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The variable viscosity and variable gravity field on the onset of convective motion in a porous layer with throughflow

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In the present article, the combined influence of the changeable gravity field and temperature-reliant viscosity on the porous bed is considered for investigation numerically by the Galerkin technique in the presence of upward vertical throughflow. The temperature-reliant viscosity is known to be exponential. The porous matrix is subjected to continuous downward gravity fluctuations varying with distance across the medium and vertical upward throughflow. Four different cases of gravity variance were discussed. A parametric analysis is conducted by adjusting the following parameters: throughflow parameter, viscosity parameter, and gravity parameter. Results show that the beginning of the convective moment would be delayed by all three parameters throughflow, temperature-reliant viscosity, and gravity variance. It has been shown that the fluidic system is more inconsistent in case (iii) and more consistent in case (iv).

Keywords: *variable viscosity; throughflow; Galerkin technique; variable gravity vector; linear stability.*

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1. Introduction

Normal convection (convection controlled by buoyancy in which gravitational power assumes a significant part) in fluid-saturated porous matrix with liquid thickness temperature reliance is of interest because of its importance in different useful applications, for example, techniques for oil recuperation in the petrol business, geothermal energy extraction, and reactor vessel protection. In different actual models for permeable construction, a few researchers investigated the convective stability problems including Nield [1, 2], Nield and Kuznetsov [3], Shivakumara et al. [4], Vafai [5], Suma et al. [6], Gangadharaiah [7], Wang and Tan [8], Ingham and Pop [9], Celli et al. [10], Mahajan and Sharma [11], and Banu and Rees [12].

In a large number of the wide-range convection situations present in the climate, the Earth's mantle, or the sea, it is known that the gravity field of the Earth alters with elevation from its surfaces Alex and Patil [13]. Different buoyancy forces will be experienced by the fluid layer at various points when the gravity field changes Alex et al. [14]. Pradhan and Samal [15] were the first researchers to study a changeable downward gravity field that affects the initiation of convection. They discovered that increasing gravity is a destabilizing effect. An extension to the porous medium with thermal diffusion and gravity gradient was made by Alex and Patil [16]. Rionero and Straughan [17] explored the effect of heat-generating porous medium with linear and nonlinear gravity field variations. In flows with heat transfer, it is well recognized that the temperature dependence of the fluid parameters can alter the flow behavior, particularly its stability features. As viscosity is more sensitive to temperature than heat capacity and thermal conductivity, it exhibits a rather obvious fluctuation concerning temperature for the majority of practical fluids. Rossby [18] calculated the viscosity and thermal conductivity values for water between 20 and 25°C and found that the difference in kinematic viscosity between 20 and 25°C is roughly 10%, while the difference in thermal conductivity of water is only 1.5%. Torrance and Turcotte [19] found that as temperature increases, liquid thickness decreases, while gases demonstrate a converse example. Numerous scholars have recently explored the effect of the thickness of boundary slab changing with temperature on thermal convection (Barletta and Nield [20], Solomatov and Barr [21], and Booker [22]). Hassan et al. [23] investigated the flow and heat transfer of fluid under shear over a sheet that is non-linearly stretching and has a variable thickness. Using the Keller–Box method, Reddy et al. [24] investigated the effect of thermal radiation on the MHD boundary layer flow of Williamson nanofluid along a stretching surface. Chabani et al. [25] used the Darcy–Brinkman–Forchheimer model and multi-physics COMSOL software to investigate the MHD flow of a hybrid nano-fluid in a triangular enclosure. In addition, the theory of throughflow is important and vital for the regulation of convective mechanisms in science, geophysics, manufacturing, etc. However, the study of variable gravity throughflow is very restricted. Suma et al. [26] and Gangadharaiah et al. [27] examined the combined impact of the internal heating and variable downward gravity effects on the device stability using the perturbation technique. Regardless, nonlinear gravity field variety with profundity can happen in sedimentary bowls, orogenic and epeirogenic developments of the crustal designs, and Earth's outside (Cordell [28], Shneiderov [29], Shi et al. [30], and Nagarathnamma et al. [31, 32]). Rao et al. [33] analyzed the outstanding, binomial, and illustrative capacities and found that most crustal constructions coordinated the allegorical model all the more intently. In the presence of heat sources and temperature profiles for composite layers was extensively studied by Manjunatha et al. [34, 35] and Yellamma et al. [36, 37]. In large-scale convection phenomena occurring in the atmosphere, the ocean, or the mantle of the Earth, it becomes imperative to consider gravity as a variable quantity varying with distance from the surface. Therefore, in this paper, we examine the mutual impact of variable viscosity, throughflow with variable downward gravity fluctuations for the four cases: (i) $H(z) = -z$, (ii) $H(z) = -z^2$, (iii) $H(z) = -z^3$, and (iv) $H(z) = -(e^{-z} - 1)$. The simulations have been conducted and tested in-depth for the throughflow parameter, the factor viscosity parameter, and the gravity variance parameter.

2. Conceptual model

The horizontal isotropic porous matrix is bounded between planes at $z = 0$ and $z = d$ with continuous constant upward throughflow of vertical velocity w_0 and downward gravity $g(z)$. Figure 1 demonstrates the physical structure of the current study. From below, the porous bed is heated; the temperature T at the bottom surface $z = 0$ is taken to be T_0 and on the top surface $z = d$ is taken to be T_l, respectively. We assume that the viscosity depends exponentially on the temperature of the form $\mu = \mu_0 \exp\left[-B(T - T_0)\right]$ and the gravity vector $\boldsymbol{g}$ is, $\boldsymbol{g} = -g_0(1 + \lambda H(z))\boldsymbol{k}$, which spreads with the vertical reverse z-direction.

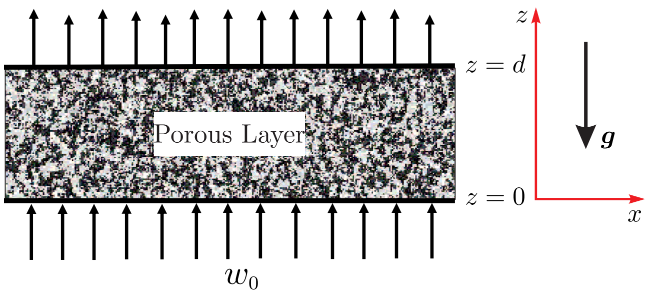


Fig. 1. Physical configuration.

3. Mathematical formulation

The appropriate basic equations of the asymmetric arrangement of the porous matrix are

$$\nabla \cdot \boldsymbol{V} = 0, \tag{1}$$

$$-\nabla p - \frac{\mu(T)}{K}\boldsymbol{V} + \rho_0\left[1 - \alpha(T - T_0)\right]\boldsymbol{g}(z) = 0, \tag{2}$$

$$A\frac{\partial T}{\partial t} + (\boldsymbol{V} \cdot \nabla)\,T = \kappa \nabla^2 T, \tag{3}$$

where $\boldsymbol{V} = (u, v, w)$ is the velocity vector, λ is the variable gravity parameter, μ is the viscosity, p

is pressure, ρ_0 is reference density, K is the permeability, α is the coefficient of thermal expansion, $\boldsymbol{g}$ is the gravity, T is the temperature, T_0 is the reference temperature, A is the heat capacity ratio, B is the viscosity parameter, κ is thermal conductivity, and μ_0 is dynamic viscosity corresponding to a temperature equal to the mean of temperature at the boundaries.

It is supposed that the basic state be time-independent and of the form:

$$[u, v, w, P, T] = [0, 0, w_0, P_b(z), T_b(z)]. \tag{4}$$

Then, the basic temperature field is

$$\kappa\frac{d^2T_b}{dz^2} - w_0\frac{dT_b}{dz} = 0. \tag{5}$$

On solving Eq. (5), we get

$$T_b(z) = \frac{e^{\mathrm{Pe}} - e^{\mathrm{Pe}\,z}}{e^{\mathrm{Pe}} - 1}, \tag{6}$$

where $\mathrm{Pe} = \frac{w_0 d}{\kappa}$ is the throughflow parameter (Peclet number), and subscript 'b' refers to the basic state. Infinitesimal disruptions are superimposed in the form to explore the stability of the basic state,

$$\boldsymbol{V} = w_0\hat{k} + \boldsymbol{V}', \quad P_b(z) + p', \quad T = T_b(z) + \theta. \tag{7}$$

Applying Eq. (7) to Eqs. (1)–(3), the linear stability equations become:

$$f(z)\nabla^2 w + f'(z)\frac{\partial w}{\partial z} = \mathrm{R}\big[1 + \lambda H(z)\big]\nabla_h^2 T, \tag{8}$$

$$\left[A\frac{\partial}{\partial t} + \mathrm{Pe}\frac{\partial}{\partial z} - \nabla^2\right]T = w\,\mathrm{Pe}\left[\frac{e^{\mathrm{Pe}\,z}}{1 - e^{\mathrm{Pe}}}\right], \tag{9}$$

where $f(z) = \exp\left[B(z - 1/2)\right]$, $B = \frac{\nu_{\max}}{\nu_{\min}}$, $\mathrm{R} = \frac{\alpha g_0 (T_l - T_u) d^3}{\nu\kappa}$ is the Rayleigh number, B is the viscosity parameter, $\nabla^2 = \nabla_h^2 + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator, and $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

4. Normal mode analysis

We assume the solution is of the form

$$(w, T) = [W(z), \Theta(z)]e^{i(lx+my)}; \tag{10}$$

where l and m are wavenumbers in x and y direction, respectively. Substituting Eq. (10) into Eqs. (8)–(9), we derive

$$f(z)\left(D^2 - a^2\right)w + f'(z)Dw = -\mathrm{R}\,a^2\left[1 + \lambda H(z)\right]\Theta, \tag{11}$$

$$\left[D^2 - \mathrm{Pe}\,D - a^2\right]\Theta = w\,\mathrm{Pe}\left[\frac{e^{\mathrm{Pe}\,z}}{1 - e^{\mathrm{Pe}}}\right], \tag{12}$$

where Θ is the disturbed temperature amplitude, a is the wavenumber, and W is the disturbed vertical velocity amplitude. The boundary conditions are

$$W = \Theta = 0 \;\text{ at }\; z = 0, 1. \tag{13}$$

5. Method of solution

Now we employ the Galerkin weighted residuals procedure to solve the system of Eqs. (11) and (12). Consequently, W and Θ are considered as

$$W = \sum_{i=1}^{n} A_i W_i \;\text{ and }\; \Theta = \sum_{i=1}^{n} B_i\Theta_i, \tag{14}$$

with the trial functions

$$W_i = \Theta_i = \sin(i\pi z), \tag{15}$$

using the governing parameters $(\mathrm{Pe}, B, \lambda, a)$, the eigenvalue critical Rayleigh number R^c can be obtained.

6. Results and discussion

Using the higher-order Galerkin process, the collective effect of the downward changeable gravity field and temperature-reliant viscosity with a steady upward throughflow on the appearance of convective motion is studied. The viscosity variation is assumed to be exponential type and the four different forms of linear and non-linear gravity field variation: (i) $H(z) = -z$, (ii) $H(z) = -z^2$, (iii) $H(z) = -z^3$, and (iv) $H(z) = -(e^{-z} - 1)$ are studied. In the present analysis, the governing parameters considered are the throughflow parameter Pe viscosity parameter (B), and gravity parameter λ. The consistency of the system is achieved in terms of critical Rayleigh number R^c and critical wavenumber a_c by referring to different values λ, B, and Pe.

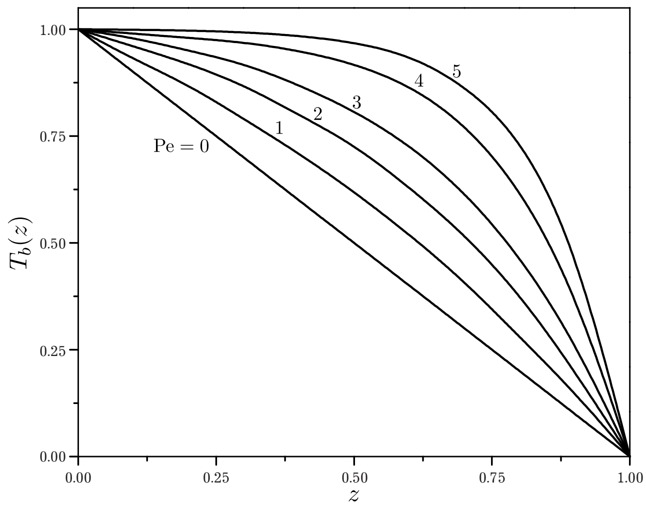


Fig. 2. The plot of the basic state temperature distributions for $\mathrm{Pe} = 0, 1, 2, 3, 4, 5$.

We have plotted Figure 2 for basic temperature distribution $T_b(z)$, it is noted that modulation of throughflow merely modifies the distribution quantitatively within the porous bed. Figures 3–6 illustrate the deviation of the R^c and a_c with respect to λ for different values of B and Pe for four types of gravity fluctuation. From these figures, we observe that the R^c increases with the increase of all three parameters Pe, B, and λ. Hence, the configuration became stable for all considered parameters. This is due to the fact that an increase in the gravity parameter, causes a decrease in the gravity fluctuation. Consequently, the system dissatisfaction and opposing tendency are notable with a reduction in a gravity field, and this causes instability in the system, and the same behavior is noted for upward throughflow impact. A rise in the viscosity parameter results in an increase in the temperature of the flow between porous walls. Therefore, viscosity has a stabilizing effect on the configuration. From these figures, it is noted that the gravity fluctuation parameter and throughflow parameter have a dual impact on wave number. In addition, the configuration is more inconsistent for case (iii), while case (iv) is found to be more consistent.

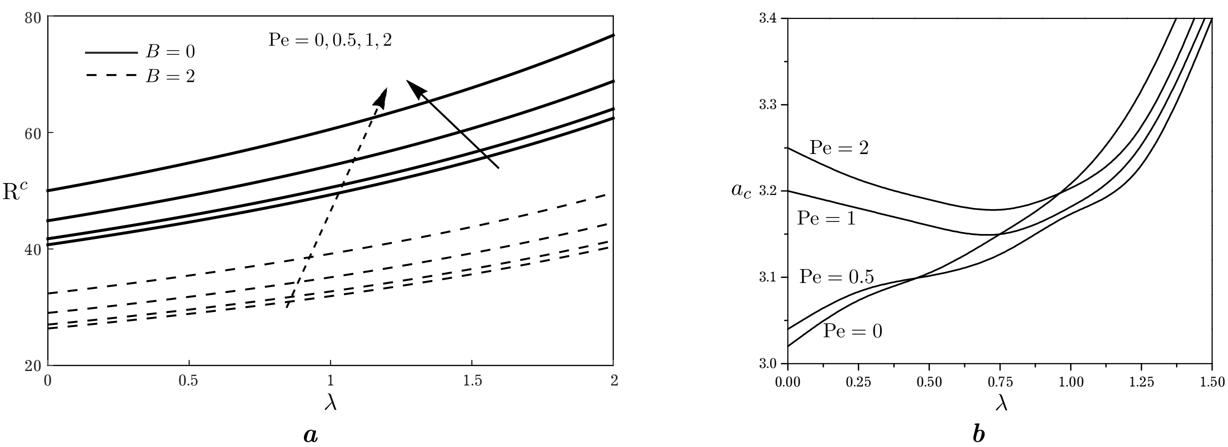
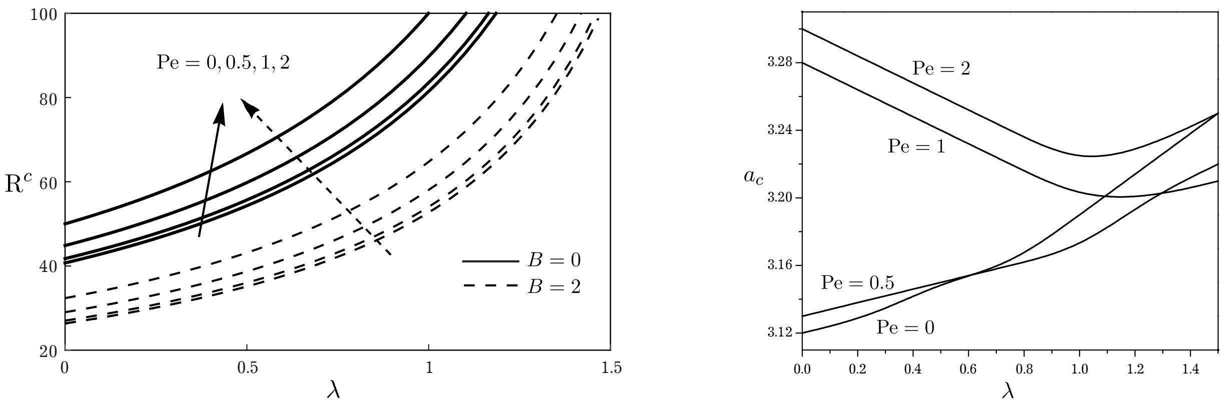


Fig. 3. The plot of (***a***) R^c and (***b***) a_c with respect to λ for $\mathrm{Pe} = 0, 0.5, 1, 2$ for case (i) $H(z) = -z$.

In order to validate the numerical approach used in the current analysis, the results are obtained in the restricted case. The persistent viscosity and lack of constant upward throughflow were compared with those stated by Rionero and Straughan [17] in Table 1. It is seen in the table that the agreement is very strong and thus confirms the precision of the tool used.

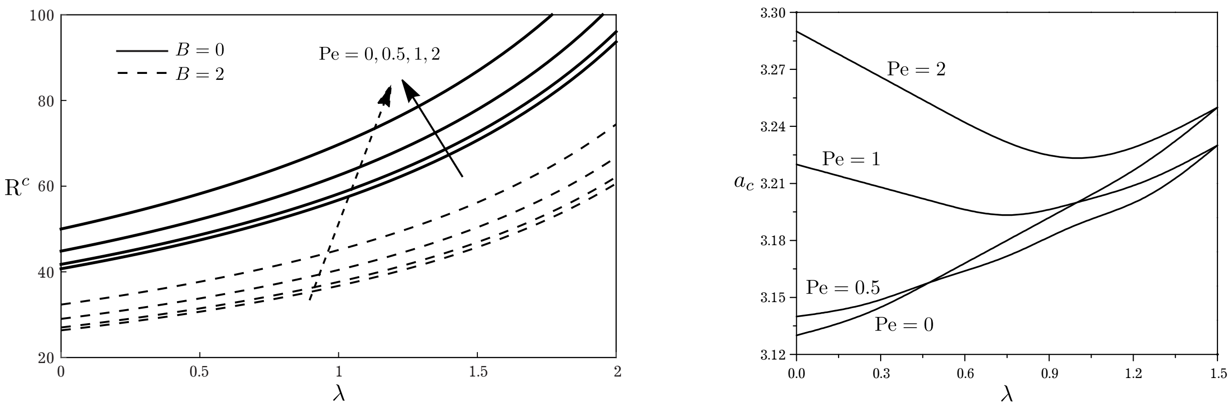
Table 1. Comparison of critical Rayleigh number R^c and critical wavenumber a_c with λ in the case of constant viscosity case $B = 0$ and nonexistence of throughflow ($\mathrm{Pe} = 0$).

$H(z)$	λ	Present study		Rionero and Straughan [17]	
		R^c	a_c^2	R^c	a_c^2
Case (i)	0	39.478	9.872	39.478	9.870
	1	77.080	10.208	77.020	10.209
	1.5	132.020	12.213	132.020	12.314
	1.8	189.908	17.198	189.908	17.198
	1.9	212.281	19.475	212.280	19.470
Case (ii)	0	39.478	9.872	39.478	9.870
	0.2	41.832	9.872	41.832	9.874
	0.4	44.455	9.885	44.455	9.887
	0.6	47.389	9.916	47.389	9.915
	0.8	50.682	9.960	50.682	9.961
	1	54.390	10.036	54.390	10.034
Case (iv)	0	39.478	9.872	39.478	9.870
	0.1	42.331	9.872	42.331	9.872
	0.2	45.607	9.885	45.607	9.883
	0.3	49.398	9.904	49.398	9.904
	0.4	53.828	9.941	53.828	9.942
	0.5	59.053	10.005	59.053	10.005



a ***b***

Fig. 4. The plot of (***a***) R^c and (***b***) a_c with respect to λ for $\mathrm{Pe} = 0, 0.5, 1, 2$ for case (ii) $H(z) = -z^2$.



a ***b***

Fig. 5. The plot of (***a***) R^c and (***b***) a_c with respect to λ for $\mathrm{Pe} = 0, 0.5, 1, 2$ for case (iii) $H(z) = -z^3$.

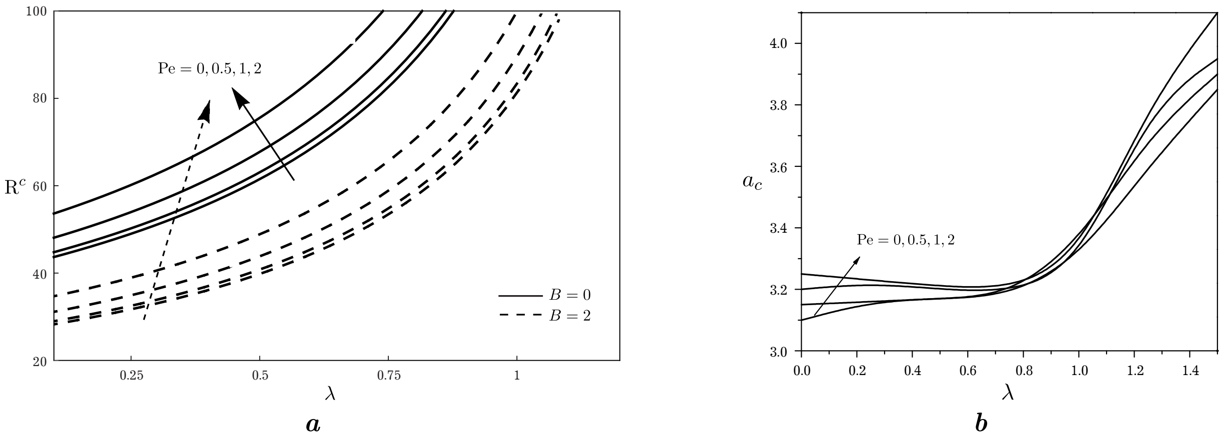


Fig. 6. The plot of $(\boldsymbol{a})$ R^c and $(\boldsymbol{b})$ a_c with respect to λ for $\mathrm{Pe} = 0, 0.5, 1, 2$ for case (iv) $H(z) = -(e^z - 1)$.

7. Conclusions

A mathematical examination of the appearance of convective unsteadiness in a porous matrix with the joint influence of the downward gravity fluctuations and viscosity with upward vertical consistent throughflow is carried out in this paper. The findings reveal that the impacts of raising the parameter of the throughflow, the parameter of gravity variance, and the viscosity parameter delay the beginning of convection, although there is a dual effect of these parameters on the convection cell size. It is noted that the fluidic system is more consistent for case (iv), while the fluidic system is more inconsistent for case (iii).

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Змінна в'язкість і змінне гравітаційне поле при настанні конвективного руху в пористому шарі з наскрізним потоком

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У цій статті розглядається комбінований вплив змінного гравітаційного поля та залежної від температури в'язкості на пористий шар для чисельного дослідження методом Галеркіна за наявності висхідного вертикального потоку. Відомо, що залежна від температури в'язкість є експоненціальною. Пориста матриця піддається постійним коливанням сили тяжіння вниз, що змінюється залежно від відстані через середовище та вертикального висхідного потоку. Було обговорено чотири різні випадки дисперсії сили тяжіння. Параметричний аналіз проводиться шляхом регулювання наступних параметрів: параметра потоку, параметра в'язкості та параметра сили тяжіння. Результати показують, що початок конвективного моменту буде затримуватися всіма трьома параметрами: потоком, залежною від температури в'язкістю та дисперсією сили тяжіння. Було показано, що рідинна система є більш суперечливою у випадку (iii) та більш узгодженою у випадку (iv).

Ключові слова: *змінна в'язкість; протікання; техніка Гальоркіна; змінний вектор сили тяжіння; лінійна стійкість.*