UKRAINIAN JOURNAL OF MECHANICAL ENGINEERING AND MATERIALS SCIENCE
Vol. 10, No. 4, 2024

Ihor Hrytsai[1], Bohdan Stoliar[2], Serhiy Hromniuk[3]

[1] Department of Robotics and Integrated Mechanical Engineering Technologies, 12, S. Bandery str., Lviv, Ukraine, E-mail: ihor.y.hrytsai@lpnu.ua, ORCID 0000-0003-3675-5897
[2] Department of Robotics and Integrated Mechanical Engineering Technologies, 12, S. Bandery str., Lviv, Ukraine, E-mail: bohdan.t.stoliar@lpnu.ua, ORCID 0009-0003-7532-3410
[3] SPREnergo limited liability company, engineer, 106, Kulparkivs'ka str., Lviv, Ukraine, E-mail: sergro2013@gmail.com, ORCID 0000-0001-9246-6390

COMPARATIVE ANALYSIS OF INVOLUTE AND SINUSOIDAL GEAR TRANSMISSIONS IN MACHINE DRIVES UNDER CONTACT AND FRICTION CONDITIONS

Received: December 1, 2024 / Revised: December 10, 2024 / Accepted: December 12, 2024

https://doi.org/

Abstract. The study results of the conditions of contact and friction in two types of gear engagements – involute and sinusoidal – are presented. The main advantages and disadvantages of traditional involute gears are analyzed, and the advantages of sinusoidal gears in some operational parameters of machine drives are noted. It is shown that the sliding speed of the profiles and the amount of contact friction are significantly lower in the sinusoidal mashing. The presented results show that reducing friction in sinusoidal engagement drives reduces power loss by a factor of 1.75 and significantly increases the efficiency of such drives.

Keywords: Gear engagements; involute gears; sinusoidal gears; sliding; friction; efficiency; advantages.

Introduction

Gears are among the most widely used components in mechanical engineering, and gear transmissions, gearboxes, and reducers are integral parts of the drives of most modern mechanisms and machines. They transmit and transform motion from the engine to machine drives or mechanisms' output (drive) shaft. These are components of increased complexity and significant labor intensity, which are manufactured in large annual volumes in all branches of mechanical engineering and in various types of production – from small-series to mass production. Statistics show that the manufacture of gears accounts for 10–12 % of the total labor intensity of machines, and for certain groups of equipment and taking into account the production of spare parts – up to 35–40 %. Under the influence of progress, speeds in transmissions increase, and loads on gear pairs increase, which leads to a steady increase in the requirements for the quality of gears and reducers.

Problem Analysis

Gear meshing is a special type of engagement of profiled regular surfaces. The conditions of moving contact during the operation of a pair of gears largely determine the durability and performance of transmissions and affect the reliability indicators of the machine as a whole. This state of affairs causes significant interest among researchers and intensive development of theory and practice in this field. The importance of gears in mechanical engineering is confirmed by many publications presenting research results on various transmissions and engagement systems.

The most common transmission, traditionally used in machine drives, is the involute gear. From publications devoted to this topic, it is known that traditional involute gears have the following advantages compared to other types of gears and engagements:

- insensitivity to fluctuations in the centre distance;
- high stability of the gear ratio;
- the ability of teeth and profiles to height and angular correction;
- the ability of longitudinal modification of teeth over the width of the gear rim;
- the possibility of using one modular tool for cutting gears with different numbers of teeth.

At the same time, involute engagement has significant drawbacks, the most important of which are:

- limited load capacity;
- insufficient bending strength;
- limited cyclic endurance;
- linear contact of teeth in engagement, which causes:
  – installation errors of shafts and gears;
  – geometric inaccuracies in the assembly process of transmissions;
  – elastic deformations of shafts and bearing assemblies lead to uneven distribution of the load and increased contact stresses during their operation.

The aforementioned disadvantages are absent in another transmission and type of gear meshing – sinusoidal, in which the tooth profiles are described by a sinusoid. The structure of such teeth provides them with higher rigidity, so they are less susceptible to elastic deformation and have a higher bending strength limit. In addition, these transmissions have:

- up to 50 % higher load capacity;
- 10–15 % (7–9 dB) lower noise level;
- greater resistance to tooth breakage;
- upper limit of cyclical fatigue;
- sinusoidal gears can be both high-speed and power [1].

The researchers of this transmission also proved that this type of transmission, compared to involute, is characterized by a lower impact speed in engagement, which is important for operation at high speeds.

In addition, with the same dimensions and module as an involute gear, a sinusoidal gear has a smaller basic axial pitch. Therefore, with the same pitch diameter, it contains a larger number of teeth and has a larger axial overlap coefficient, which is an additional factor in increasing the smoothness of operation and lower acoustic emission during operation.

Increased interest in sinusoidal gears arose after the first publications on this topic, which presented the results of their research and technologies for cutting sinusoidal profiles [5, 9].

Today, many scientists and author teams study sinusoidal gears and technologies for their manufacture [5–14].

An analysis of the sources shows that the most common research objects are the following transmission parameters:

- load capacity;
- contact strength;
- cyclic fatigue;
- bending endurance of teeth.

At the same time, the following are left out of attention:

- conditions of tooth slip in transmissions;
- friction, which is the result of moving contact and significantly affects their wear resistance.

At high rotational speeds, the slip of tooth profiles, which are under significant load, causes them to heat up to high temperatures, especially during irreversible long-term transmission operation, as well as with insufficient lubrication. High temperature causes local hardening of the working surfaces, resulting in crumbling and/or chipping of the tooth material. In addition, at high slip speeds, the liquid lubricant is pressed into microcracks under high pressure, which causes a wedging action in the damaged surface and promotes crack growth.

At sufficiently high temperatures, wear of tooth surfaces can also occur through diffusion and adhesion (pressure welding), and break points can be in both the gear and the pinion. The intensity of profile slip and the pressure on their surfaces also determines such a common type of gear tooth failure as abrasive wear, which is caused by particles that separate from the tooth surfaces and fall onto the moving surfaces.

Based on this state of affairs, **the aim** is to study the contact conditions of teeth and compare the conditions of slip and friction in two types of transmissions – involute and sinusoidal.

Investigated Parameters and Obtained Results

1. Sliding velocity

1.1. Involute gearing is currently dominant in transmissions. The characteristic of the involute curve is that the length of the tangent line from the circle to the involute curve and the length of the arc to the tangent point are equal. Based on the diagram (Fig. 1), the polar equation of this curve is:

$$\rho = \frac{R_0}{\cos\alpha}, \tag{1}$$

R_0 – base circle radius; α – pressure angle (the angle between its radius vector and line tangent to the involute); ρ – radius vector of the corresponding point of the involute.

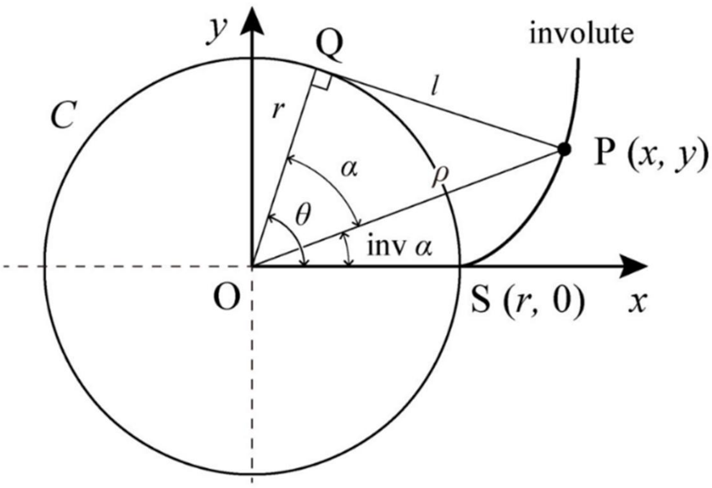


Fig. 1. Involute curve diagram

∠POS is defined as the involute function invα[rad]. When ∠POQ is α[rad], the length of the tangent line QP (l) and arc $\widehat{QS}$ is equal to the definition of the involute curve:

$$l = r \cdot \tan\alpha = \widehat{QS} = r \cdot \theta, \tag{2}$$

θ – roll angle[rad].

Therefore $\tan\alpha = \theta$, and

$$inv\alpha = \theta - \alpha = \tan\alpha - \alpha. \tag{3}$$

In equation (1) $R_0 = \frac{m \cdot Z_1}{2} \cdot \cos\alpha_\omega$,

Z_1 – number of teeth; α_ω – the gear engagement angle.

Let’s set the following initial data for calculations:

- *module*: m_1 = 2.5 mm; m_2 = 3.5 mm and m_3 = 5 mm (three options);
- *number of teeth*: gear Z_1 = 74, pinion Z_2 = 21;
- *rotational speed* of the gear is 500 rpm, pinion – 1762 rpm.

Accordingly:

- *angular velocity*: gear $500 \cdot \frac{2\pi}{60} = 52.3$ rad/s, pinion $1762 \cdot \frac{2\pi}{60} = 184.4$ rad/s;
- *values of the radii of the gear*: base circle R_o= 173.85 mm; root circle R_b= 178.75 mm; addendum circle R_a = 190 mm.
- *pressure angle of the transmission* α_ω =20°.

The limiting values of the angle α within the angle of involute development of the gear, which correspond to the radii of the root (R_b) and addendum (R_a) of the tooth, based on formula (1), are:

$$\alpha_{min} = \cos^{-1}\frac{R_0}{R_b};\ \alpha_{max} = a \cdot \cos^{-1}\frac{R_0}{R_a}. \tag{4}$$

According to the theory of gear meshing, the sliding velocity V_s of the involute profiles is determined from the dependence:

$$V_s = \frac{R_0 \cdot (\omega_1+\omega_2) \cdot (\tan\alpha_i - \tan\alpha_\omega) \cdot 60}{1000}, {}^{m}/_{min} \tag{5}$$

where the sliding path length $L_S = R_0 \cdot \tan\alpha$, mm.

Having the limiting values of α_{min} and α_{max}, within which the involute angle changes, we determine the sliding velocity of the profiles for gears with modules of 2.5 mm, 3.5 mm, and 5 mm (Fig. 2).

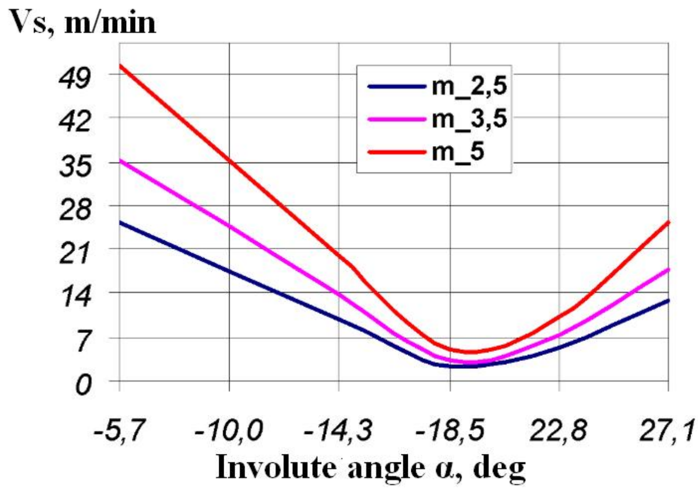


Fig. 2. Sliding velocity of involute tooth profiles

It should be noted that the direction of the sliding velocity at the pitch point of the gears changes to the opposite (the graphs show only the absolute value of the parameter V_s).

1.2. Sinusoidal gear transmission

The relationship between the normal module in an involute gear and the eccentricity of a sinusoidal gear is determined by the ratio:

$$e = 0.5m \cdot \cot\alpha_\omega \tag{6}$$

Thus, the eccentricities of the sinusoidal gears corresponding to the modules of 2.5 mm, 3.5 mm, and 5 mm will be equal to 3.43 mm, 4.81 mm, and 6.87 mm.

The coordinates of the points of the sinusoidal profile in the coordinate system of the gear are described by the system of parametric equations:

$$\begin{cases} x_i = (R_\omega + e \cdot \cos\varphi) \cdot \cos\frac{\varphi}{Z}, \\ y_i = (R_\omega + e \cdot \cos\varphi) \cdot \sin\frac{\varphi}{Z}, \end{cases} \tag{7}$$

φ – the rotation angle of the gear; R_ω – the pitch circle radius; Z – the number of teeth of the gear.

Fig. 3 shows the profile of the tooth space between the sinusoidal teeth.

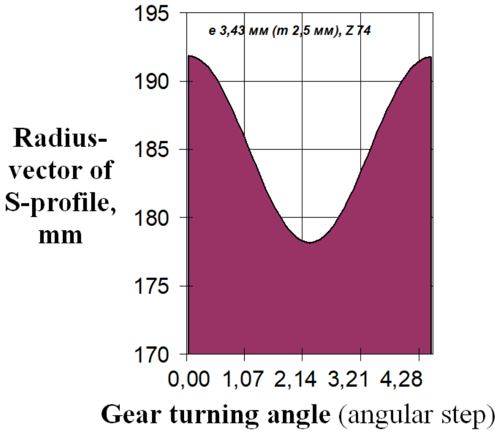


Fig. 3. Tooth space between sinusoidal teeth

For a sinusoidal gear, the length of the contact path is:

$$L_S = \rho \cdot \tan\varphi \cdot \cos\alpha_\omega, \text{ mm.} \tag{8}$$

The sliding velocity in sinusoidal engagement is equal to:

$$V_s = \frac{\rho \cdot \tan\varphi \cdot \cos\alpha_\omega \cdot (\omega_1 + \omega_2)}{(60 \cdot 1000)}, {}^{m}/_{min}. \tag{9}$$

Fig. 4 shows how the sliding velocity changes in the contact of sinusoidal gears for the above initial data.

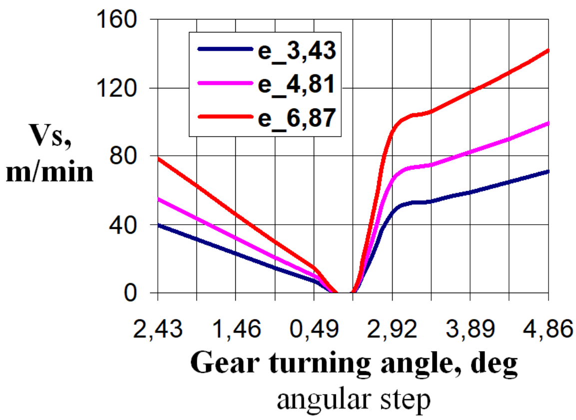


Fig. 4. Sliding velocity of profiles in sinusoidal engagement

2. Power required to overcome friction

As noted above, mutual sliding of profiles at high relative velocity under significant loads and during prolonged operation will cause intense friction on the contact surfaces, ultimately resulting in energy losses in the drive and increasing the wear rate of the teeth.

If the power supplied to the gear is N kW, then at the rotational speed of the gear n_1 rpm, the torque on its axis will be:

$$T = 9550 \cdot \frac{N}{n_1}, \text{ Nm} \tag{10}$$

and the normal force acting on the tooth of the gear and corresponding to this moment is equal to:

$$P_n = \frac{10^3 \cdot T \cdot \cos\alpha_i}{\rho_i}, \text{ N.} \tag{11}$$

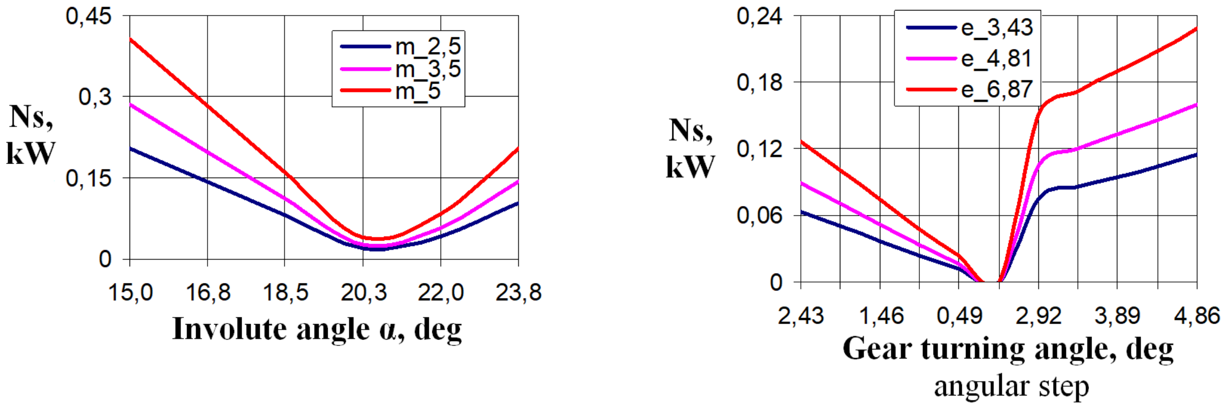
The current values of the parameters α_i and the radius vector ρ_i, determined above, make it possible to establish a change in force (11) for the involute angle α.

Knowing the value of P_n, one can determine the friction force acting in the direction opposite to the sliding velocity as $F_s = P_s\,\mu$.

The coefficient of friction μ in a "steel on steel" pair for ground surfaces is in the range of 0.45–0.5. Based on this, the power spent to overcome friction on the surfaces of the teeth in the transmission will be described by the equation:

$$N_s = \frac{10^{-3} \cdot P_n \cdot V_s \cdot \mu}{60},\ \text{kW}. \qquad (12)$$

Assuming that the power transmitted by a pair of gears equals 4.1 kW, the torque on the pinion axis will be 80 Nm. Based on the above initial data and formulas (5)–(12), the graphs of the powers that must be developed to overcome friction in involute (a) and sinusoidal (b) transmissions are shown in Fig. 5.



a *b*

Fig. 5. Power to overcome friction in involute (a) and sinusoidal (b) transmissions

The value of N_s characterizes the power losses, from which the efficiency of the transmissions can be determined. As can be seen from the calculation results, the maximum power required to overcome friction in a sinusoidal transmission is 1.75 times less than the corresponding power in an involute transmission. Accordingly, the efficiency of the involute transmission is 0.9–0.95 versus 0.97–0.98 for the sinusoidal transmission.

The calculated values relate to a pair of teeth; however, several teeth are simultaneously in engagement, and their number can be determined from the dependence:

$$Z_{nr} = \frac{W}{\pi \cdot m}, \qquad (13)$$

$$W = [\pi \cdot (Z_2 - 0.5) + Z_2 \cdot inv\alpha] \cdot m \cdot \cos\alpha, \qquad (14)$$

W – the length of the common normal; $\pi\ m$ – the base pitch.

For the data used in this article and an average module of 3.5 mm, we obtain $W = 3$, and the actual maximum energy losses will be 1.3 J (in the involute transmission) and 0.7 J (in the sinusoidal transmission) per hour. This means that friction losses over one day during continuous operation of the drive will be significant, but the sinusoidal transmission will be much more efficient than the involute one.

Conclusions

Calculations made based on the basic provisions of the theory of gearing show that sinusoidal transmission is significantly more efficient than the traditional involute transmission. In addition to its known positive properties, it has been established that this transmission has better sliding and friction conditions than the involute transmission. As a result, the loss of energy during its transmission from the engine to the working body of the machine is significantly reduced by 1.75 times, and the efficiency of the transmission increases according to the parameter of power losses, which arise due to the need to overcome contact friction between the teeth of the drive gears.

References

[1] I. E. Hrytsay, "Sinusoidal Gears and Alternative Method of Tooth Generation", *In: Advances in Gear Theory and Gear Cutting Tool Design*, Springer, Cham, doi.org/10.1007/978-3-030-92262-7_8, pp. 233–254, 2022.

[2] I. E. Hrytsay, E. M. Blahut, V. Z. Yavorskyy, "Synusoyidalni zubchasti kolesa ta novyy efektyvnyy metod yikh vyhotovlennya" [Sinusoidal gears and a new efficient method of their production], *UYTS "Nauka. Tekhnyka. Tekhnolohyya"*, pp. 47–50, 2004.

[3] Y. M. Lytvynyak, I. E. Hrytsai, I. I. Yurchyshyn, "Tekhnolohichni osoblyvosti otrymannya zubchastykh kolis iz synusoyidalʹnym profilem bichnykh poverkhonʹ zubtsiv" [Technological features of obtaining gears with a sinusoidal profile of the side surfaces of the teeth]. *In: 7th International Symposium of Ukrainian Engineers – Mechanics in Lviv. Abstracts of reports*. Lviv, pp. 86–87, May 18–20, 2005.

[4] S. I. Pasternak, Yu. M. Danylchenko, M. H. Storchak, A. V. Kryvosheia, "Ekonomichne obgruntuvannia metodu konturnoi obrobky detalei z periodychnymy profiliamy dyskovym instrumentom" [Economic justification of the method of contour processing of parts with periodic profiles using a disk tool]. *Visnyk NTU "KhPI": Problemy mekhanichnoho pryvodu, 19,* ISSN 2079-0791, pp.118–126, 2009.

[5] Wei Chen, Yuzhou Tian, Qiang Sun and Pengfei Zhu, "A Novel Model of an S-Shaped Tooth Profile Gear Pair with a Few Pinion Teeth and Tooth Contact Analysis Considering Shaft Misalignments". *School of Mechanical Engineering, Shijiazhuang Tiedao University, Shijiazhuang 050043, China. Appl. Sci., 13(15), 8959*. doi.org/10.3390/app13158959, 2023.

[6] Y. H. Sun, Q. Sun, "A Kind of Special Type Profile Gear". CN104455315A, 25 March 2015.

[7] S. H. Yahaya, M. S. Salleh, J. M. Ali, "Spur Gear Design With an S-shaped Transition Curve Application Using MATHEMATICA and CAD Tools". *In Proceedings of the 2009 Second International Conference on Machine Vision, Dubai, United Arab Emirates*, pp. 273–276, 28–30 December 2009.

[8] Z. Damijan, K. Simon, T. Joze, D. Joze, "Different teeth profile shapes of polymer gears and comparison of their performance". *J. Adv. Mech. Des. Syst. Manuf.*, JAMDSM0083, 11–2017.

[9] G. Hlebanja, "Gradual development of S-shaped gears. MATEC Web Conf. 2022, 366, 01001. Wu, X. T. Gear Geometry and Applied Theory, 2nd ed."; *China Machine Press: Beijing, China*, 2009.

[10] S. H. Yahaya, M. S. Salleh, J. M. Ali, "Spur Gear Design With An S-Shaped Transition Curve Application Using MATHEMATICA And CAD Tool". *In: International Conference on Computer Technology and Development*, doi.org/10.1109/ICCTD.2009.76, pp. 426–429, 2009.

[11] J. Hlebanja, G. Hlebanja, "Anwendbarkeit der S-Verzahnung im Getriebebau: Nichtevolventische Verzahnungen weiterentwickelt. Antriebstechnik", 44, No. 2, pp. 34–38, 2005.

[12] J. Hlebanja, G. Hlebanja, M. Umberger,"S-gear design rules". Ventil, No. 4, 26, pp. 254–263, 2020.

[13] G. Hlebanja, M. Erjavec, M. Hriberšek, L. Knez, S. Kulovec, "Theory and applications based on S-gear geometry". *In S. P. Radzevich (Ed.), Recent advances in gearing: scientific theory and applications, 1st ed. Springer Nature*, pp. 51–87, 2022.

[14] Luo Shanming, Yue Wub, Jian Wang, "The generation principle and mathematical models of a novel cosine gear drive". *In: Mechanism and Machine Theory, 43,* pp. 1543–1556, 2008.