond-shaped plates on the frequency of disturbance in the range 0 - 1000 Hz are shown in Fig. 6.

From Fig. 6 it can be seen that, unlike rectangular plates, in diamond-shaped continuous sections, the greatest stresses occur when the plates are disturbed by a harmonic force at the first, third, and fifth natural frequencies of oscillation. At the thickness of the diamond-shaped plate, the stresses that occur during harmonic disturbance at the fifth natural frequency are greater than those that occur at the third natural frequency. At the same time, it is possible to observe some increase in the stresses that occur during the operation of the diamond-shaped plate at frequencies of 700–800 Hz. In this frequency range, there are the seventh, eighth, and ninth natural forms of oscillation. This makes it possible to obtain one continuous gentle stress peak.

It can also be seen that the stresses that occur in the center of the plate are significantly greater than the stresses at the ends and between the hinge supports.

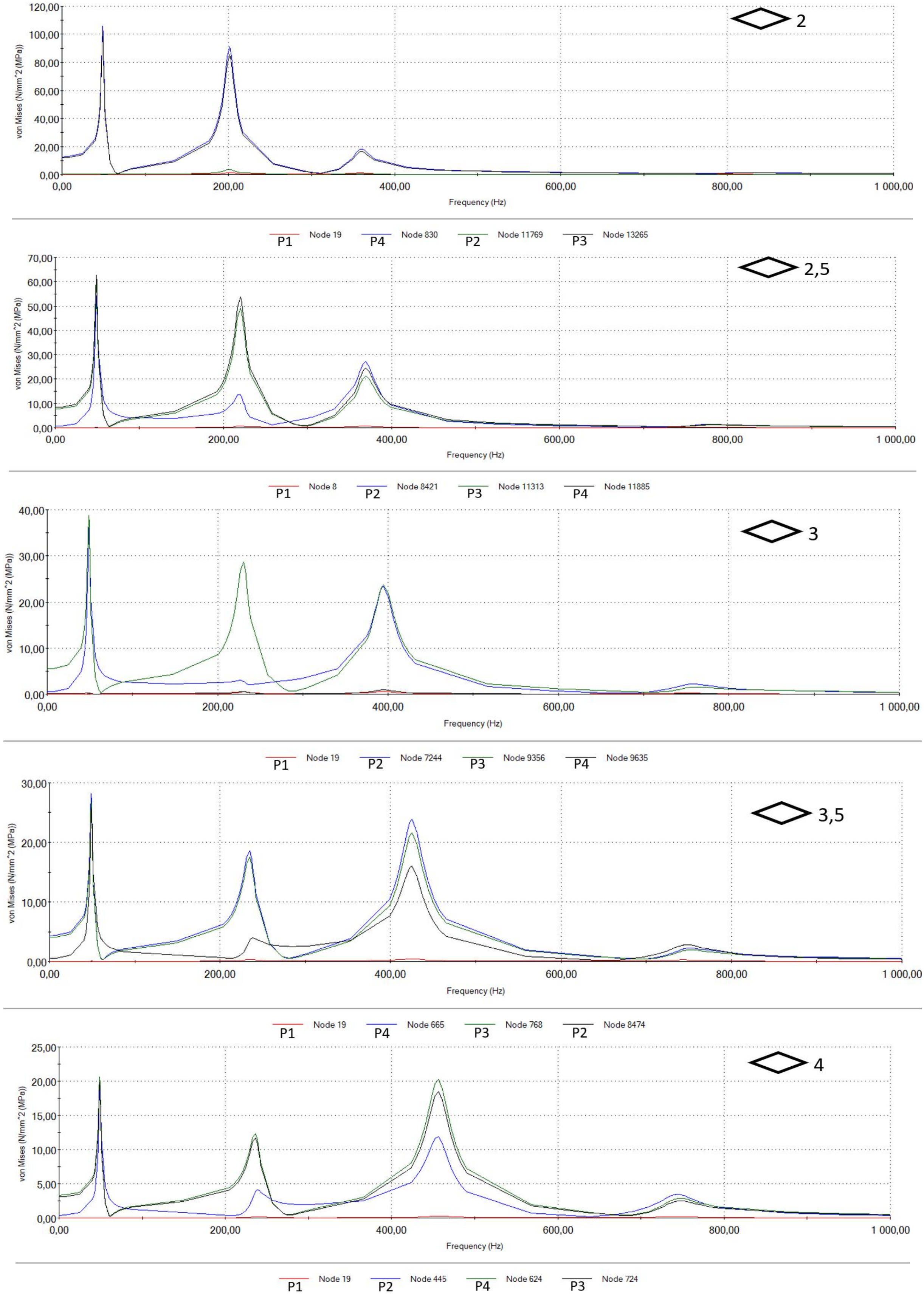

Fig. 6. Graphs of the dependence of stresses at control points P_1 - P_4 of diamond-shaped plates on the frequency of disturbance

X-shaped plate. A type of continuous section with variable width, for which 5 control points are set for the study of stresses (Fig. 7): at the apex of the polygon (P_1), in the middle of the side of the wing of the plate (P_2), between the hinge supports along the width (P_3), at the point of intersection of the diagonals of the central section between the hinge supports (P_4) and between the hinges supports along the length (P_5).

The expansion of the ends in a diamond-shaped plate causes a decrease in length compared to a rectangular plate at the same values of the first natural oscillation frequency. X-shaped plates are less effective than rectangular and diamond-shaped plates when disturbing the working body of a discrete-continuous inter-resonance vibration machine at frequencies close to the first natural one [11].

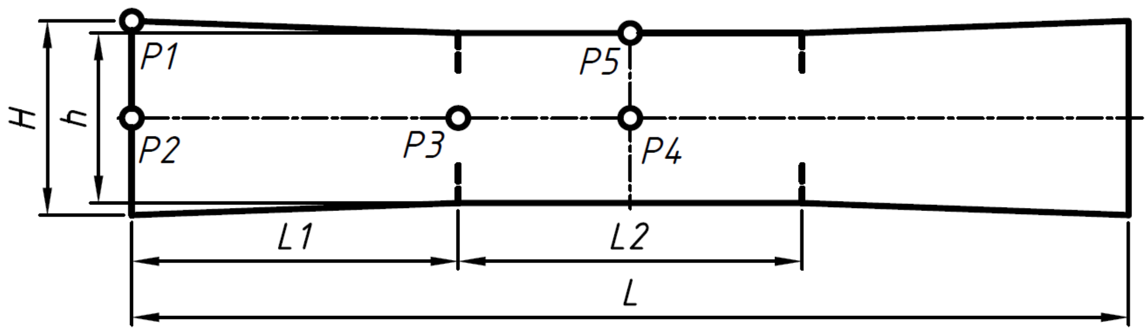

Fig. 7. Calculation scheme of the X-shaped plate

The parameters of the X-shaped plate samples designed concerning the system of conditions (1) are presented in the Table. 3.

Table 3

Geometric dimensions and natural frequencies of oscillations of X-shaped plates

No	Thick-ness *b*, mm	Width *h*, mm	Width *H*, mm	Length L_1, mm	Length L_2, mm	Length *L*, mm	Natural frequencies of oscillations, Hz				
							ω_{n1}	ω_{n2}	ω_{n3}	ω_{n4}	ω_{n1}
1	2	87.5	100	125.5	176	427	49.05	72.20	239.12	275.52	281.81
2	2.5	87.5	100	145	176	466	49.11	69.90	277.56	294.82	299.70
3	3	87.5	100	163	176	502	48.94	68.02	308.62	311.41	315.47
4	3.5	87.5	100	179	176	534	49.07	67.03	328.13	331.78	332.69
5	4	87.5	100	194.5	176	565	48.93	65.87	342.57	345.89	347.69

In X-shaped plates, as in the case of rectangular plates with increasing thickness, when the first natural frequencies of oscillation are close in value, the values of the third (ω_{n3}), fourth (ω_{n4}), and fifth (ω_{n5}) natural frequencies increase, and the values of the second (ω_{n2}) natural frequency increase oscillations decrease.

Graphs of the dependence of the stresses at the control points P_1 - P_4 on the disturbance frequency in the range 0 - 1000 Hz for X – similar plates are shown in Fig. 8.

From Fig. 8, it is possible to notice a significant increase in stresses during the operation of the continuous section at frequencies close to the first, third, and, partially (with increasing thickness), the seventh natural oscillation frequencies. The highest stresses in the X-shaped plate occur when disturbed at the first natural frequency.

Similarly to the diamond-shaped plate, in the X-shaped plate, the stresses arising in the center of the plate are significantly greater than the stresses at the ends and between the hinge supports.

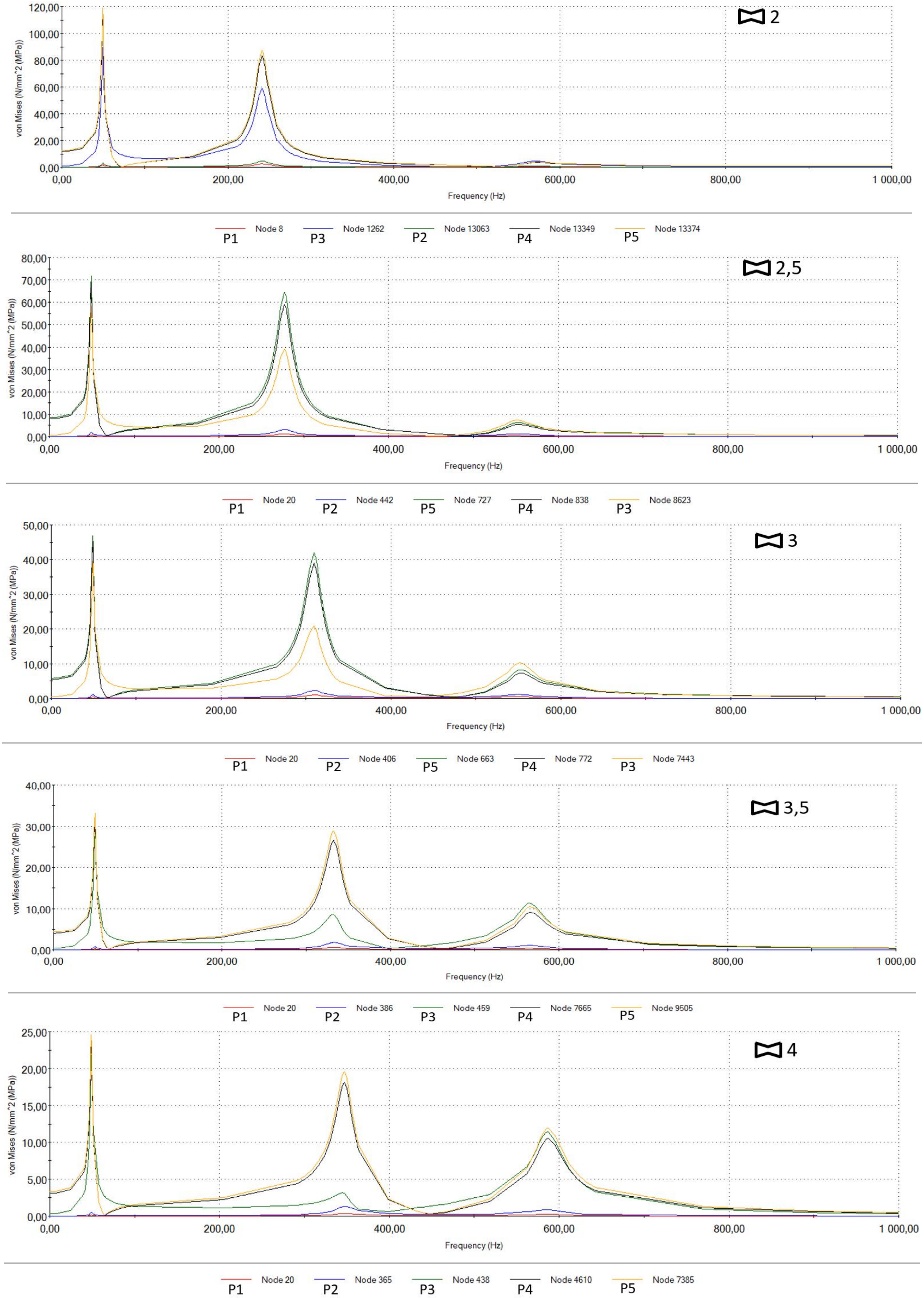

Fig. 8. Graphs of the dependence of stresses at control points P_1 - P_5 of X-shaped plates on the frequency of disturbance

Parabolic convex plate. A type of continuous section with variable width, the sides of which are parabolas and intersect at points with coordinates $(0,0)$ and $(0, L)$, and can be described using the set of equations:

$$y(x)=\left(-\frac{1}{2}\cdot\frac{h\cdot x^2}{L_1(L_1+L_2)}\right)+\left(\frac{1}{2}\cdot\frac{(2L_1+L_2)h\cdot x}{L_1(L_1+L_2)}\right), \tag{2}$$

$$y(x)=\left(\frac{1}{2}\cdot\frac{h\cdot x^2}{L_1(L_1+L_2)}\right)-\left(\frac{1}{2}\cdot\frac{(2L_1+L_2)h\cdot x}{L_1(L_1+L_2)}\right). \tag{3}$$

For this continuous section, 4 control points are set for the study of stresses (Fig. 9): at the point of intersection of parabolas (P_1), between the hinge supports along the width (P_2), in the middle of the plate, (P_3) and between the hinge supports along the length (P_4).

The narrowing of the ends in a parabolic convex plate, similar to a diamond-shaped plate, causes an increase in length compared to a rectangular plate at the same values of the first natural oscillation frequency. A parabolic convex plate can more effectively disturb the working body of a discrete-continuous inter-resonance vibration machine [11] compared to a rectangular one. The calculation scheme of the parabolic convex plate is presented in Fig. 9.

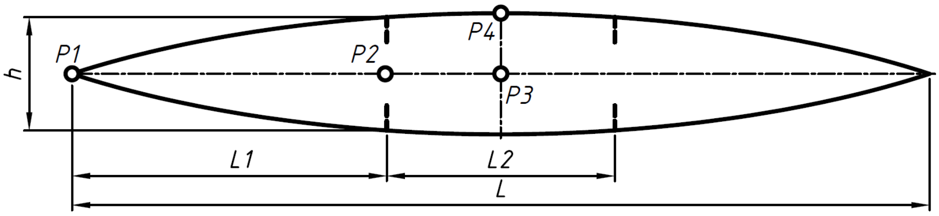

Fig. 9. Calculation scheme of a parabolic convex plate

The parameters of the designed samples of the parabolic convex plate corresponding to the system of conditions (1) are shown in the Table. 4.

Table 4

Geometric dimensions and natural frequencies of oscillations of parabolic convex plates

No	Thick-ness *b*, mm	Width *h*, mm	Length L_1, mm	Length L_2, mm	Length *L*, mm	Natural frequencies of oscillations, Hz				
						ω_{n1}	ω_{n2}	ω_{n3}	ω_{n4}	ω_{n5}
1	2	87.5	186.5	176	549	48.95	67.23	205.75	330.21	377.37
2	2.5	87.5	213	176	602	49.05	65.59	231.77	322.36	389.59
3	3	87.5	237	176	650	49.05	64.40	247.79	315.93	413.31
4	3.5	87.5	259	176	694	49.08	63.53	256.42	311.13	444.47
5	4	87.5	280	176	736	48.94	62.62	259.99	306.21	477.94

In the parabolic convex plate, as in the case of diamond-shaped plates, with an increase in thickness, when the first natural frequencies of oscillation are close in value, the values of the third (ω_{n3}) and fifth (ω_{n5}) natural frequencies increase. The value of the second (ω_{n2}) and fourth (ω_{n4}) natural oscillation frequencies decreases.

Graphs of the dependence of stresses at control points $P_1 - P_4$ on the disturbance frequency in the range $0 - 1000$ Hz for parabolic convex plates are shown in Fig. 10. From Fig. 10, it is possible to observe a significant increase in stresses during the operation of the continuous section at frequencies close to the first, third and fifth natural frequencies of oscillation. Partially, with an increase in thickness, there is an increase in stresses during operation in the frequency range $800 - 900$ Hz. The largest stresses in parabolic convex plates with a 2–4 mm thickness occur when the first natural frequency is disturbed.

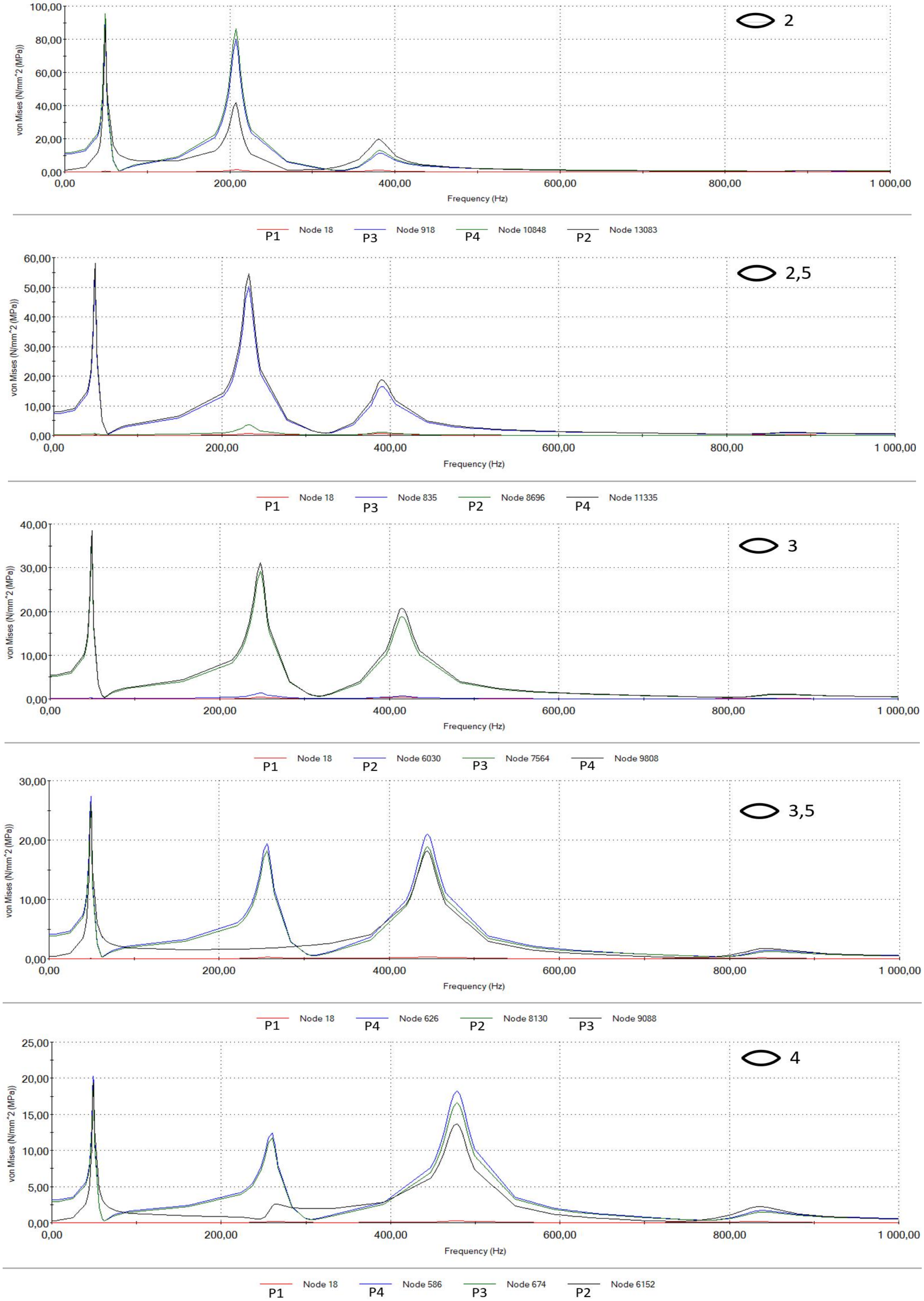

Fig. 10. Graphs of the dependence of stresses at control points P_1 - P_4 of parabolic convex plates on the frequency of disturbance

Parabolic concave plate. A type of continuous section with variable width whose sides are parabolas and do not intersect. They can be described using equations:

$$y(x)=\left(\frac{1}{2}\cdot\frac{(H-h)x^2}{L_1(L_1+L_2)}\right)+\left(-\frac{1}{2}\cdot\frac{(2HL_1-2hL_1+L_2H-L_2h)x}{L_1(L_1+L_2)}\right)+\frac{1}{2}\cdot H, \tag{4}$$

$$y(x)=\left(-\frac{1}{2}\cdot\frac{(H-h)x^2}{L_1(L_1+L_2)}\right)+\left(\frac{1}{2}\cdot\frac{(2HL_1-2hL_1+L_2H-L_2h)x}{L_1(L_1+L_2)}\right)-\frac{1}{2}\cdot H. \tag{5}$$

The calculation scheme of the parabolic concave plate is shown in Fig. 11. For a continuous section in the form of a parabolic concave plate, 5 control points have been established for studying stresses: at the apex (P_1), in the middle of the wing side of the plate (P_2), between the hinged supports in width (P_3), at the point of intersection of the diagonals of the central section between the hinged supports (P_4) and between the hinged supports in length (P_5).

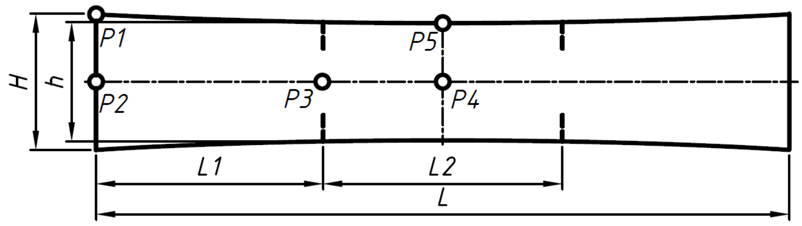

Fig. 11. Calculation scheme of a parabolic concave plate

The geometric dimensions and natural frequencies of oscillations of the studied samples of parabolic convex plates with thicknesses of 2–4 mm, corresponding to the conditions of the system (1) are presented in the Table. 5.

Table 5

Geometric dimensions and natural frequencies of oscillations of parabolic concave plates

No	Thickness *b*, mm	Width *h*, mm	Width *H*, mm	Length L_1, mm	Length L_2, mm	Length *L*, mm	Natural frequencies of oscillations, Hz				
							ω_{n1}	ω_{n2}	ω_{n3}	ω_{n4}	ω_{n5}
1	2	87.5	100	125	176	426	49.08	72.43	239.52	278.63	284.57
2	2.5	87.5	100	145	176	466	48.94	69.73	277.33	297.12	301.77
3	3	87.5	100	162.5	176	501	49.08	68.31	309.08	315.20	319.19
4	3.5	87.5	100	179	176	534	49.01	66.97	331.17	332.18	334.71
5	4	87.5	100	194.5	176	565	48.91	65.85	345.97	347.11	349.22

As can be seen from the Table 5, the parameters of parabolic concave plates are close in value to the corresponding parameters of X-shaped plates. Therefore, as in the case of X-shape plates and rectangular plates, with an increase in thickness, when the first natural frequencies of oscillation are close in value, the values of the third (ω_{n3}), fourth (ω_{n4}), and fifth (ω_{n5}) natural frequencies increase, and the value of the second (ω_{n2}) natural frequency of oscillations decreases.

Graphs of the dependence of stresses at control points $P_1 - P_5$ on the disturbance frequency in the range $0 - 1000$ Hz for parabolic concave plates are shown in Fig. 12.

From Fig. 12, it can be seen that a significant increase in stress occurs during the operation of the continuous section at frequencies close to the first, third, and, partially (with increasing thickness), the seventh natural oscillation frequencies. The greatest stresses occur during the disturbance at the first natural frequency.

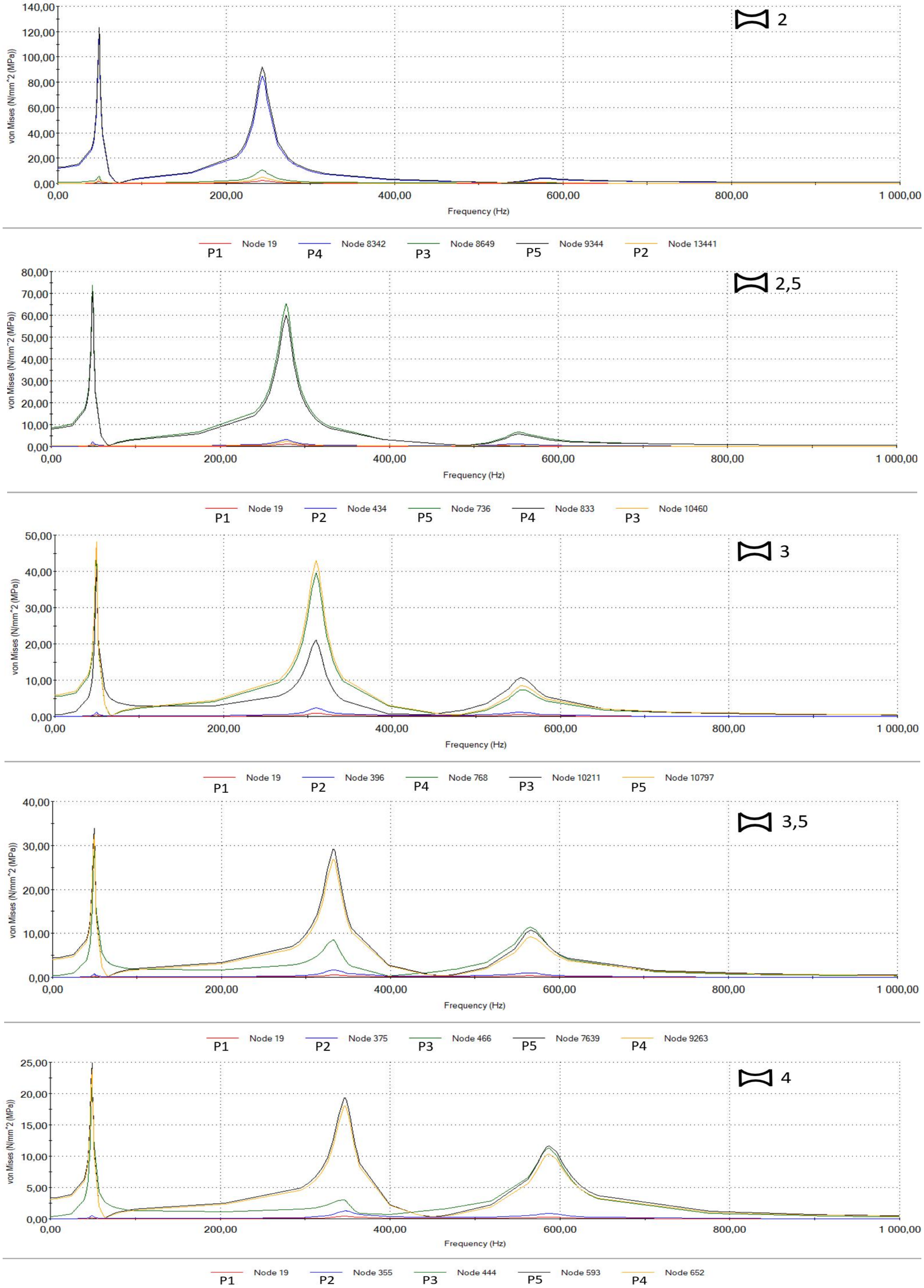

Fig. 12. Graphs of the dependence of stresses at control points P_1 - P_5 of parabolic concave plates on the frequency of disturbance

Results and Discussion

The results obtained from the conducted research, in general, indicate the presence of stress amplifications occurring in plates of various shapes and sizes at certain frequencies of harmonic disturbance. However, not every natural frequency of plate oscillation causes a sharp increase in stress. In all investigated types of plates, there is an increase in stresses at the first and third natural frequencies of oscillations. Amplifications often occur at the seventh natural frequency of oscillation. In the diamond-shaped plate, it is also possible to observe some increase in the stresses that occur when working at 700–800 Hz frequencies, where the seventh, eighth, and ninth natural forms of oscillations are located, forming one continuous gentle stress peak.

The dependence of the increase in the values of the maximum stresses on the decrease in the thickness of the plate is obvious, which is valid for all forms of plates. At the same time, for all plates with a thickness of 2 mm, the largest stresses occur in the parabolic concave plate and the smallest – in the parabolic convex. If we compare the two most energy-efficient types of plates – diamond-shaped and parabolic convex, with similar concepts of geometry change, we can see slightly lower maximum stresses occurring in parabolic convex plates with thicknesses of 2 and 2.5 mm. When the plate thickness increases to 3 mm, the maximum stress parameters for diamond-shaped and parabolic convex continuous sections become close in value (difference $<1\ \text{MPa}$).

Also, from the results of the conducted research, it can be seen that the third peak of the amplification of the maximum stresses arising in the plates, as their thickness increases, becomes more noticeable compared to the first and second peaks. The graph of the dependence of the change in maximum stresses during the third peak of stress amplification on the thickness of the plates is shown in Fig. 13.

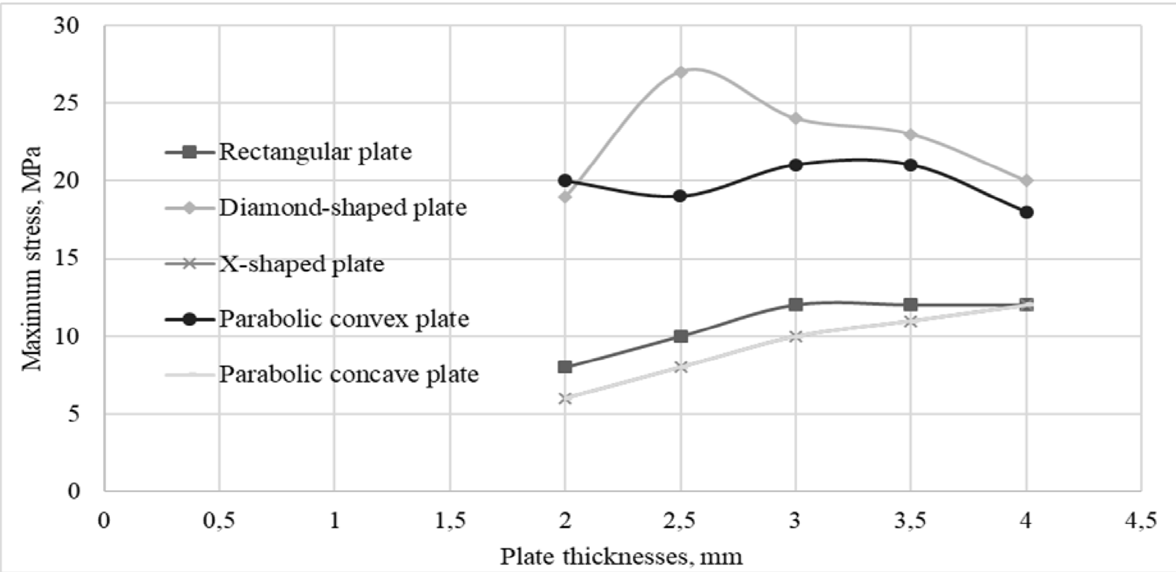

Fig. 13. Graph of the dependence of the change in maximum stresses during the third peak of stress amplification on the thickness of the plates

As shown in Fig. 13, diamond-shaped and parabolic convex plates form the highest stresses at the third amplification peak. The stress values for X-shaped and parabolic concave plates are identical.

Considering the fact that the yield point for the plate material, AISI 1020 steel, is $\sigma_m = 350\,\text{MPa}$, all types of plates can be briefly disturbed in the entire frequency range $0 - 1000\ \text{Hz}$ [12]. However, considering the material's fatigue strength curve and the plates' operating conditions (absence of high temperatures, aggressive environment), we can consider permissible stresses $\sigma < 32\ \text{MPa}$, as those at which the plates could theoretically be operated for an infinite number of load cycles.

Conclusions

A study of the influence of the shape and size of continuous sections on their stress in the frequency range $0 - 1000\ \text{Hz}$ was carried out. Continuous sections in the form of plates have many of their natural shapes and frequencies of oscillation. However, not every natural frequency of plate oscillation causes a sharp increase in stress. It was found that when the plates are harmonically disturbed at frequencies close in value to the first and third natural frequencies, a significant increase in the maximum stresses is observed. As their thickness increases, the third peak of the amplification of the maximum stresses arising in the plates becomes more noticeable compared to the first and second peaks.

Considering the material's fatigue strength curve and the plates' operating conditions, it is possible to consider permissible stresses $\sigma < 32\ \text{MPa}$, as those at which the plates could theoretically be operated for an infinite number of load cycles. Therefore, it is recommended to use plates with a thickness of at least 3 mm as a continuous section of discrete-continuous vibration machines with a distance between hinge supports of 176 mm. It is not recommended to disturb plates with a thickness of less than 4 mm at a distance between hinge supports of 176 mm at a frequency of forced oscillations, which is close in value to the first and third natural frequencies. The optimal options for use as a continuous section are diamond-shaped and parabolic convex plates. The reaction of continuous sections to the simultaneous interaction of several forces with different frequencies and amplitudes of disturbance requires further research.

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