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Machine learning and similar image-based techniques based on Nash game theory

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The use of computer vision techniques to address the task of image retrieval is known as a Content-Based Image Retrieval (CBIR) system. It is a system designed to locate and retrieve the appropriate digital image from a large database by utilizing a query image. Over the last few years, machine learning algorithms have achieved impressive results in image retrieval tasks due to their ability to learn from large amounts of diverse data and improve their accuracy in image recognition and retrieval. Our team has developed a CBIR system that is reinforced by two machine learning algorithms and employs multiple clustering and low-level image feature extraction, such as color, shape, and texture, to formulate a Nash game. Consequently, we are faced with a multicriteria optimization problem. To solve this problem, we have formulated a three-player static Nash game, where each player utilizes a different strategy (color descriptor, Zernike descriptor, and SFTA descriptor) based on their objective function. The Nash equilibrium is defined as the membership classes of the query image.

Keywords: *image retrieval; game theory; multicriteria optimization; machine learning; color, Zernike and SFTA descriptors.*

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1. Introduction

Similar images retrieval is a fundamental problem in computer vision, image processing, and pattern recognition, with numerous applications in fields such as multimedia retrieval, medical diagnosis, surveillance, and more.

With the exponential growth of digital image data, there is an increasing need for efficient and accurate techniques to retrieve similar images [1]. Many approaches were suggested to tackle these issues. The most traditional and frequently used is image retrieval techniques based on keywords or descriptions of the images so that retrieval can be performed over the annotation words. They are referred to as Text-based Image Retrieval (TBIR) systems [2,3]. The problem with these techniques is that they have the demerits of efficiency, loss of information, more expensive tasks, and time consumption. To overcome these problems, the researchers were oriented to Content-Based Image Retrieval (CBIR) systems [4]. Over the last few years, machine learning has become a critical component and a powerful technique for solving this problem [5]. The availability of large datasets for training and testing and the ability of machine learning algorithms to be trained to automatically recognize and classify images based on their visual features have enabled significant improvements in the speed and efficiency of image retrieval systems [6].

The presented study proposes a new method for searching for similar images among a database of images, based on the multiple representations, formulated as Nash static game with complete information, and reinforced by machine learning algorithms: the clustering algorithm K-means and the classification algorithm K-nearest neighbors (KNN) algorithm.

As first step in constructing our CBIR system, we represented each image of the dataset with three vectors of representation of visual features of low level: a vector indicating the color using a compressed color histogram, a vector representing the texture (using Segmentation-based Fractal Texture Analysis

(SFTA)), and a vector representing the shape represented by Zernike Moments. Hence, the database will be presented by three Matrices: matrix one for color feature, second matrix for SFTA descriptors and the third one for Zernike moments.

Looking through an entire database to locate similar images may not seem practical, specially for a large database. Therefor, we took a more sophisticated approach by using machine learning to divide the training dataset into clusters. We achieved this by applying the clustering algorithm K-means to three levels of representation, which split the images into various classes. This approach automatically grouped the images into several predefined classes at each representation level, making the process more efficient.

This work primary aim is to propose a resolution of image retrieval problem, using Machine Learning and game theory strategy. The fact that each image of the training database is presented by three vectors naturally leads us to three-player Nash game with complete information. The first player acts according to his objective function J_C using the first strategy (color descriptor). The second player uses the second strategy (Zernike descriptor) to minimize his objective function J_F and the third player uses the third strategy (SFTA descriptor) to minimize his objective function J_T. At this stage we obtain a mutlidisciplinary optimization problem. Its optimal solution for each level of representation is used to detect the classes of membership. Then we apply the KNN algorithm on the intersection of classes of membership of the query image in order to obtain fine-grained results.

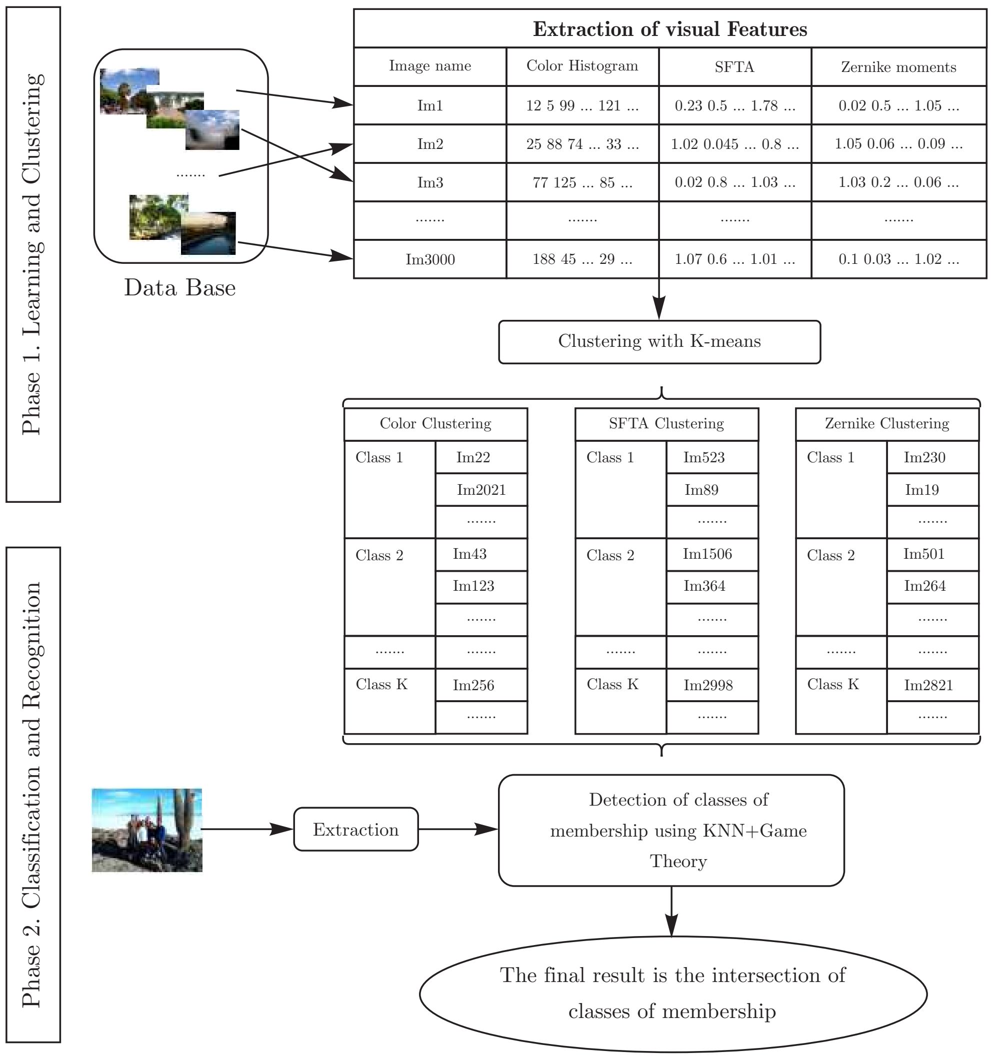


Fig. 1. Proposed CBIR system.

2. Pretreatment

Noisy images are common on the web due to various factors such as low-quality cameras, compression artifacts, and image resizing. These images can be difficult to interpret and can affect the overall user experience. Denoising techniques can help to remove unwanted noise from images and improve their visual quality [1, 7].

In our proposed CBIR system, we worked with a Tikhonov regularization-based algorithm that is capable of effectively handling images with noise.

Let U represent the original image, which is considered as function mapping an open and bounded domain Ω, and N represent the image with noise defined as

$$N = U + \eta, \tag{1}$$

where η is white additive Gaussian noise.

We reconstructed U the original image from N by minimizing the quadratic mist [8],

$$J(U) = \min_{U} \frac{1}{2}\|U - N\|^2 + \frac{\varepsilon}{2}\|\nabla U\|^2 \quad \forall U \in H_0^1(\Omega), \quad N \in L^2(\Omega), \tag{2}$$

where ε is a parameter to adjust.

a Original image

b Noisy image

c Restored image

Fig. 2.

3. Extraction of visual characteristics of the dataset

Since working with huge databases of images is a painful mission, we used the extraction process to maintain the large amount of information that represents each image.

Three following descriptors form the ground for our work [9]:

- Compressed Color Histogram;
- Zernike Moments;
- Segmentation-based Fractal Texture Analysis (SFTA).

3.1. Color histogram

A color histogram is a record of the frequency of pixels in image data that have the same color and it is the most used color descriptor in CBIR systems. Our work is based on the reduced histogram of colors developed from the regular histogram of color [9, 10].

Algorithm 1 Compressed histogram of color

Require: Image: U;
Ensure: Matrix: $[S_r\ S_g\ S_b]$;

$S_r \leftarrow U(:\,,\ :\,,\ 1)$;
$S_g \leftarrow U(:\,,\ :\,,\ 2)$;
$S_b \leftarrow U(:\,,\ :\,,\ 3)$;
$[k\ ,h] \leftarrow size(U)$;
for $j \leftarrow 1, k$
  for $j \leftarrow 1, h$
    one compute the occurrence of each value of pixel between 0 and 255;
$S_r \leftarrow \frac{1}{16} * \sum_{m=1}^{16} hr(m,1)$;
$S_g \leftarrow \frac{1}{16} * \sum_{m=1}^{16} hg(m,1)$;
$S_b \leftarrow \frac{1}{16} * \sum_{m=1}^{16} hb(m,1)$;
for $l \leftarrow 1, 15$
  $S_r(l+1) \leftarrow \frac{1}{16} * \sum_{m=16*l}^{16*l+1} hr(m,1)$
  $S_g(l+1) \leftarrow \frac{1}{16} * \sum_{m=16*l}^{16*l+1} hg(m,1)$
  $S_b(l+1) \leftarrow \frac{1}{16} * \sum_{m=16*l}^{16*l+1} hb(m,1)$

Where hr, hg and hb are respectively the red color, green color and blue color histograms.

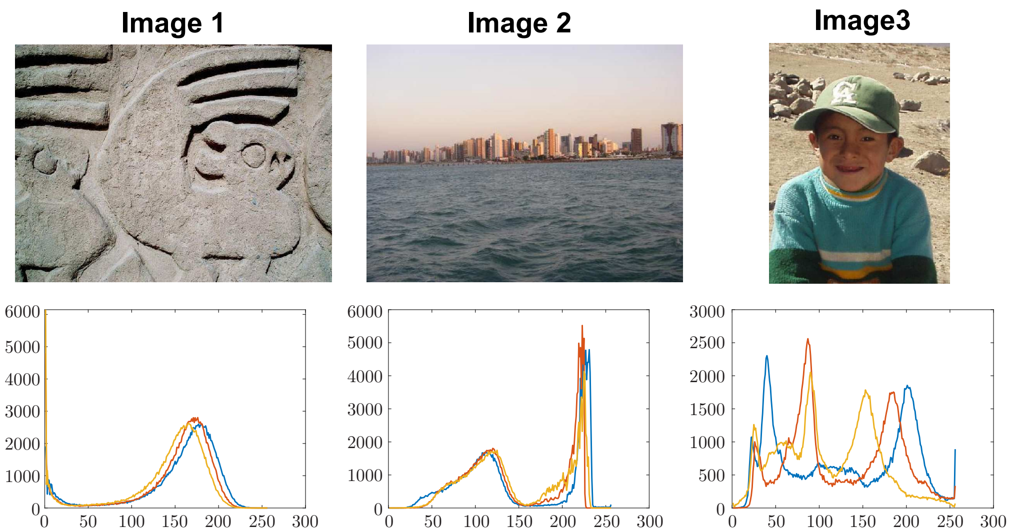


Fig. 3. Example of the compressed color histogram.

3.2. Zernike moments

Zernike moments serve as means of describing complex shapes. They are employed in similarity-based image retrieval applications, providing theoretical invariance properties with regards to translation, rotation, and scaling. Zernike moments establish a vector space where the image of the shape is projected, allowing for a compact and straightforward code for computation [11, 12].

3.3. SFTA descriptors

Segmentation-based Fractal Texture Analysis is a technique that leverages texture patterns in gray-scale images to estimate the shape or surface properties of objects of scenes. By analyzing the statistical properties of texture, SFTA algorithm can offer valuable insights into three-dimensional structure of the observed environment [10].

3.4. Preparation of the training database

We have a database containing 3000 images; for each image we calculate three vectors of representation, a color vector $CC(1,48)$, Zernike vector $CFf(1,18)$ and SFTA vector $CFt(1,36)$.

Finely the training data base is composed as follow:

- $MC(3000,48)$, a matrix that collects all CC. (3000 lines per every picture, 48 the number of color components that we have chosen).
- $MF(3000,18)$, a matrix that collects all CFf.
- $MT(3000,36)$, a matrix that collects all CFt.

4. Clustering of the database

Searching for a similar image within a vast database is impractical, especially when dealing with large dataset. To address this issue, we employed a clustering algorithm called K-means in our research [13, 14]. K-means is a widely used unsupervised clustering algorithm in machine learning. Its purpose is to group similar data points together based on their characteristics. The algorithm operates by repeatedly assigning each data point to the nearest cluster center and then updating the cluster centers by calculating the mean of the assigned data points. This iterative process continues until the cluster centers stabilize or the maximum number of iterations is reached.

To perform the K-means algorithm, the essential steps are:

Algorithm 2 K-means Algorithm

Require: Y a set of M data. K, the number of desired clusters;
Ensure: $\{P_1, P_2; \cdots P_K\}$, a partition of K clusters;
Initialize the P_K centroids with random values;
repeat
  Assignment: Assign each object to the group whose centroids is the closest, and generate a new partition;
  $y_i \in P_K$ if $\|y_i - \eta_k\| = \min_j \|y_i - \eta_j\|$ $\forall j$ where η_K is the centroids of the K classes;
  Representation: Compute the associated centroids to the new partition; $\eta_K = \frac{1}{M}\sum_{y_i \in P_K} y_i$
until the algorithm converges to a stable partition;

To evaluate the working of the K-means clustering algorithm, we applied it on a dataset of images that contains 24 images (Figure 4), and for the number of desired clusters, we chose $K = 2$.

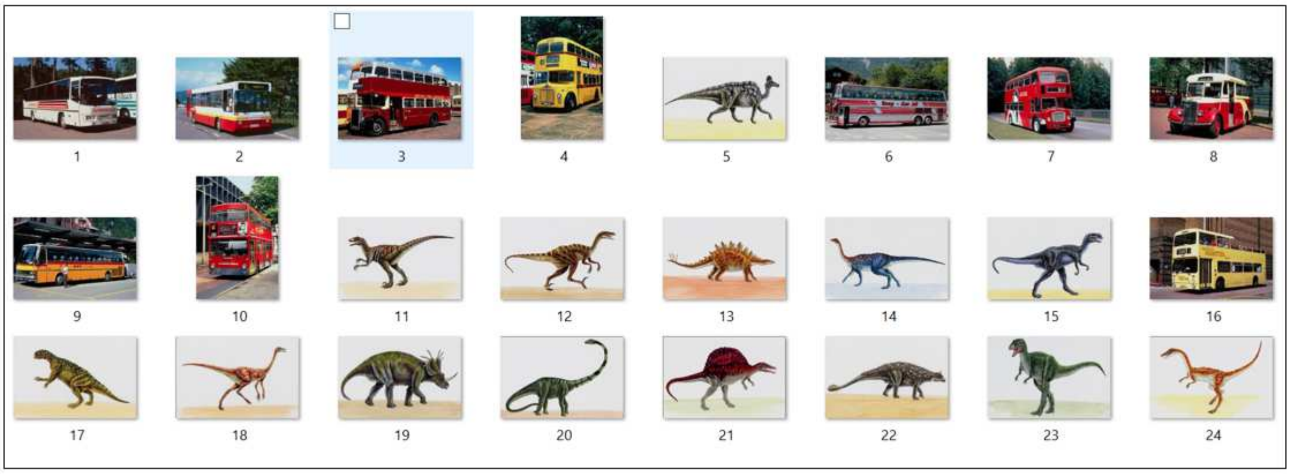


Fig. 4. DataSet.

In this example, the clustering analysis is done on the basis of color feature, where we tried to partition the dataset. As a result, we achieved our goal and obtained two homogeneous subgroups within the data (Figures 5 and 6).

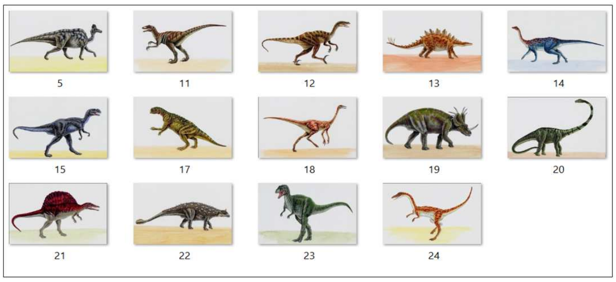


Fig. 5. Cluster 1.

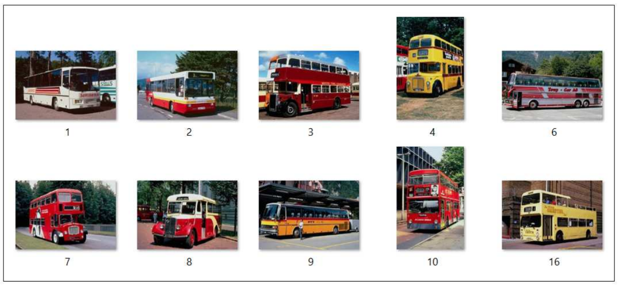


Fig. 6. Cluster 2.

5. Proposed system

The proposed CBIR system involves splitting the initial optimization variable [15] into three distinct design strategies: color (C), Zernike (F), and SFTA (T). We refer to this information as the primitive image ($U(C,F,T)$) information. The images in the database are categorized into three classes (color, Zernike, and SFTA) based on their corresponding vectors (N_C, N_F, and N_T).

Our CBIR system is designed as a three-player static Nash game of complete information [13, 16], where three players act following different objectives. The first player uses the first strategy, i.e. color descriptor, in order to minimize to his objective function J_C. The second player uses the second strategy, i.e. Zernike descriptor, to minimize his objective function J_F, while the third player uses his strategy, i.e. SFTA descriptor, to minimize his objective function J_T.

There are considered the functionals $J_C(U(C,F,T))$, $J_F(U(C,F,T))$ and $J_T(U(C,F,T))$ defined by

$$J_C(U(C,F,T)) = \frac{1}{2}\|C - N_C\|^2 + \frac{\varepsilon}{2}\left(\|\nabla C\|^2 + \|\nabla F\|^2 + \|\nabla T\|^2\right), \tag{3}$$

$$J_F(U(C,F,T)) = \frac{1}{2}\|F - N_F\|^2 + \frac{\varepsilon}{2}\left(\|\nabla C\|^2 + \|\nabla F\|^2 + \|\nabla T\|^2\right), \tag{4}$$

$$J_T(U(C,F,T)) = \frac{1}{2}\|T - N_T\|^2 + \frac{\varepsilon}{2}\left(\|\nabla C\|^2 + \|\nabla F\|^2 + \|\nabla T\|^2\right). \tag{5}$$

We address the problem of the splitting of the optimization variable between three players where the first player acts according to his objective function $J_C(C,F,T)$ using the first strategy (color descriptor). The second player uses the second strategy (Zernike descriptor) to minimize his objective function $J_F(C,F,T)$ and the third player uses the third strategy (SFTA descriptor) to minimize his objective function $J_T(C,F,T)$.

The proposed system is designed by the following optimization problem (A Game):

$$\mathbb{P}\colon \begin{cases} \text{Find} \quad U(C^*,F^*,T^*) \quad \text{such that:} \\ \min\limits_{C} J_C(C,F^*,T^*) = J_C(C^*,F^*,T^*), \\ \min\limits_{F} J_F(C^*,F,T^*) = J_Z(C^*,F^*,T^*), \\ \min\limits_{T} J_T(C^*,F^*,T) = J_T(C^*,F^*,T^*). \end{cases}$$

According to *Aubin* (1979) this problem has at least one solution (Nash equilibrium).

Theorem 1. *There exists a Nash equilibrium* (C^*,F^*,T^*) *solution of the problem* $\mathbb{P}$.

Proof. Because the functionals J_C, J_F, and J_T represent the compliance of color, Zernike, and SFTA equations, respectively, they can always be regarded as the highest envelopes of convex functions concerning C, F, and T, respectively. Thus, these three objective functions are also convex. Additionally, the strategy spaces, which are convex, exhibit weak-star compactness, while the objectives are weak-star lower semi-continuous and weakly compact concerning their strategies. Notably, the weak-star lower semi-continuity of J_C regarding C stems from using compact filters, which maintain convexity through the linearity of the filters. All assumptions necessary to apply the Nash existence theorem are satisfied, which guarantees the presence of a Nash equilibrium. For more details, see [17]. ■

5.1. Algorithm of Nash equilibrium

In order to find the Nash equilibrium we needed to solve the last problem $\mathbb{P}$ using the following decomposition algorithm 3.

Algorithm 3 Nash equilibrium Algorithm

Require: An initial strategy $U^{(0)} = (C^{(0)}, F^{(0)}, T^{(0)})$. Set $m = 0$.

1: Step 1:

Phase 1: Determine a resolution to the problem $\min\limits_{C} J_C(C, F^{(m)}, T^{(m)}) \longrightarrow C^{(m+1)}$

Phase 2: Determine a resolution to the problem $\min\limits_{F} J_F(C^{(m)}, F, T^{(m)}) \longrightarrow F^{(m+1)}$

Phase 3: Determine a resolution to the problem $\min\limits_{T} J_T(C^{(m)}, F^{(m)}, T) \longrightarrow T^{(m+1)}$

2: Step 2:

Iterate parallel phases 1, 2 and 3, set $U^{(m+1)} = (C^{(m+1)}, F^{(m+1)}, T^{(m+1)})$ until we achieve to convergence.

5.2. Similar images recognition

In order to maintain consistency, it is necessary to subject the query images to the same preprocessing procedures as the images in the training database. This entails creating descriptors for each of three representation levels and subsequently calculating the Nash equilibrium solution. Once the solution is determined, we employ it to establish membership classes using the K-nearest neighbors (KNN) algorithm [14].

When working with a new input image, denoted as I, the output is estimated by analyzing the overlap or intersection of three membership classes.

6. Tests and results

To test the proposed system, we have used a database of 3000 images from ipartc 12 database [18].

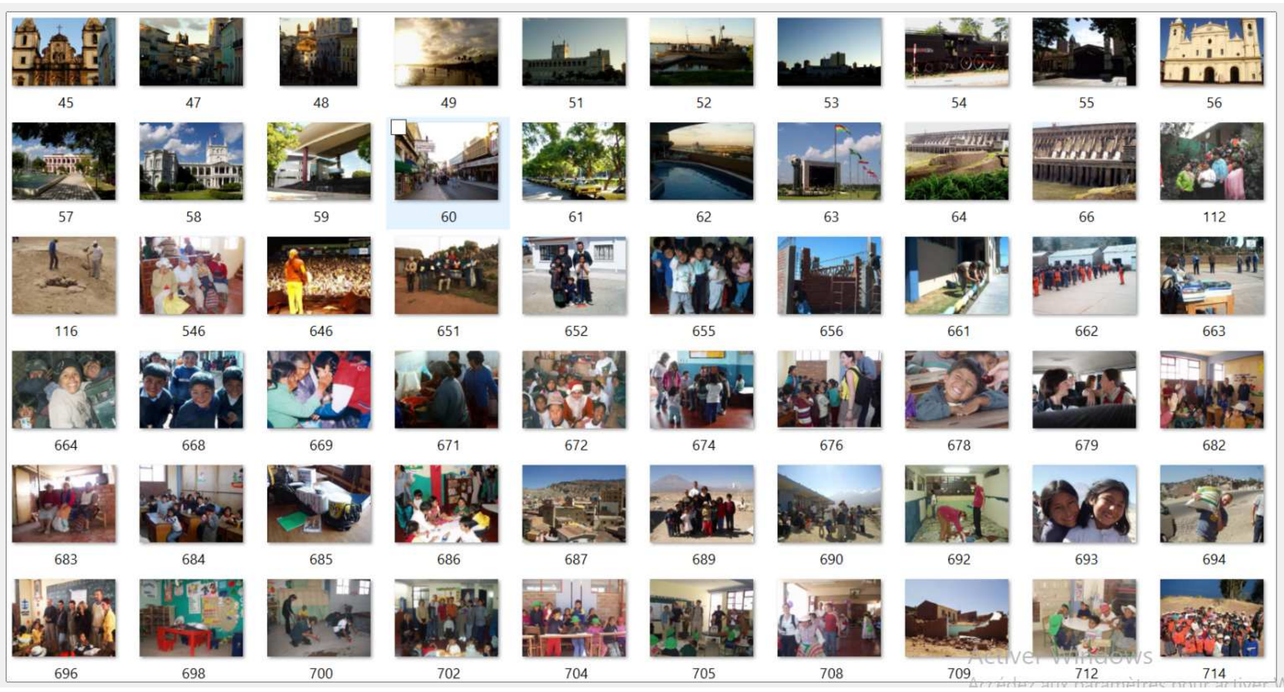


Fig. 7. Sample images from the ipartc 12 database.

Our primary goal was to create a system that could handle large amounts of data quickly, so we focused on minimizing execution time. Rather than using traditional methods that involve calculating the distance between the query image and all images in the database, we developed a system that narrows down the number of images to be checked to less than 30. We achieved this by working with different numbers of clusters for Zernike, SFTA, and compressed color histogram, specifically using $K = 12$, $K = 10$, and $K = 12$ respectively.

To highlight the effectiveness of our system we have followed two paths to retrieve similar images: the first one is searching for similar images of a **noisy image** as the query, and the second path is retrieving images where the query is the **restored image** obtained using the Tikhonov regularisation presented in section 2.

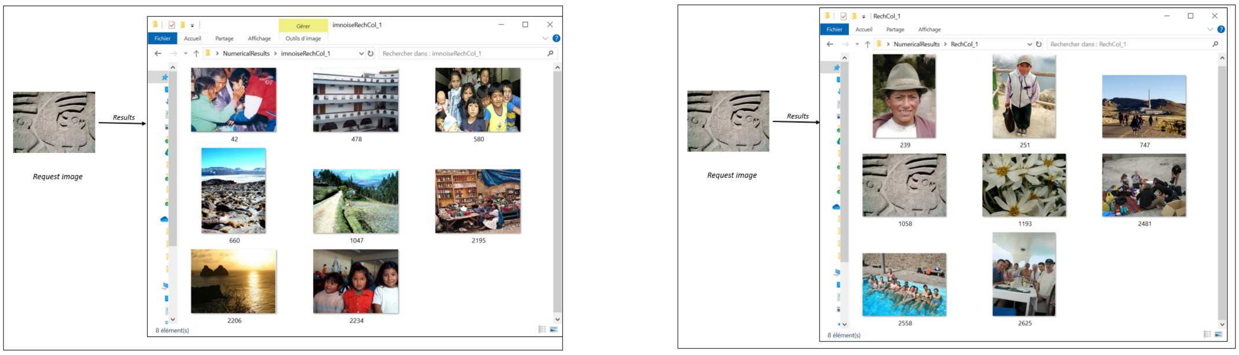
We present in the following sections examples of the wanted results.

6.1. Retrieving similar images: Results of image test 1

At the first stage of the tests performed, we added a Gaussian noise to the query image (the original) and then searched for similar images using, at each test, one of three feature descriptors.

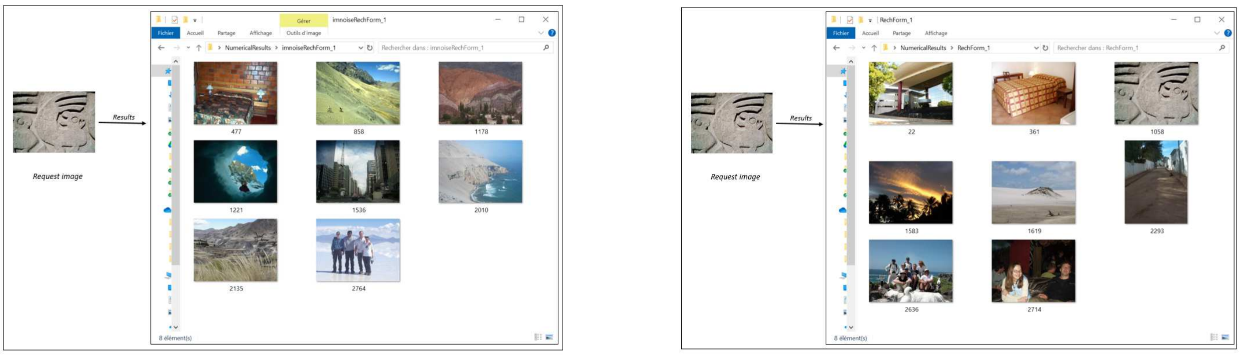
Figures 8a, 9a, and 10a situated in the first column at left present the results of searching for similar images of **noisy image** using respectively three approaches: first approach is based on a color descriptor, the second one is based on Zernike descriptor and the last one is based on SFTA descriptor.

The following figures showcase the outcomes that correspond to eight images that are most similar to the query.



a ***b***

Fig. 8. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using color histogram.

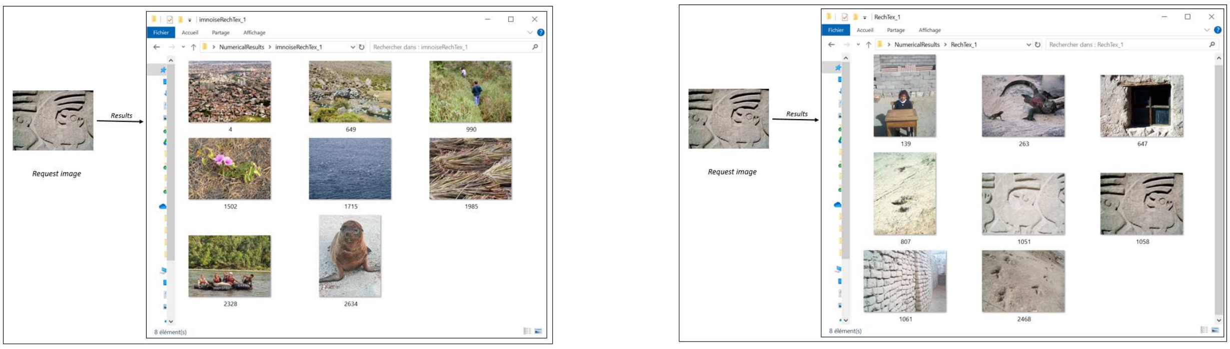


a ***b***

Fig. 9. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using the Zernike moments.

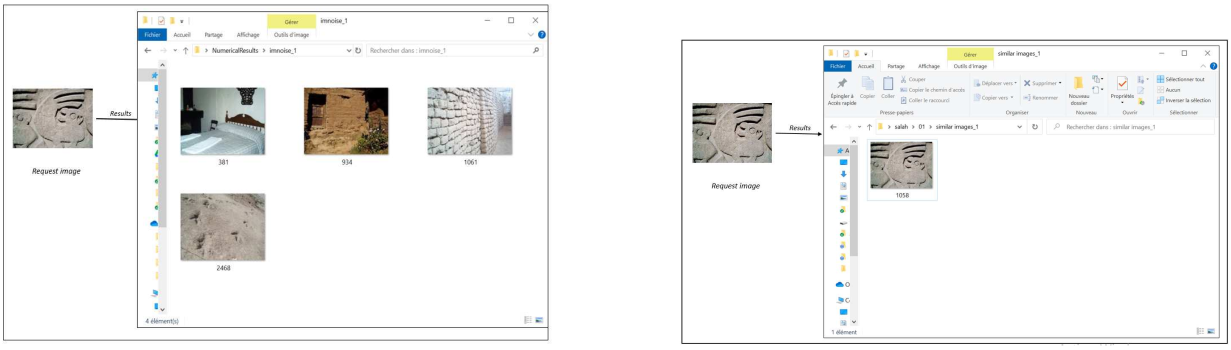
Based on the previous results, we conclude that three methods on their own produce less than satisfactory results depending on the query image, and the desired image has not been returned, whereas in Figures 8b, 9b, and 10b we can conclude that the query (restored) image has been returned, and moreover, the results are satisfactory compared to the first ones, allowing us to say that the utilization of the Tikhonov regularization in our search is indispensable and highly important to guarantee accurate results.

After observing slightly inconsistent results with previous techniques, we suggest a novel approach based on machine learning and game theory. This approach aims to determine the most pertinent im-



a *b*

Fig. 10. Results of similar images detection of the **noisy image** (*a*) and **restored image** (*b*) using SFTA.



a *b*

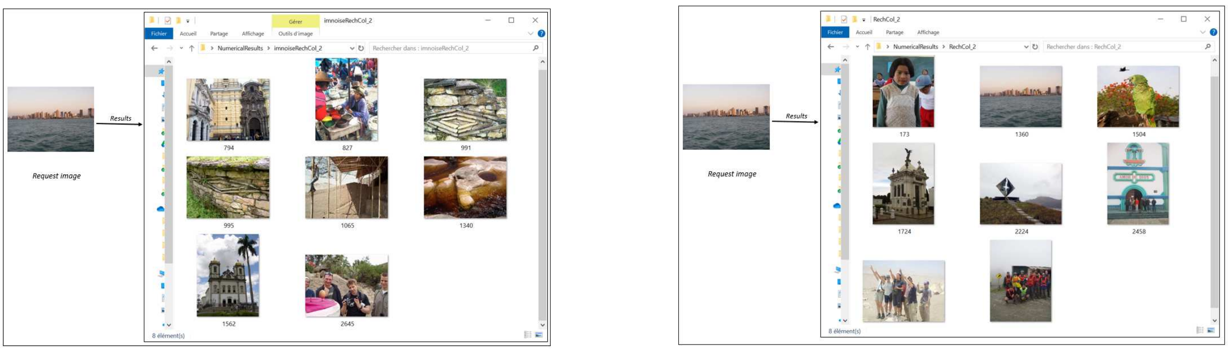
Fig. 11. Results of similar images detection of the **noisy image** (*a*) and **restored image** (*b*) using the proposed system.

ages by intersecting the set of images obtained through the utilization of the color descriptor (condensed color histogram), the shape descriptor (Zernike moments), and the texture descriptor (SFTA).

In Figure 11*b*, we can see a considerable reduction in the number of images returned, and provide the query image with an accuracy of 99.9%.

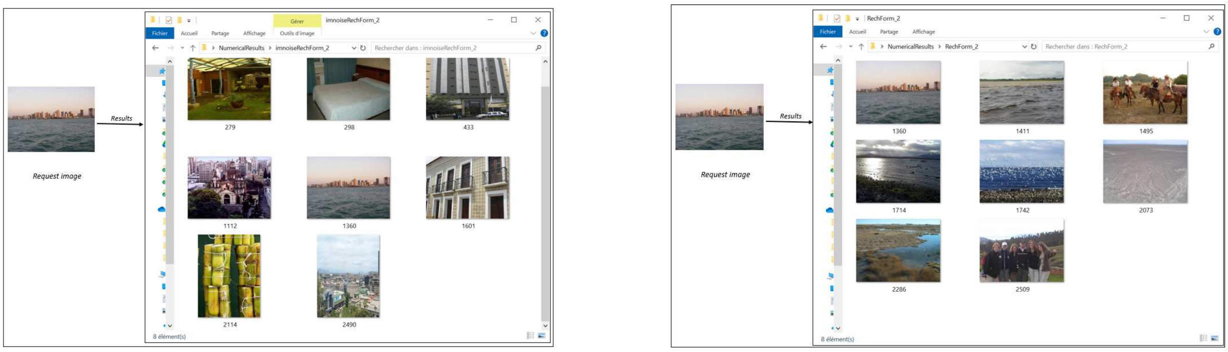
6.2. Retrieving similar images: Results of image test 2

In the second example we followed the same process as the first one. We always remark some unsatisfactory results when using the classical methods utilized in the previous example or when not using the Tikhonov algorithm. For example in Figure 13*a*, the searched image has been correctly returned but the other returned images are not similar to the query, while we have the opposite in Figure 13*b*.



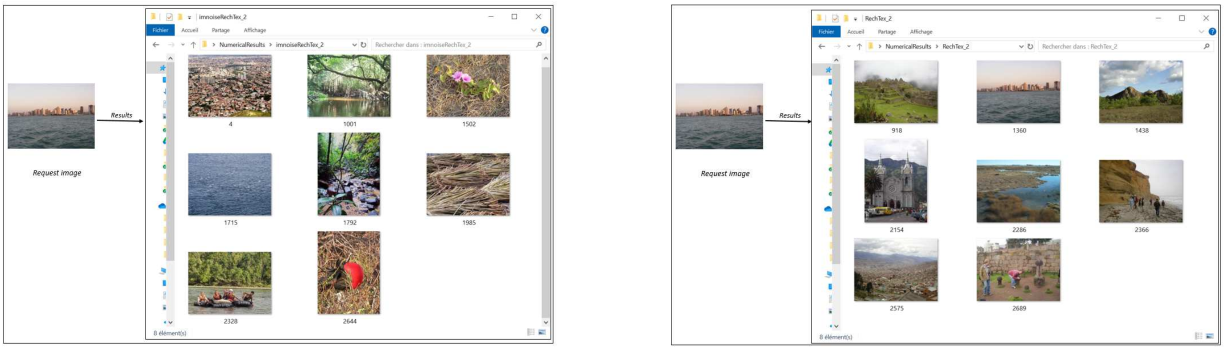
a *b*

Fig. 12. Results of similar images detection of the **noisy image** (*a*) and **restored image** (*b*) using color histogram.



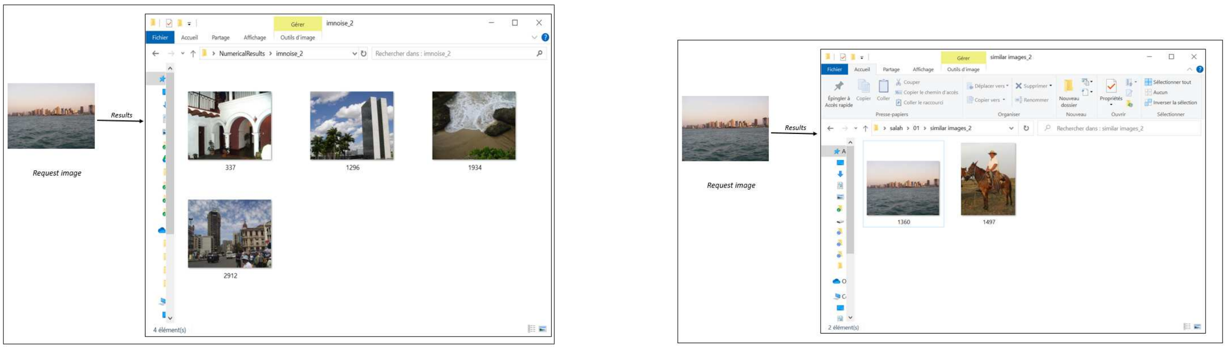
a ***b***

Fig. 13. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using Zernike moments.



a ***b***

Fig. 14. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using SFTA.



a ***b***

Fig. 15. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using the proposed system.

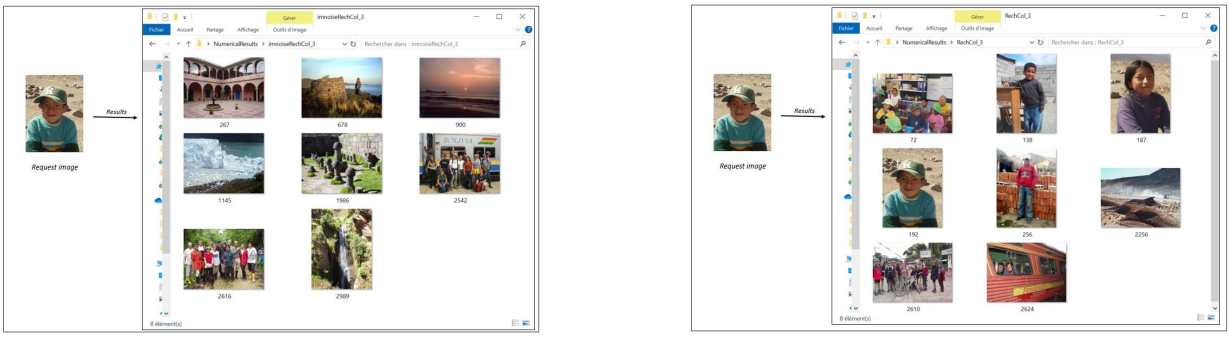
6.3. Retrieving similar images: Results of image test 3

Our algorithm shows between 1 to 6 images including the searched image. Then we manually evaluate if the number of images judge similar to the query is more than 4 the results are considered correct.

To assess the effectiveness of the suggested approach, we conducted a comparative analysis between our algorithm and combining K-means and KNN methods using the descriptors we presented.

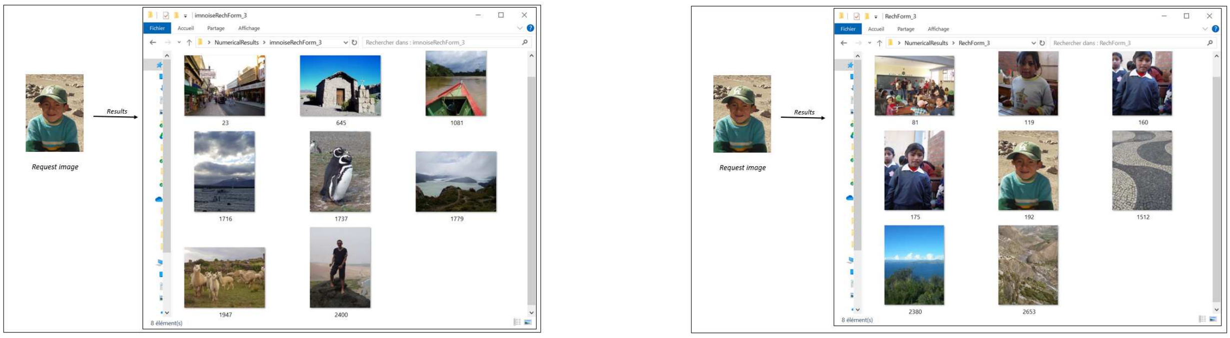
Table 1 resumes the maximum time and the detection rates required to recognize an image of the previous tests.

The presented data in the table below are a solid proof to the effectiveness of our system. We observed that the integration of game theory and machine learning was able to increase the rate of classification achieved by KNN and K-means from 89% to 99%.



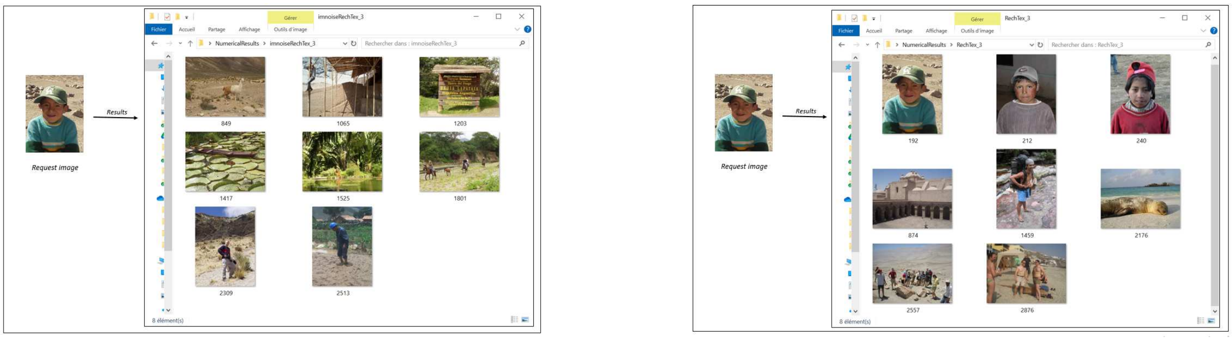
a ***b***

Fig. 16. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using color histogram.



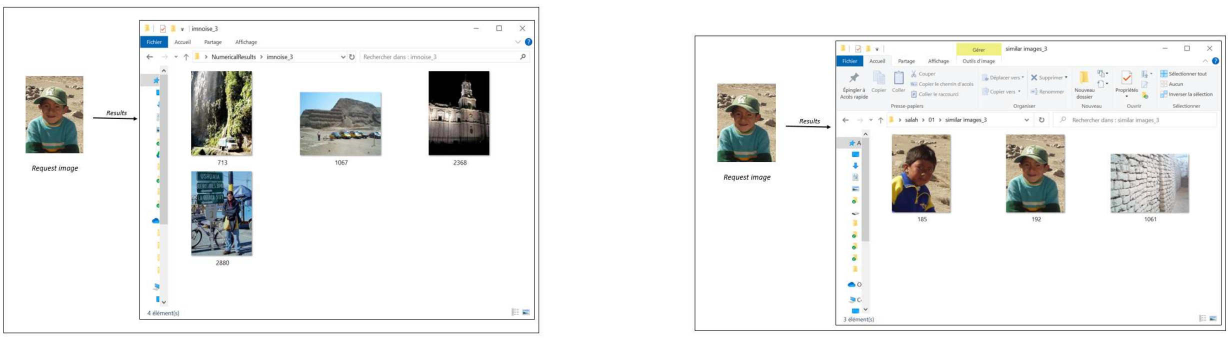
a ***b***

Fig. 17. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using Zernike moments.



a ***b***

Fig. 18. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using SFTA.



a ***b***

Fig. 19. Results of similar images detection of the **noisy image** (***a***) and **restored image** (***b***) using the proposed system.

Table 1. A comparative analysis: The classical approaches Versus the proposed system.

		K-means+KNN+HCC		K-means+KNN+Zernike		K-means+KNN+SFTA		K-means+KNN+(3 descriptors)		Proposed system	
		detection rate	max time	detection rate	max time	detection rate	max time	detection rate	max time	detection rate	max time
IMAGE 1	noisy	12%	12s	18%	16s	20%	10s				
	restored	75 %	50 s	50%	49 s	93%	46 s	40%	10 s	**100%**	**41 s**
IMAGE 2	noisy	30%	13 s	37%	13 s	10%	11 s				
	restored	62%	49 s	87.6%	57 s	75%	47 s	15%	9 s	**91%**	**45 s**
IMAGE 3	noisy	23 %	10 s	37.5%	14 s	34.7%	9 s				
	restored	87.5%	45 s	75%	50 s	85%	44 s	12.4%	9 s	**89%**	**42 s**

7. Conclusion

In this study, we proposed an image retrieval system based on Nash game theory and improved by machine learning. According to the performed tests and the obtained results, the integration of game theory proved efficiency of the developed CBIR system compared to existing methods including KNN-based methods. Also it was shown that using machine learning has allowed to improve the speed and accuracy in searching and retrieving a vast number of images similar to the request.

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Машинне навчання та подібні методи на основі зображень, засновані на теорії ігор Неша

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Використання методів комп'ютерного зору для вирішення завдання пошуку зображень відоме як система пошуку зображень на основі вмісту (Content-Based Image Retrieval, CBIR). Ця система призначена для пошуку та вилучення відповідного цифрового зображення з великої бази даних за допомогою зображення–запиту. За останні декілька років алгоритми машинного навчання досягли вражаючих результатів у задачах пошуку зображень завдяки своїй здатності навчатися на великих обсягах різноманітних даних і підвищувати точність розпізнавання та пошуку зображень. Наша команда розробила систему CBIR, яка підсилена двома алгоритмами машинного навчання і використовує множинну кластеризацію та виділення низькорівневих ознак зображення, таких як колір, форма та текстура, для формулювання гри Неша. Отже, ми зіткнулися з проблемою багатокритеріальної оптимізації. Щоб вирішити цю проблему, сформульовано статичну гру Неша для трьох гравців, де кожен гравець використовує свою стратегію (дескриптор кольору, дескриптор Церніке та дескриптор SFTA) на основі своєї цільової функції. Рівновага Неша визначається як класи належності зображення запиту.

Ключові слова: *пошук зображень; теорія ігор; багатокритеріальна оптимізація; дескриптори: колір, Церніке та SFTA*