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Accelerated residual new iterative method for solving the generalized Burgers–Huxley equation

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Recently, Batiha B. et al. in Symmetry **15** (3), 688 (2023), propose the New Iterative Method (NIM) for solving the generalized Burgers–Huxley equation. In order to give an extended version of this work, we rewrite NIM method in an elegant form in the first step, and introduce a controlled parameter in the second step, called the Accelerated Residual New Iterative Method (ARNIM). We apply the established framework to solve the generalized Burgers–Huxley equation and then we give a convergence study according to the values of the control parameter.

Keywords: *new iterative method; Adomian decomposition method; accelerated residual method; generalized Burgers–Huxley equation.*

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1. Introduction

The majority of problems in various fields such as chemistry, biology, physics and engineering are modeled by nonlinear partial differential equations (NPDEs). These equations are crucial for describing phenomena such as heat transfer, fluid dynamics, etc.

However, solving nonlinear models for real-world problems proves to be a difficult task, both theoretically and numerically. The search for reliable solutions is complicated by the complexity and non-linearity of the models. Several numerical methods are available in the literature to solve NPDEs, but they all have their limitations and the development of new approaches to solve NPDEs will remain an active area of research and development in various fields [1, 2].

In this paper, we combine the Burgers equation, which studies the dynamics of viscous fluids, and the FitzHugh–Nagumo model, which studies the behavior of excitable cells and called the Burgers–Huxley equation. Solving this equation can be challenge due to its complexity and nonlinear nature, but it's has some properties such as symmetry, invariance under Galilean transformations and symmetry in the case of time reversal, that allows us to better understand and predict the behavior of complex systems [3, 4].

Let us consider the generalized nonlinear Burgers–Huxley equation:

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1-u^\delta)(u^\delta - \gamma), \quad 0 \leqslant x \leqslant 1, \quad t \geqslant 0. \tag{1}$$

where the coefficients $\alpha, \beta, \delta \geqslant 0$ and $\gamma \in (0,1)$ are given parameters. Equation (1) models the interaction between reaction mechanisms, convection effects and diffusion transports; see [5]. The exact solution of Eq. (1) subject to the initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(\sigma\gamma x)\right]^{1/\gamma},$$

and using nonlinear transformations [2], is given by

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x - \left\{\frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\gamma},$$

where $\sigma = \delta(\rho-\alpha)/4(1+\delta)$ and $\rho = \sqrt{\alpha^2 + 4\beta(1+\delta)}$.

Several approaches to solving the generalized Burgers–Huxley problem exist such as the finite difference method or the finite element method [6, 7], but also semi-analytical methods such as the Adomian decomposition method (ADM) [8, 9] and the variational iteration method [10]. In [11, 12], the authors introduced a new method called new iterative method (NIM) to solve linear and nonlinear functional equations. This method is an effective tool for dealing with nonlinear equations such as integral equations, algebraic equations, and ordinary or partial differential equations of fractional and integer order. In [13], the authors applied NIM method to solve the generalized Burgers–Huxley equations and give some comparisons with ADM and VIM.

In order to give an extended version of [13], we decompose our paper as follow: in Section 2, we recall and rewrite NIM method in an elegant form and give our new formulation. In Section 3, we introduce a controlled parameter to our formulation called the Accelerated Residual New Iterative Method (ARNIM). We apply the established framework to solve the generalized Burgers–Huxley equation and then we give a convergence study according to the values of the control parameter. Finally a conclusion will be given in Section 4.

2. Our formulation

2.1. New iterative method recall

Initially introduced by Daftardar–Gejji et al. [11] and applied to solve the generalized Burgers–Huxley equations in Batiha et al. [13]. We give the basic idea of this work: let us consider

$$u = f + L(u) + N(u). \tag{2}$$

In the equation above, f is a known function, and L and N are linear and nonlinear operators, respectively. The NIM solution for equation (2) has the form

$$u = \sum_{i=0}^{\infty} u_i.$$

Since L is linear, then

$$L\Big(\sum_{i=0}^{\infty} u_i\Big) = \sum_{n=0}^{\infty} L(u_i). \tag{3}$$

The nonlinear operator N in Eq. (2) is decomposed as

$$N\Big(\sum_{i=0}^{\infty} u_i\Big) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N\Big(\sum_{j=0}^{i} u_j\Big) - N\Big(\sum_{j=0}^{i-1} u_j\Big) \right\} = \sum_{i=0}^{\infty} G_i, \tag{4}$$

where

$$\begin{aligned} &G_0 = N(u_0),\\ &G_1 = N(u_0 + u_1) - N(u_0),\\ &G_2 = N(u_0 + u_1 + u_2) - N(u_0 + u_1),\\ &\quad \dots\\ &G_i = N\Big(\sum_{j=0}^{i} u_j\Big) - N\Big(\sum_{j=0}^{i-1} u_j\Big), \quad i \geqslant 1. \end{aligned} \tag{5}$$

Using equations (3)–(5) in Eq. (2), we get

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} G_i.$$

The solution of Eq. (2) can be expressed as

$$u = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \ldots + u_n + \ldots,$$

where

$$\begin{aligned} u_0 &= f, \\ u_1 &= L(u_0) + G_0, \\ u_2 &= L(u_1) + G_1, \\ &\dots \\ u_n &= L(u_{n-1}) + G_{n-1} \\ &\dots \end{aligned}$$

2.2. New formulation of NIM

We introduce a new formulation of New Iterative Method described as follows.

Denote the $(n+1)$-term approximate solution of (2) by S_n, i.e.

$$S_n = \sum_{i=0}^{n} u_i, \quad n \in \mathbb{N}. \tag{6}$$

In view of the recurrence relation (6), we get the following algorithm for computing S_n's:

$$\begin{cases} S_0 = f, \\ S_n = S_0 + F(S_{n-1}), \quad n = 1, 2, \dots, \end{cases} \tag{7}$$

where $F(v) = L(v) + N(v)$ and $\lim\limits_{n\to\infty} S_n = u$, being the required solution.

Indeed, $S_0 = f = u_0$, and $S_1 = u_0 + u_1 = u_0 + L(u_0) + N(u_0) = S_0 + F(S_0)$.

Then, $S_2 = u_0 + u_1 + u_2 = f + L(u_0) + A_0 + L(u_1) + A_1 = f + L(u_0 + u_1) + A_0 + A_1 = S_0 + F(S_1)$.

The iterative scheme (7) therefore leads to

$$S = f + F(S), \tag{8}$$

with $S = \lim\limits_{n\to\infty} S_n$. We formulate the following contraction context theorem.

Theorem 1. *Let F be an operator from a Hilbert space H onto H and u is exact solution of (2). Assume $\exists \mu < 1$ such that $\|u_{i+1}\| \leqslant \mu \|u_i\|$, $\forall i \in \mathbb{N}$, $\{S_n\}_{n=0}^{\infty}$ converges to S which is obtained by (8).*

Proof. We have

$$\begin{aligned} S_0 &= u_0, \\ S_1 &= u_0 + u_1, \\ S_2 &= u_0 + u_1 + u_2, \\ &\dots\dots \\ S_n &= u_0 + u_1 + \dots + u_n. \end{aligned}$$

Then

$$\|S_{n+1} - S_n\| = u_{n+1} \leqslant \mu u_n \leqslant \mu^2 u_{n-1} \leqslant \dots \leqslant \mu^n u_1 = \mu^n \|S_1 - S_0\|.$$

For every n, $m \in \mathbb{N}$, $n \geqslant m$,

$$\begin{aligned} \|S_n - S_m\| &= \|S_n - S_{n-1} + S_{n-1} - S_{n-2} + \dots + S_{m+1} - S_m\| \\ &\leqslant \|S_n - S_{n-1}\| + \|S_{n-1} - S_{n-2}\| + \dots + \|S_{m+1} - S_m\| \\ &\leqslant (\mu^{n-1} + \mu^{n-2} + \dots + \mu^m)\|S_1 - S_0\| \leqslant \frac{\mu^m}{1-\mu}\|S_1 - S_0\|. \end{aligned}$$

Then $\{S_n\}_{n=0}^{\infty}$ is a Cauchy sequence in the Hilbert space H and it implies that $S = \lim\limits_{n\to\infty} S_n$ is a solution of (8). ∎

3. Accelerated residual new iterative method

3.1. Basic idea

Without loss a generality, we take the nonlinear partial differential equation given by

$$u - F(u) = f \quad \text{(with some initial and boundary conditions)} \tag{9}$$

and define the Accelerated Residual New Iterative Method (ARNIM) by the iterative scheme as follows

$$\begin{cases} \overline{S}_0 = S_0, \\ \overline{S}_n = \overline{S}_{n-1} - \omega\left(\overline{S}_0 + F(\overline{S}_{n-1}) - \overline{S}_{n-1}\right), \quad n = 1, 2, \dots, \end{cases}$$

where ω is a control parameter.

Remark 1. If we take $\omega = -1$ then $\overline{S}_n = S_n$, $\forall n > 0$.

3.2. Convergence analysis of ARNIM

To study the convergence of the ARNIM method, we place ourselves in a Hubert space H, equipped with the norm denoted $\|\cdot\|$ and a scalar product noted $\langle , \rangle$. In order to solve the problem (9), we define the operator $\mathcal{T}$, by $\mathcal{T}(u) = F(u) - u$ and propose the following theorem.

Theorem 2. *Assume that:*
H1: $\langle \mathcal{T}(u) - \mathcal{T}(v), u - v\rangle \geqslant K(u,v)\|u - v\|^2$, $K(u,v) > 0$, $\forall u, v \in H$.
H2: For any $M > 0$, $\exists C(M) > 0$, *such that for* $u, v \in H$, *with* $\|u\| \leqslant M$, $\|v\| \leqslant M$, *we have* $\langle \mathcal{T}(u) - \mathcal{T}(v), w\rangle \leqslant C(M)\|u - v\|\|w\|$ *for every* $w \in H$.

Then the ARNIM method is convergent for a suitable ω, *and converges towards the solution of the problem (9).*

Proof. Let S be the solution to the problem (9) and define $E_{n+1} = \overline{S}_{n+1} - S$. Then,

$$\begin{aligned} \|E_{n+1}\|^2 &= \langle E_{n+1}, E_{n+1}\rangle \\ &= \langle \overline{S}_{n+1} - S, \overline{S}_{n+1} - S\rangle = \|\overline{S}_{n+1} - S\|^2 \\ &= \|\overline{S}_n - \omega\left(\overline{S}_0 + F(\overline{S}_n) - \overline{S}_n\right) - S + \omega\left(S_0 + F(S) - S\right)\|^2 \\ &= \langle \overline{S}_n - \omega\left(\overline{S}_0 + F(\overline{S}_n) - \overline{S}_n\right), S - \omega\left(S_0 + F(S) - S\right)\rangle \\ &\leqslant \|E_n\|^2 - 2\omega\langle \mathcal{T}(\overline{S}_n) - \mathcal{T}(S), E_n\rangle + \omega^2\|\mathcal{T}(\overline{S}_n) - \mathcal{T}(S)\|^2. \end{aligned}$$

Let M_0 be such that $\|S\| \leqslant \frac{M_0}{2}$. Let us make the recurrence hypothesis

$$\|E_n\| = \|S_n - S\| \leqslant \frac{M_0}{2},$$

leading, in particular

$$\|S_n\| \leqslant M_0.$$

With the previous hypotheses H1 and H2, we will obtain

$$\|\mathcal{T}(\overline{S}_n) - \mathcal{T}(S)\|^2 \leqslant C(M_0)\|\mathcal{T}(\overline{S}_n) - \mathcal{T}(S)\|\|E_n\|,$$

it therefore results

$$\|\mathcal{T}(\overline{S}_n) - \mathcal{T}(S)\| \leqslant C(M_0)\|E_n\|,$$

and consequently

$$\|E_{n+1}\|^2 \leqslant \left(1 - 2\omega K(S_n, S) + C(M_0)\omega^2\right)\|E_n\|^2 = \mu\|E_n\|^2.$$

If we choose ω such that $\mu < 1$ the recurrence hypothesis will be verified. It follows that $E_{n+1} \to 0$, when $n \to \infty$ which completes the justification. ∎

4. Application to solve the generalized Burgers–Huxley

In order to solve the generalized Burgers–Huxley, let us take the Hilbert space

$$H = L^2((a,b)\times[0,T]),$$

we define

$$y\colon (a,b)\times[0,T]\to\mathbb{R} \text{ with } \int_{(a,b)\times[0,T]} y^2(x,s)\,ds\,dx < +\infty,$$

the scalar product

$$\langle y,z\rangle_H = \int_{(a,b)\times[0,T]} y(x,s)\,z(x,s)\,ds\,dx,$$

and the associated norm

$$\|y\|_H^2 = \int_{(a,b)\times[0,T]} y^2(x,s)\,ds\,dx,$$

Now write Eq. (1) in the operator form

$$\mathcal{T}(y) = -\alpha y^\delta \frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2} + \beta y(1-y^\delta)(y^\delta-\gamma).$$

Theorem 3 (Sufficient condition of convergence). *Under the assumptions in Theorem 2, the ARNIM method applied to the generalized Burgers–Huxley Eq. (1) converges towards a solution.*

Proof. To study the convergence of the method, we prove that the assumptions of Theorem 2 are verified.

First, we will verify hypothesis $H1$ for the operator $\mathcal{T}(y)$.

We have

$$\begin{aligned}\mathcal{T}(y)-\mathcal{T}(z) &= -\alpha\left[y^\delta\frac{\partial y}{\partial x} - z^\delta\frac{\partial z}{\partial x}\right] + \left[\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 z}{\partial x^2}\right]\\ &\quad + \beta(1+\gamma)(y^{\delta+1}-z^{\delta+1}) - \beta(y^{2\delta+1}-z^{2\delta+1}) - \beta\gamma(y-z)\\ &= -\frac{\alpha}{\delta+1}\frac{\partial}{\partial x}(y^{\delta+1}-z^{\delta+1}) + \left[\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 z}{\partial x^2}\right] + \beta(1+\gamma)(y-z)\sum_{i=1}^{\delta+1} y^{\delta-i+1}z^{i-1}\\ &\quad - \beta(y-z)\sum_{i=1}^{\theta+1} y^{\theta-i+1}z^{i-1} - \beta\gamma(y-z),\end{aligned} \tag{10}$$

where $\theta = 2\delta$. Therefore,

$$\begin{aligned}\langle\mathcal{T}(y)-\mathcal{T}(z), y-z\rangle &= \frac{\alpha}{\delta+1}\left\langle -\frac{\partial}{\partial x}(y^{\delta+1}-z^{\delta+1}), y-z\right\rangle + \left\langle \frac{\partial^2}{\partial x^2}(y-z), y-z\right\rangle\\ &\quad - \beta(1+\gamma)\left\langle -(y-z)\sum_{i=1}^{\delta+1} y^{\delta-i+1}z^{i-1}, y-z\right\rangle\\ &\quad + \beta\left\langle -(y-z)\sum_{i=1}^{\theta+1} y^{\theta-i+1}z^{i-1}, y-z\right\rangle - \beta\gamma\langle y-z, y-z\rangle.\end{aligned} \tag{11}$$

Using the properties of the differential operator $\partial^2/\partial x^2$ and the definition of the scalar product in H, there exists a constant $\delta_1 > 0$ such that

$$\left\langle \frac{\partial^2}{\partial x^2}(y-z), y-z\right\rangle \geqslant \delta_1\|y-z\|^2. \tag{12}$$

By the definition of the scalar product and the properties of the differential operator $\partial/\partial x$ and the Schwartz inequality in H, we have

$$\left\langle \frac{\partial}{\partial x}\big(y^{\delta+1}-z^{\delta+1}\big), y-z\right\rangle \leqslant \left\|\frac{\partial}{\partial x}\big(y^{\delta+1}-z^{\delta+1}\big)\right\| \|y-z\|$$

$$\begin{aligned}
&\leqslant \delta_2 \|y^{\delta+1} - z^{\delta+1}\| \|y-z\| \\
&= \delta_2 \left\| (y-z) \sum_{i=1}^{\delta+1} y^{\delta-i+1} z^{i-1} \right\| \|y-z\| \\
&\leqslant \delta_2 (\delta+1)\, M^{\delta} \|y-z\|^2. \qquad (13)
\end{aligned}$$

Here $\|y\| \leqslant M$ and $\|z\| \leqslant M$. Hence

$$\left\langle -\frac{\partial}{\partial x}(y^{\delta+1} - z^{\delta+1}), y-z \right\rangle \geqslant -\delta_2(\delta+1)\, M^{\delta} \|y-z\|^2. \qquad (14)$$

Again, by the Schwartz inequality we have

$$\begin{aligned}
\left\langle (y-z) \sum_{i=1}^{\delta+1} y^{\delta-i+1} z^{i-1}, y-z \right\rangle &\leqslant \left\| (y-z) \sum_{i=1}^{\delta+1} y^{\delta-i+1} z^{i-1} \right\| \|y-z\| \\
&\leqslant (\delta+1)\, M^{\delta} \|y-z\|^2. \qquad (15)
\end{aligned}$$

Thus

$$\left\langle -(y-z) \sum_{i=1}^{\delta+1} y^{\delta-i+1} z^{i-1}, y-z \right\rangle \geqslant -(\delta+1)\, M^{\delta} \|y-z\|^2. \qquad (16)$$

Similarly,

$$\left\langle -(y-z) \sum_{i=1}^{\theta+1} y^{\theta-i+1} z^{i-1}, y-z \right\rangle \geqslant -(\theta+1) M^{\theta} \|y-z\|^2. \qquad (17)$$

Substituting (12)–(17) into (11) gives

$$\begin{aligned}
\langle \mathcal{T}(y) - \mathcal{T}(z), y-z \rangle &\geqslant \left[\delta_1 - \delta_2 \alpha M^{\delta} + \beta(1+\gamma)(\delta+1) M^{\delta} - \beta(\theta+1) M^{\theta} - \beta\gamma \right] \|y-z\|^2 \\
&= K\|y-z\|^2, \quad (K > 0).
\end{aligned}$$

Where we require

$$K = \delta_1 - \delta_2 \alpha M^{\delta} + \beta(1+\gamma)(\delta+1) M^{\delta} - \beta(\theta+1) M^{\theta} - \beta\gamma > 0,$$

that implies

$$\delta_1 > \delta_2 \alpha M^{\delta} - \beta(1+\gamma)(\delta+1) M^{\delta} + \beta(\theta+1) M^{\theta} + \beta\gamma.$$

Thus, hypothesis H1 holds.

We now verify hypothesis H2 for the operator $\mathcal{T}(u)$.

Using the Schwartz inequality, we have

$$\begin{aligned}
\langle \mathcal{T}(y) - \mathcal{T}(z), w \rangle = \frac{\alpha}{\delta+1} &\left\langle -\frac{\partial}{\partial x}(y^{\delta+1} - z^{\delta+1}), w \right\rangle + \left\langle \frac{\partial^2}{\partial x^2}(y-z), w \right\rangle \\
&+ \beta(1+\gamma) \left\langle (y-z) \sum_{i=1}^{\delta+1} y^{\delta-i+1} z^{i-1}, w \right\rangle \\
&\beta \left\langle (y-z) \sum_{i=1}^{\theta+1} y^{\theta-i+1} z^{i-1}, w \right\rangle - \beta\gamma \langle y-z, w \rangle \\
&\leqslant \alpha M^{\delta} \|y-z\| \|w\| + \delta_3 \|y-z\| \|w\| + \beta(1+\gamma)(\delta+1) M^{\delta} \|y-z\| \|w\| \\
&\quad + \beta(\theta+1) M^{\theta} \|y-z\| \|w\| + \beta\gamma \|y-z\| \|w\| \\
&= \left[\delta_3 + \beta\gamma + \{\alpha + \beta(\delta+1)\} M^{\delta} + \beta(\theta+1) M^{\theta} \right] \|y-z\| \|w\| \\
&= C(M) \|y-z\| \|w\|,
\end{aligned}$$

where

$$C(M) = \delta_3 + \beta\gamma + \{\alpha + \beta(\delta+1)\} M^{\delta} + \beta(\theta+1) M^{\theta},$$

and therefore H2 is fulfilled. This completes the proof. ∎

5. Conclusion

In three sections of this paper, we have taken up Bahita's work on the application of the NIM method to solve the generalized Burgers–Huxley equation. And we gave another formulation of the NIM method as well as an extension of this method by the introduction of a control parameter called ARNIM. Then we showed the convergence of our method as a function of the parameter ω under certain hypotheses. In Section 4, we applied the proposed method for the solution of the generalized Burgers–Huxley equation and we established the convergence of this method by verifying that the hypotheses of Theorem 2 are true in this case and the new method converges to required solution.

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Новий прискорений ітераційний метод залишків для розв'язування узагальненого рівняння Бюргерса–Хакслі

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Нещодавно Batiha B. et al. в Symmetry **15** (3), 688 (2023) запропонували новий ітераційний метод (NIM) для розв'язання узагальненого рівняння Бюргерса–Хакслі. Щоб надати розширену версію цієї роботи, переписуємо метод NIM в елегантній формі на першому етапі та вводимо контрольований параметр на другому етапі, який називається новим прискореним ітераційним методом залишку (ARNIM). Застосовуємо встановлену структуру для вирішення узагальненого рівняння Бюргерса–Хакслі, а потім проводимо дослідження збіжності відповідно до значень контрольного параметра.

Ключові слова: *новий ітераційний метод; метод декомпозиції Адомяна; прискорений метод залишків; узагальнене рівняння Бюргерса–Хакслі.*