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DYNAMIC MODEL OF THE DURATION OF GASEOUS ENVIRONMENT PUMPING FROM A LIMITED VOLUME

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Abstract. An analytical equation for the duration of air pumping from a limited volume has been developed. The equation of the mechanical energy of air movement takes into account the law of mass conservation for a gas in a controlled volume, the work of the energy of air movement and the work to overcome air friction. Gaseous medium is Newtonian. The duration of pumping (filling) the limited volume of the pneumatic chamber with air was calculated by comparing the mass flow per second and the increase in mass, as a differential of the change in air density. The mathematical model enables the simulation of air pumping time depending on pressure, as a density parameter and at different Mach numbers in the subsonic range. The $\boldsymbol{K_1^*}$ proportionality coefficient, which characterizes the ratio of the dynamic force of gas mass displacement to the static pressure relative to the diameter of the air pipeline, is proposed as a criterion for evaluating the dynamics of the flow. It should be noted that the analytical dependence works for Newtonian media and Mach numbers of $M < 1$, the gas flow is caused by the pressure difference, the gas itself is limited by a chamber space characterized by a volume as design parameter of structure.

Key words: air pumping; mathematical model; air friction; dynamics of the air flow; pneumatic chamber.

Introduction

The purpose of the research was to derive analytical dependences of the duration of air pumping from a limited volume, taking into account the coefficient of friction for the gas medium due to changes in pressure and density of the gas in the process of pumping it out.

Pneumatic systems are widely used in engineering. This is due to the low cost of the air used in machinery and the availability of developed innovative technical solutions. They are used to convert the power of compressed air into mechanical power, for example in automobiles, air-powered pneumatic machines [1–3].

In order to improve the efficiency of pneumatic systems, they are being studied for increased productivity. Shen et al. [4] investigated the dynamic characteristics of a pneumatic pump during the air injection process. Takeuchi et al. [5] developed an expansion type pump using the expansion energy and they proved the efficiency of the new air booster structure. Shaw et al. [6] design a pneumatic motor system that is driven by compressed air. These authors obtained the ratio of speed and efficiency, but did not investigate the method of improving the parameters of the system.

An important parameter of the efficiency of the pneumatic system is the time of air pumping up (air pumping out) to a given pressure, which characterizes its dynamics. Weixiang Ni et al. [7] developed a mathematical model of a small pneumatic chamber to determine time parameters. The authors modeled numerically using Van der Waals and Redlich – Kwong equations for a given interval of input parameters.

Jones et al. [8] confirmed that the gas approximately conforms to the law of the equation of state of an ideal gas only at low temperature. At high pressure, any gas will not completely follow the inverse polytropic dependence. Gas characteristics affect the polytropy index of the thermodynamic process. This indicator is equal to n = 1.41 [9]. Also, the authors assumed a value of 1.2 in the calculations of [10, 11], but this value is not theoretically justified. Transient processes during experiments showed that n was approximately 1.4 in closed pneumatic chambers, and the thermodynamic regime is close to adiabatic with zero heat transfer [12, 13].

Compressor design requires a mathematical model. Differential equations are solved numerically. Tuhovák et al. experimentally verified the results of numerical modeling [14]. But time parameters were not modeled.

Also Liang et al. showed a mathematical model for formulating the influence of structural, geometric and control parameters on the output pulsation. The finite difference method was used to solve nonlinear equations [15].

The time parameters of the pneumatic system are important for the design and research of robot systems [16–18]. To model the airflow control system, Wu et al. conducted dynamic simulations in SIMULINK [19]. The simulation results were compared with the data of experimental studies for a laboratory ventilation unit. Rakova et al. analyzed the pneumatic system using a simulation model [20]. The analysis involved building a model of each type of pneumatic system component. Also, Dmytriv et al. calculated the time parameters of the pneumatic generator using a regression model [21].

Materials and methods

In the tasks of implementing adaptive control systems, the problem of analytical identification of process parameters arises. Such systems are characterized by processes whose parameters are simultaneously influenced by several factors characterizing the technological process.

The purpose of the research is analytical modeling of the time of pumping air, as a Newtonian medium, from a limited space of a given volume.

Derivation of the analytical equation for the duration of air pumping from a limited volume

Pumping and filling the limited space of technological equipment with a gaseous medium is a problem of the dynamics of the movement of this medium caused by pressure changes.

Consider the dynamics of transient processes of an isolated chamber. Let's take air as a gaseous medium. Air will move in two directions, either by pumping out of the isolated chamber to a given vacuum and filling the isolated chamber to atmospheric pressure, or by filling the isolated chamber with air to a given pressure and venting to atmospheric pressure.

To derive the analytical dependence, we consider the section of the pneumatic system. Taking into account the law of conservation of mass for a gas in a controlled volume through the equation of the mechanical energy of air movement [22] and the work to overcome air friction [23], the equation of the mechanical energy of air movement can be written in the following form:

$$dQ - d(PW_{AR}) - dL - dL_{fr} = dU + d\left(\frac{v^2}{2g}\right) + gdz, \qquad (1)$$

where Q – the amount of heat supplied to the system (under normal conditions T = const, $Q = 0$, $dQ = 0$); z – the height difference of the position of two different sections of the air flow, which characterizes the potential energy (for the analyzed case $z = 0$, $dz = 0$); L – technical work that characterizes the change in the physical position in area of the mechanical elements of the system; L_{fr} – the work spent on overcoming the frictional forces of the air to the elements of the system.

The movement of mechanical elements does not occur, therefore the mechanical work from the movement of air is spent on changing the internal energy of the air in the system $dL = dU + pdW_{AR}$.

The equation of work to overcome air friction forces can be written in the following form [23]:

$$dL_{fr} = \lambda \frac{l_{fr}}{D_{fr}} d\left(\frac{v^2}{2g}\right). \tag{2}$$

Taking into account the above statements, dependence (1) will take the form:

$$-W_{AR} \cdot dp = d\left(\frac{\upsilon^2}{2g}\right) + \lambda \cdot \frac{l_{fr}}{D_{fr}} \cdot d\left(\frac{\upsilon^2}{2g}\right) \tag{3}$$

where $W_{AR} = 1/(g \cdot \rho_{AR})$ – specific volume of air; λ – coefficient of air friction during its transportation.

The coefficient of air friction during its transportation in the air pipeline will be written in the form of [23]

$$\lambda = \frac{1-M^2}{M^2} \cdot \frac{2 \cdot D}{x \cdot \sqrt{K_1^*}} \cdot \left(\ln\frac{\rho}{\rho_i} - \frac{m}{V} \cdot \left(\frac{1}{\rho_i} - \frac{1}{\rho}\right)\right) \tag{4}$$

where M – the Mach number; D – the inner diameter of pipeline, m; x – the length of the pipeline section for which the coefficient of air friction is calculated, m; ρ – the air density at given pressure, kg/m^3; ρ_i – the air density at pressure of vacuum, kg/m^3; m – the mass flow of air in the pipeline, reduced to atmospheric pressure, kg/sec; V – the volume flow of air at a given pressure, m^3/sec; K_1^* – the aspect ratio characterizes the ratio of forces.

After substitution in equation (4) and simplification, equation (2) will have the form:

$$dL_{AR} = \frac{1-M^2}{M^2} \frac{2}{\sqrt{K_1^*}} \left(\ln\left(\frac{P_i}{P_a}\right)^{\frac{1}{n}} + \frac{m_\Gamma}{R_0 \cdot T} \cdot \frac{P_a}{\rho_a} \cdot \left(1 - \left(\frac{P_a}{P_i}\right)^{\frac{1}{n}}\right)\right) \cdot d\left(\frac{v^2}{2 \cdot g}\right). \tag{5}$$

Taking into account the above, dependence (3) will take the form:

$$-\frac{dP}{g \cdot \rho} = d\left(\frac{v^2}{2g}\right) + \frac{1-M^2}{M^2} \frac{2}{\sqrt{K_1^*}} \left(\ln\left(\frac{P_i}{P_a}\right)^{\frac{1}{n}} + \frac{m_\Gamma}{R_0 \cdot T} \cdot \frac{P_a}{\rho_a} \cdot \left(1 - \left(\frac{P_a}{P_i}\right)^{\frac{1}{n}}\right)\right) \cdot d\left(\frac{v^2}{2g}\right). \tag{6}$$

We denote the components of equation (6) as follows,

$\xi = \frac{1-M^2}{M^2} \frac{2}{\sqrt{K_1^*}} \left(\ln\left(\frac{P_i}{P_a}\right)^{\frac{1}{n}} + \frac{m_\Gamma}{R_0 \cdot T} \cdot \frac{P_a}{\rho_a} \cdot \left(1 - \left(\frac{P_a}{P_i}\right)^{\frac{1}{n}}\right)\right)$ – coefficient of frictional resistance [23].

Let's rewrite equation (6) as follows:

$$d\left(\frac{v^2}{2g}\right) = -\frac{1}{1+\xi} \cdot \frac{dP}{g \cdot \rho}. \tag{7}$$

Let's integrate equation (7) given the limits $P \to$ from P_i to P_a and $v \to$ from v_1 to v_2 and consider the equation of state of an ideal gas, which is subject to polytropic dependence with the polytropy index n, which will determine the change in the speed of air movement, provided that $v_1 = 0$ at the time of pumping air from the system

$$v_2 = \sqrt{\frac{2}{1+\xi} \frac{P_i}{\rho_i} \left(1 - \left(\frac{P_a}{P_i}\right)^{\frac{n-1}{n}}\right)}. \tag{9}$$

The mass flow per second of air is calculated as the product of the speed of air movement (8), the cross-sectional area of the hole through which the air is pumped, and its density of $m = \upsilon_2 \cdot S_D \cdot \rho_i$ and after taking into account of $\rho_i = \rho_a \cdot \left(\frac{p_i}{p_a}\right)^{\frac{1}{n}}$, we will obtain a dependence for modeling the flow of second air supply during pumping:

$$\mathrm{M} = \frac{\pi \cdot D^2}{4}\sqrt{\frac{2}{1+\xi} \cdot P_i \cdot \rho_i \left(\left(\frac{P_i}{P_a}\right)^{\frac{2}{n}} - \left(\frac{P_i}{P_a}\right)^{\frac{n+1}{n}}\right)}. \tag{9}$$

We form the differential equation for pumping out the V given volume of air at the condition that the M mass of air that passes during time τ and taking into account the expression for ρ_i, then the differential equation will be as:

$$d\mathrm{M} = \frac{\pi \cdot D^2}{4} \cdot \psi \cdot \sqrt{P_i \cdot \rho_i} \cdot d\tau, \tag{10}$$

where $\psi = \sqrt{\frac{2}{1+\xi} \cdot \left(\left(\frac{P_i}{P_a}\right)^{\frac{2}{n}} - \left(\frac{P_i}{P_a}\right)^{\frac{n+1}{n}}\right)}$ – speed coefficient of proportionality, which characterizes the pressure ratio for air pumping out; p_i and ρ_i – current value of pressure and density in the V volume.

After taking into account the expression for ρ_i, we will perform the transformation in the dependence of (10):

$$d\mathrm{M} = \frac{\pi \cdot D^2}{4} \cdot \psi \cdot \sqrt{P_a \cdot \rho_a} \cdot \sqrt{\left(\frac{P_i}{P_a}\right)^{\frac{n+1}{n}}} \cdot d\tau\,. \tag{11}$$

Also, after taking into account that the change in air mass is $d\mathrm{M} = V \cdot d\rho_i$ and the expression for ρ_i, we will get:

$$d\mathrm{M} = V \cdot \rho_a \cdot d\left[\left(\frac{P_i}{P_a}\right)^{\frac{1}{n}}\right] = \frac{V}{n} \cdot \rho_a \cdot \left(\frac{P_i}{P_a}\right)^{\frac{1-n}{n}} \cdot d\left(\frac{P_i}{P_a}\right). \tag{12}$$

After equating formulas of (11) and (12), taking into account the sign and shortening, we will obtain the differential equation of pumping air from the V volume in the following form:

$$\frac{1}{n} \cdot \int_1^{\frac{P_i}{P_a}} \left(\frac{P_i}{P_a}\right)^{\frac{1-3n}{2n}} \cdot d\left(\frac{P_i}{P_a}\right) = -\frac{\pi \cdot D^2}{4} \cdot \frac{\psi}{V} \cdot \sqrt{\frac{P_a}{\rho_a}} \cdot \int_0^{\tau} d\tau\,. \tag{13}$$

The integration results, which characterize the duration of pumping air from the V volume from the p_a pressure to the p_i pressure, will be in the following form:

$$\tau = -\frac{8 \cdot V}{(1-n) \cdot \pi \cdot D^2 \cdot \psi} \cdot \sqrt{\frac{\rho_a}{P_a}} \cdot \left(\left(\frac{P_i}{P_a}\right)^{\frac{1-n}{2n}} - 1\right), \tag{14}$$

where V – the volume of the chamber from which the air is pumped, m^3; D – the diameter of the air pipeline, m; ρ_a – the air density in V volume at atmospheric pressure, kg·sec^2/m^4; P_a – the atmospheric pressure of the air in the V volume, kg/m^2; P_i – the pressure value at the i-th moment of time, kg·sec^2/m^4; m_e – the molar mass of air, kg/mol; R_0 – the universal gas constant, kg·м^2/(mol·K·sec^2); T – air temperature, К; n – the polytropy index, $n = 1.41$.

Results

Simulation of pumping time to a given pressure in a limited volume

Air pumping time was modeled as a function of pressure, as a density parameter, and at various Mach numbers. Gaseous medium is Newtonian. The speed of air movement in the air pipeline is subsonic and was assumed to be no higher than the Mach number of $M = 0.5$. Accordingly, the pumping pressure was taken from atmospheric to vacuum pressure up to 10 KPa. The results of modeling under the condition of changing the pressure from atmospheric to vacuum at the level of 10 KPa are shown in Fig. 1.

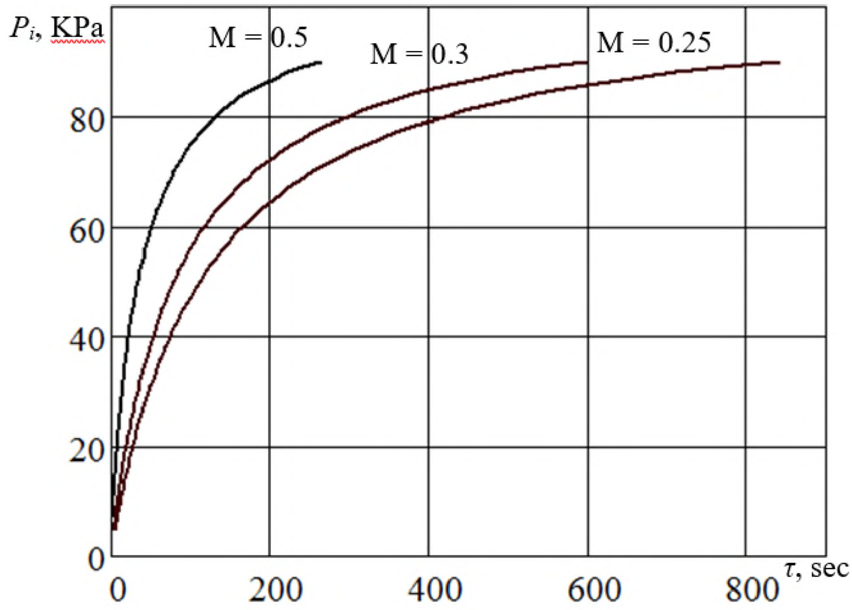

Fig. 1. The τ duration of pumping air to the P_i pressure in a limited volume depending on the air flow rate in the air pipeline (*M* – Mach numbers)

The volume of the chamber from which air was pumped was of 2 m^3, the molar mass of air was $m_г = 0.02896$ kg/mol, the universal gas constant was $R_0 = 8.31441$ kg·m^2/(mol·K·sec^2), the air temperature was $T = 288$ K. Air density of $\rho_a = 0.125$ kg·s^2/m^4 was taken at a given temperature.

The simulation results of the time of air pumping from a chamber with a volume of 2 m^3 from atmospheric pressure to a vacuum of 10 KPa are as follows: at a Mach number of $M = 0.25$, the duration of pumping is 840.2 sec, at a Mach number of $M = 0.3$, the duration of pumping is 596.9 sec, and at a Mach number of $M = 0.5$ pumping duration is 263.7 sec. The diameter of air pipeline was assumed to be 1 inch.

The pressure change in the variable pressure volumes, which regulates the volume of the variable pressure chambers $V = 0.00063$, 0.00056, 0.00032, 0.00012 m^3, was modeled. The results are shown in Fig. 2 and 3.

Analysis of the graphs (Figs. 2 and 3) shows that it takes 0.112 sec to pump air from the variable pressure volumes to the vacuum pressure $p_i = 48$ kPa. The nature of air pumping, as shown by the simulation results, is linear.

Similarly, as for a vacuum, under excess pressure, with an increase in the Mach number during air transportation, the duration of pumping from a given chamber volume decreases.

It should be noted that when modeling the air pumping time from a given chamber in the analytical dependence, the formula of the air friction resistance coefficient is used, which takes into account its change in dynamics during the air transportation process. The ratio of forces is taken into account through the proportionality factor of K_1^* [23].

For example, for the Mach number of $M = 0.005$, under vacuum at the air flow of $V = 5.751 \cdot 10^{-3}$ m^3/sec and the speed of air transportation in the pipeline $\boldsymbol{v} = 1.497$ m/sec, the proportionality factor was $\boldsymbol{K_1^*} = 0.0799$, under excess pressure at the Mach number of $M = 0.005$ these parameters were, respectively: $V = 5.681 \cdot 10^{-3}$ m^3/sec, $\boldsymbol{v} = 1.469$ m/sec, $\boldsymbol{K_1^*} = 0.0794$.

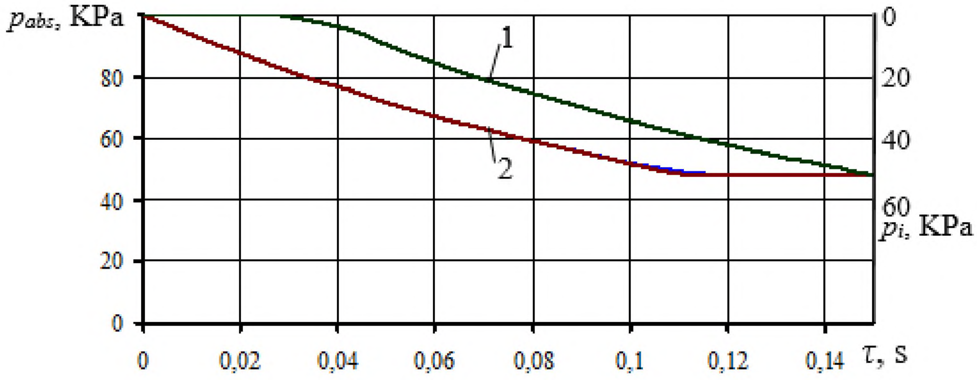

Fig. 2. The pressure change schedule in variable pressure chambers:
1 – V = 0.00063 m³; 2 – 0.00032 m³

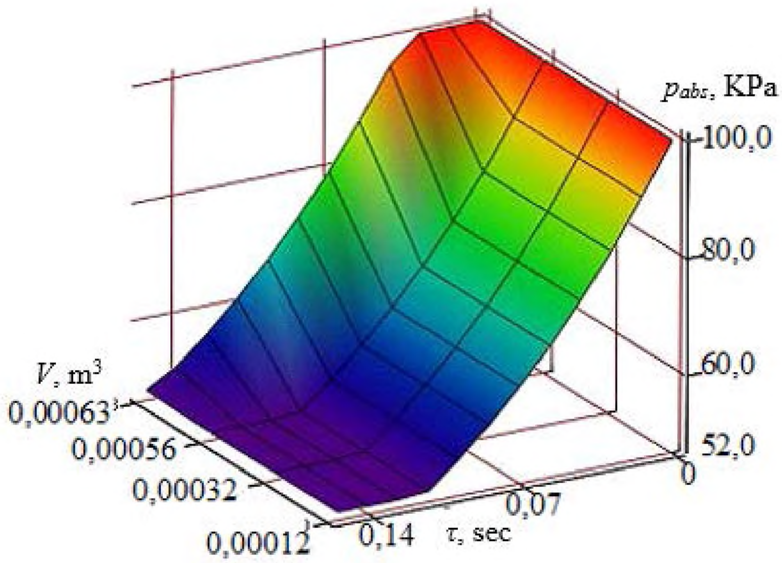

Fig. 3. Model of pressure change pads in variable pressure chambers depending on their volume V

Let us consider the results for other Mach numbers. For the Mach number of $M = 0.01$, respectively, under vacuum of $V = 2.951 \cdot 10^{-3}$ m³/sec, $\boldsymbol{v} = 2.9$ m/sec, $\boldsymbol{K_1^*} = 0.11163$; under excess pressure of $V = 2.987 \cdot 10^{-3}$ m³/sec, $\boldsymbol{v} = 2.936$ m/sec, $\boldsymbol{K_1^*} = 0.1123$.

At the conditions of Mach number of $\boldsymbol{M} = 0.1$, respectively, under vacuum of $V = 33.375 \cdot 10^{-3}$ m³/sec, $\boldsymbol{v} = 29.01$ m/sec, $\boldsymbol{K_1^*} = 0.353$; under excess pressure of $V = 33.759 \cdot 10^{-3}$ m³/sec, $\boldsymbol{v} = 29.348$ m/sec, $\boldsymbol{K_1^*} = 0.3551$.

Conclusions

The analysis of the simulation results shows that at vacuum and overpressure with a decrease in the Mach number, the duration of gas pumping increases. As the diameter of the pipeline increases, it is clear that the pumping time decreases. The nature of the change in pumping time approaches an exponential characteristic, which corresponds to the physical reality of the process of pumping air from a limited space.

The nature of the increase in the dynamic parameters of the gas flow, speed and flow regimes – from laminar to turbulent, respectively, leads to an increase in the Mach number. The proportionality coefficient evaluates the relationship of the flow kinematics to the dynamic characteristics of the environment and characterizes the flow density.

The developed analytical dependence for modeling and calculating the time of air pumping from a given space of chamber reflects the physics of the process of the flow of media in pneumatic systems.

It should be noted that the analytical dependence works for Newtonian media and Mach numbers $M < 1$, the gas flow is caused by the pressure difference; the gas itself is limited by a chamber space characterized by a design parameter – volume.

As a criterion for evaluating the dynamics of the flow, the $\boldsymbol{K}_1^*$, proportionality coefficient is proposed, which characterizes over time the ratio of the dynamic force of gas mass movement to the static pressure in relation to the diameter of the air pipeline. As the $\boldsymbol{K}_1^*$ proportionality factor increases, the pumping time of the gaseous medium decreases.

The developed analytical dependence of pumping time can be applied to the design of pipeline transport of gaseous media and pneumatic and dynamic technological systems, under vacuum and excess pressures in the system, which are subject to Newton's laws and Mach numbers less than one.

It should be noted that the developed analytical dependence works for Newtonian media and $M < 1$ Mach numbers, the gas flow is caused by the pressure difference in the space with design parameters of reduced diameter and length of transportation.

References

[1] J. Li, C. F. Li, Y. X. Zhang and H. G. Yue, "Compressed Air Energy Storage System Exergy Analysis and its Combined Operation with Nuclear Power Plants", *Applied Mechanics and Materials*, Vol. 448, pp. 2786–2789, 2014.

[2] M. Cai, Y. Wang, Y. Shi and H. Liang, "Output dynamic control of a late model sustainable energy automobile system with nonlinearity", *Advances in Mechanical Engineering*, Vol. 8(11), 2016. https://doi.org/10.1177/1687814016672784

[3] Y. Shi, Y. Wang, H. Liang and M. Cai, "Power characteristics of a new kind of air- powered vehicle", *International Journal of Energy Research*, Vol. 40(8), pp. 1112–1121, 2016.

[4] Y. T. Shen and Y. R. Hwang, "Design and implementation of an air-powered motorcycles", *Applied Energy*, Vol. 86(7), pp. 1105–1110, 2009.

[5] O. Takeuchi, T. Fujita and T. Kagawa, "Characteristics Analysis of Expanding-type Booster", *Proceedings of the Autumn Symposium on Hydraulics and Pneumatics*, Tokyo (Japan), pp. 69–72, 1995.

[6] D. Shaw, J. J. Yu and C. Chieh, "Design of a Hydraulic Motor System Driven by", *Compressed Air. Energies*, Vol. 6(7), pp. 3149–3166, 2013.

[7] W. Ni, J. Zhang, L. Shi, T. Wang, X. Zhang and S. Chen, "Mathematical Model of Small-Volume Air Vessel Based on Real Gas Equation", *Water*, Vol. 12, p. 530, 2020. https://doi.org/10.3390/w12020530

[8] J. B. Jones and R. E. Dugan, *Engineering Thermodynamics*; Prentice Hall Inc.: New Jersey, USA, 1996.

[9] Y. G. Shin, "Estimation of instantaneous exhaust gas flow rate based on the assumption of a polytropic process", *J. Automob. Eng.*, Vol. 25, pp. 637–643, 2001.

[10] H. Najm, P. H. Azoury and M. Piasecki, "Hydraulic ram analysis: A new look at an old problem", *J. Power Energy*, Vol. 213, pp. 127–141, 1999.

[11] V. Filipan, Z. Virag and A. Bergant, "Mathematical modelling of a hydraulic ram pump system", *J. Mech. Eng.*, Vol. 49, pp. 137–149, 2003.

[12] L. Zhou, D. Liu and B. Karney, "Investigation of hydraulic transients of two entrapped air pockets in a water pipeline", *J. Hydraul. Eng.*, Vol. 139, pp. 949–959, 2013.

[13] L. Zhou, D. Liu, B. Karney and P. Wang, "Phenomenon of white mist in water rapidly filling pipeline with entrapped air pocket", *J. Hydraul. Eng.*, Vol. 139, pp. 1041–1051, 2013.

[14] J. Tuhovák, J. Hejík, M. Jícha and L. Šnajdárek, "Experimental validation of mathematical model for small air compressor", *EPJ Web of Conferences*, Vol. 143, p. 02133, 2017.

[15] H. Liang, G. Zhi-jian, G. Tian-yi, R. Xiao-dong and F. Xin, "Modeling of pneumatic dual reciprocating bellows pump with flexible linkage", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 235(6), pp. 1085–1096, 2021. DOI:10.1177/0954406220936725

[16] W. Huang, J. Xiao and Z. Xu, "A variable structure pneumatic soft robot", *Sci. Rep.*, Vol. 10, p. 18778, 2020. DOI: 10.1038/s41598-020-75346-5

[17] A. Djurkov, J. Cloutier and M.P. Mintchev, "Mathematical model and simulation of a pneumatic apparatus for in-drilling alignment of an inertial navigation unit during horizontal well drilling", International Journal "Information Technologies and Knowledge", Vol. 2, pp. 147–156, 2008.

[18] P. Cępa and E. Lisowski, "Application of pneumatic suction cup as a positioning element for thin metal sheets in technological processes", Technical Transactions. Mechanics, Vol. 1-M(5), 2013. DOI: 10.4467/2353737XCT.14.001.1927

[19] Z. Wu, R. V. N. Melnik and F. Borup, "Model-based analysis and simulation of airflow control systems of ventilation units in building environments", Building and Environment, Vol. 42, pp. 203–217, 2007. DOI: 10.1016/j.buildenv.2005.08.031

[20] E. Rakova and J. Weber, "Process simulation of energy behaviour of pneumatic drives", Procedia Engineering, Vol. 106, pp. 149–157, 2015.

[21] V. T. Dmytriv, I. V. and P. P. Yatsunskyi, "Experimental pulse generator combined with the milking machine collector", INMATEH – Agricultural Engineering, Vol. 59(3), pp. 219–226, 2019. https://www.scopus.com/authid/detail.uri?authorId=57195630274

[22] V. T. Dmytriv, O. S. Lanets, I. V. Dmytriv and I. I. Horodetskyy, "Modelling of work of the rotor-type blade pump with revolving stator", International Journal of Applied Mechanics and Engineering, Vol. 26(4), pp. 17–28, 2021.

[23] V. Dmytriv, I. Dmytriv, I. Horodetskyy and T. Dmytriv, "Analytical dynamic model of coefficient of friction of air pipeline under pressure", Diagnostyka, Vol. 20(4), pp. 89–94, 2019.