UKRAINIAN JOURNAL OF MECHANICAL ENGINEERING AND MATERIALS SCIENCE
Vol. 9, No. 4, 2023

Ihor Hrytsai[1], Dmytro Herus[2]

[1] Department of Robotics and Integrated Mechanical Engineering Technologies, Lviv Polytechnic National University, 8, S. Bandery str., Lviv, Ukraine, e-mail: ihor.y.hrytsai@lpnu.ua, ORCID 0000-0003-3675-5897
[2] Department of Robotics and Integrated Mechanical Engineering Technologies, Lviv Polytechnic National University, 8, S. Bandery str., Lviv, Ukraine, e-mail: dmytro.a.herus@lpnu.ua, ORCID 0009-0005-0203-9165

STUDY OF THE GEAR CUTTING PROCESS BY THE RADIAL-CIRCULAR METHOD

Received: November 22, 2023 / Revised: November 28, 2023 / Accepted: December 6, 2023

https://doi.org/10.23939/ujmems2023.04.020

Abstract. The results of the study of the radial-circular method of gear cutting are presented. The cutting and shaping of teeth is carried out by a thin disc cutter under conditions of continuous generating, with a combination of axial feed of the cutter and its constructive radial movement due to eccentric installation on the spindle axis. The value of the eccentricity is determined by the gear module. A graph-analytical model was used to calculate the parameters of the cuts, and the cutting force and torque on the cutter axis were modeled using these data. It was found that the cutting process is characterized by significant load variations. The areas of maximum tool wear were predicted. It is shown that for under the same initial conditions, the cutting time using this method is two times less than the time of hobbing. Since a disc cutter can cut gears in a wide range of modules, this process is one of the most versatile. This method opens up great possibilities for the restoration and repair of gears and drives manufactured to standards other than those adopted in Europe.

Key words: radial-circular method; gear cutting; thin disc cutter; chip; cutting power; torque.

Introduction

Gears and gearing are integral components of modern machines. Given their importance and widespread use, intensive research is underway to improve existing and create new technologies and methods for manufacturing these parts. For example, in recent years, the machine building industry has introduced Power Skiving technology and its variant Hard Scudding for cutting gears after surface hardening of the teeth. This technology is gradually replacing traditional gear manufacturing methods such as worm hobbing, inner crown hobbing, and broaching due to its significant advantages and high efficiency. At the same time, there is another method of gear cutting that has all the signs of high efficiency and versatility, which is called the "radial-circular method" (RCM). Its versatility lies in the ability to cut gears of various parameters – modulus, number of teeth, type of engagement, shape, and direction of teeth – with one simple tool, a thin disc cutter. The invention of this method [1] concerned gears with a sinusoidal profile, but it was later adapted to gears of any type – involute, circular, arched, etc. Its advantages, as well as the advantages of sinusoidal engagement, have been substantiated by previous studies [2–4]. At the same time, several simplifications and assumptions have been made in well-known studies, which lead to certain deviations and inaccuracies in the description of this process and its implementation. Based on the advances in computer technology and the development of application programs, it is possible to significantly clarify the theoretical foundations of RCM and practical conclusions and recommendations for its optimal use.

The purpose of this paper is to develop the basics of modeling the process of gear cutting by the radial-circular method and to analyze the factors that affect its efficiency.

Research results

The sequence of studying this process is based on the following algorithm: creation of a refined graph analytical model of the cut layers ⇒ analysis of the cutting force and its components ⇒ study of elastic deformations and vibrations in the elastic system of the machine ⇒ study of friction and thermal processes on contact surfaces ⇒ prediction of tool wear ⇒ selection and optimization of parameters that determine the efficiency of the radial-circular method: minimizing time and achieving maximum productivity In this formulation, the problem is formulated as a sequence of interrelated and interdependent processes and phenomena that constitute the essence of the cutting process and accompany it, ensuring systematic and comprehensive research.

Modeling the parameters of the cut layers

By the above algorithm, at the first stage, a graphical and analytical model was created that reflects the cutting process with a disc cutter. The analysis of primary sources describing similar approaches in traditional tooth processing methods showed that several simplifications and assumptions are made in known studies that reduce the adequacy of the developed models.

Thus, due to the complexity of the described processes, some authors simplify the kinematics of a particular method of tooth processing [5–15]. Often, the thickness of the cut layer is used instead of the complex parameter of the cross-sectional area of the cuts [6, 16–20], although this is a partial indicator of the actual shape and size of the chips. When modeling the cutting force, some researchers use the specific cutting force [9–12], which is generally acceptable. However, the specific cutting force is a function of the chip deposition rate, and in non-primary generating gearmaking methods, the slice thickness is continuously changing, so the shear rate also changes. As a result, the static parameter of the specific force does not correspond to the actual conditions of cutting and the formation of the cutting force. In some works [7], to compensate for this deviation, the authors resort to introducing additional correction factors, but this way also does not provide the required accuracy. In studies based on experimental data [22], the analytical dependencies obtained are adequate, but they are tied to the conditions under which the experiments were conducted.

The most appropriate methodology for use is the one developed for worm gear milling [23] and used in studies on this problem [2–4]. However, these works also used simplifications when estimating the cross-sectional area of the cuts, which was calculated by the thickness of the cut for a rectangular tooth and its width. Improvement of this technique has become possible nowadays based on modern CAD systems and the development of applied computer programs.

The methodology used in this study is as follows. The continuous cutting process is divided into discrete sequential positions of the tool's cutting edge and its rake face relative to the surface to be machined, i. e., the surface of the insertion between the gear teeth. The radial and angular change in these positions corresponds to the kinematics of a particular gearing method. In each position, the contour position of the tooth that makes the cut and the position of this tooth in its previous positions are described or defined graphically, according to the kinematics of the method. The superposition of these traces provides an instantaneous cross-section of the layer to be cut quantifies the cross-sectional area and allows you to determine other parameters of the cut – thickness, width, and length of contact between the tooth and the workpiece.

The kinematics of the radial-circular method is that the tool is mounted eccentrically relative to its kinematic axis, and during continuous rotation of the cutter and gear, a single tool rotation removes the allowance in one gap. In the axial feed cycle, when the cutter moves over the full width of the gear, all the gaps and tooth profiles of the workpiece are formed. The combination of uniform rotation of the workpiece and an eccentrically mounted cutter forms a sinusoid on the tooth surface, but by changing the law of radial movement of the disc cutter, teeth of any gear profile can be cut.

The law of sinusoidal motion in plane coordinates is defined by the system:

$$\begin{cases} x_2 = (R_b + e\cdot\cos\varphi_1)\cdot\cos\varphi_2 - \delta\cdot\sin\varphi_2; \\ y_2 = -(R_b + e\cdot\cos\varphi_1)\cdot\sin\varphi_2 - \delta\cdot\cos\varphi_2; \\ z_2 = e\cdot\sin\varphi_1 + s_o, \end{cases}$$

where e – is the eccentricity of the cutter; φ_1 – is the angle of rotation of the tool; φ_2 – is the angle of rotation of the coordinate system associated with the gear; s_o – is the axial feed rate of the cutter; δ – is the error of the cutter installation in the axial direction; R_b – is the radius of the initial circle of the gear.

Fig. 1 shows how this method was used to determine the instantaneous cross-sectional area of chips at certain characteristic angular positions of the cutter and workpiece for two tool tooth profiles – a standard rectangular and a rounded one.

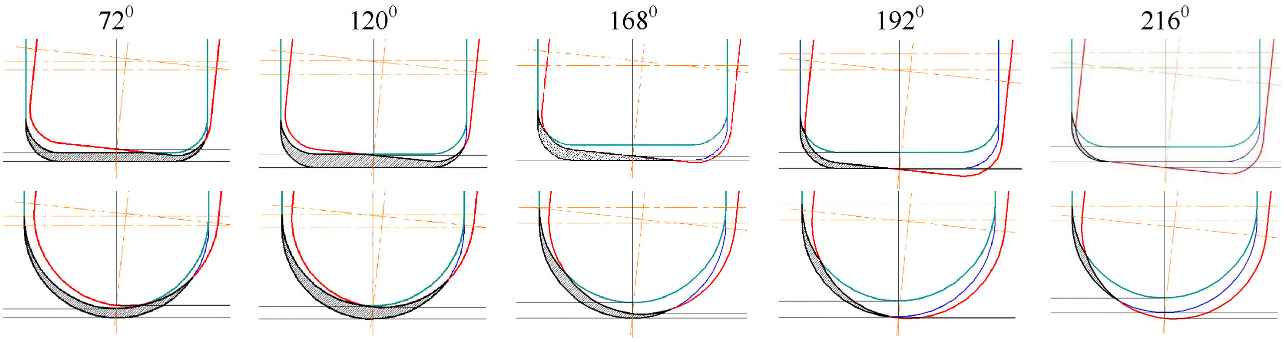


Fig. 1. Cross-sections sections of disc cutters with a straight and rounded apex blade in certain angular positions of the cutter

Based on the data obtained, Fig. 2 shows the graphs of the cross-sectional area of the cuts for these two cutters. The following parameters were taken: involute spur gear; module 2.5 mm; number of teeth: 36 for the gear and 60 for the disc cutter; rake angle 0°, relief angle 15°; axial feed 0.75 mm/rev; cutting speed 42 m/min; cutter material high-speed steel R6M5, gear's material is steel AISI 1040 before heat treatment, tensile strength is 660 MPa.

The graphs in Fig. 2 show that, taking into account the change in the area at the input and output sections, both cutters have the same productivity

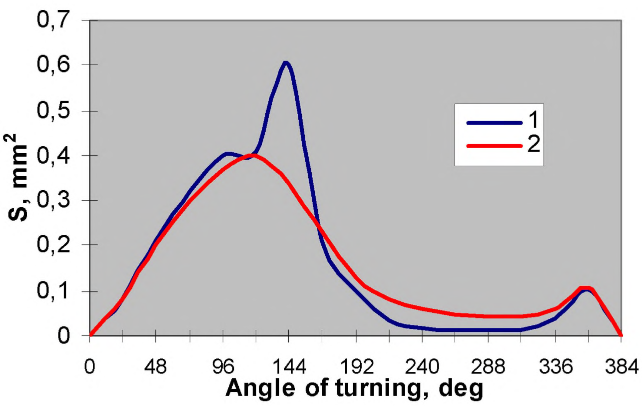
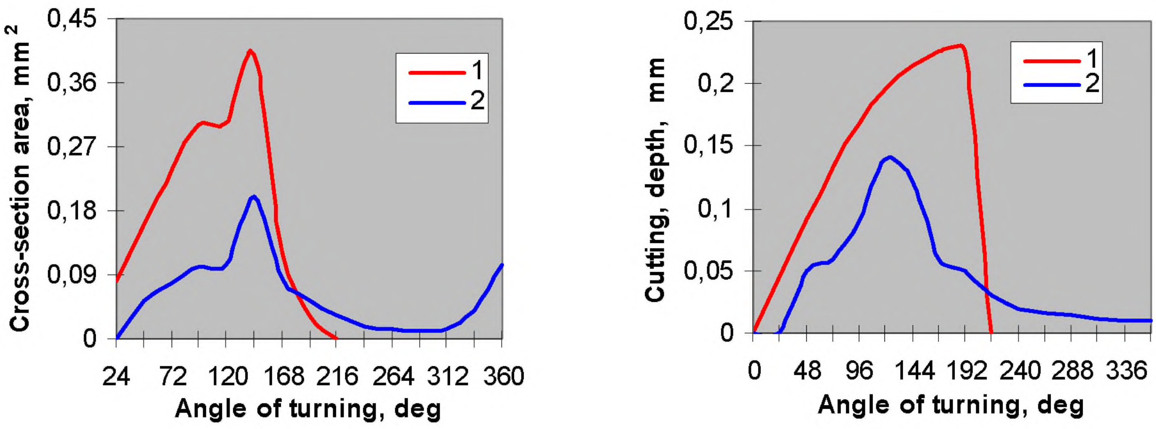


Fig. 2. Cross-sectional area of sections 1 – straight blade; 2 – rounded blade

As you can see from the graphs in Fig. 1, the milling cutters have top and side output blades. Fig. 3 shows the graphs of the area and thickness of the cuts on these blades for a standard cutter of the first type. At the bottom of the gap, the thickness of the cuts is very small due to the peculiarities of the sinusoidal formation – a large number of cuts at a small angle of rotation of the workpiece.



a b

Fig. 3. Cut area (a) and thickness of sections (b) on the blades along the angle of rotation of the cutter in one gap: 1 – upper edges; 2 – trailing edges

The slice thicknesses on the outlet blades are also significantly less than those on the apex blades. This means that the apex blades are subjected to considerable pressure in terms of absolute load, while the side blades, especially in the second phase of the hollowing process, experience significant shear deformations. In addition, when the thickness of the layers to be cut is small, the micro-cutting process turns into a scraping process, which is accompanied by intensive wear of the side blades. Since the right-hand hollowing profiles are formed within the thickness of the cuts, and the left-hand profiles have wider cuts, the quality of the right-hand profiles will be worse.

Cutting power of the circular cutter

Based on the basic principles of cutting theory, the main component of the cutting force acting in the direction of the cutting speed vector is represented by the following relationship [24]:

$$P_o = P_\tau \cdot \cos\Phi = [\tau] \cdot S \cdot ctg\Phi, \qquad (1)$$

where P_τ – is the shear force acting in the plane of maximum shear deformation during cutting; Φ – is the shear angle; [τ] – is the shear strength of the workpiece material.

As is well known, the shear angle Φ is related to the chip deposition coefficient ξ and the rake angle γ of the tool by the following relationship:

$$\xi = ctg\Phi \cdot \cos\gamma + \sin\gamma\ . \qquad (2)$$

At zero rake angle $\xi = ctg\Phi$, and the force P_o will be equal to [20]:

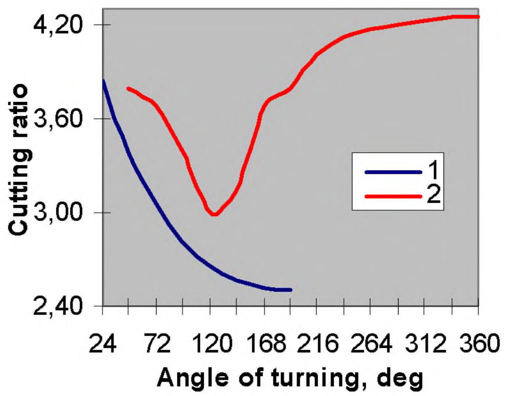
$$P_o = P_z = \tau \cdot \xi \cdot S\ . \qquad (3)$$

According to the third theory of strength, or the theory of maximum tangential stress, the shear yield strength [τ] is equal to half the tensile strength of the material: $[\tau] = 0{,}5\,[\sigma]$, in our case [τ] = 340 MPa.

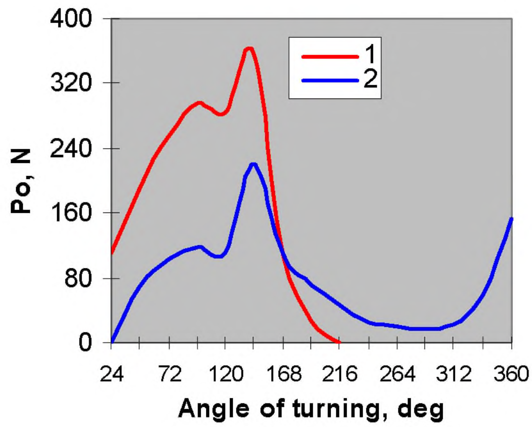
Cutting in the RCM occurs with a continuous change in the thickness of the cuts. To determine the chip deposition coefficient depending on the cut thickness, we used the *Deform 2D* computer system for rheological modeling of the cutting process, and the desired function has the following form:

$$\xi = -23.19 \cdot x^3 + 32.64 \cdot x^2 - 17.12 \cdot x + 3.97\ . \qquad (4)$$

The plots of the deposition coefficient for the blades as a function of thickness on these blades are shown in Fig. 4, *a*. Based on these results, the cutting force of the RCM is modeled, the plots of which are shown in Fig. 4, *b*. The torque acting on the axis of the disc cutter is shown in Fig. 5.



a



b

Fig. 4. Dependence of the chip deposition coefficient on the thickness of the cuts (a) and the change in cutting force on the cutter blades (b) along the angle of its rotation in one gap: 1 – upper edges; 2 – trailing edges

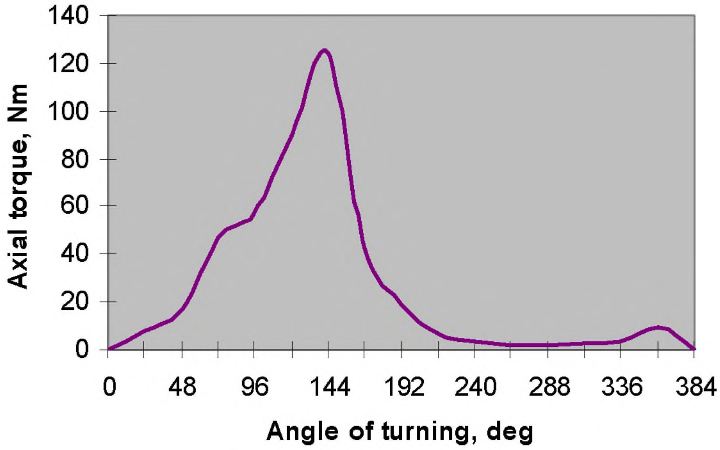


Fig. 5. Torque on the cutter axis

Analysis of the results

The torque shown in the figure causes an angular torsional deformation of the spindle with the cutter; its value is determined taking into account the angular overlap coefficient, which varies from 1 to 3. If for an average gear milling machine of normal accuracy the torsional stiffness of the spindle is 1.5 kN/deg, then the maximum torque on the cutter axis of 124 Nm will cause an angular circular deformation on the cutter axis equal to 0.11°, which will form an arc of 0.15 mm in length at the corresponding cutter radius (78.8 mm). This is a periodic elastic deformation of the spindle with a cutter that occurs during one revolution of the cutter, which is repeated during each revolution of the tool and occurs on the same group of teeth located in the area of maximum eccentricity. The negative effect of this deformation is to increase the cutting path. For example, if the width of the gear rim is 20 mm, then with an axial feed rate of 0.75 mm per revolution of the workpiece, the gear will complete 35 revolutions before the gear is completely cut (taking into account tool entry and exit), and for several gear teeth equal to 36, this path will increase by 106 mm. When machining a batch of parts, this additional cutting path can be significant, resulting in reduced cutter stability.

To evaluate the efficiency of the studied process, let us determine the time for the operation. According to the kinematics of the RCM, at a nominal cutting speed of 42 m/min and a dividing radius of the cutter of 75 mm, the tool rotation frequency is 90 min^{-1}. The machining time of one gap is equal to the time of one revolution, i. e. 0.67 s, and the total time of gear cutting will be 14 minutes.

Let's compare this time with worm gear milling. The diameter of a standard worm cutter with a 2.5-mm module is 80 mm, which means that the rotational speed will increase to 170 min^{-1}. At the same time, the maximum permissible axial feed rate should not exceed 0.15 mm per revolution of the workpiece, a limitation due to the high cutting force and unevenness of the worm cutter cutting process. Then, for the same conditions, the time for complete machining of the gear with a worm cutter will be 31 minutes, i.e., 2 times longer. Thus, in terms of productivity, RCM is significantly higher than worm hobbing, but inferior to Power Skiving. On the other hand, compared to Power Skiving, RCM is much more economical due to the use of universal gear milling machines instead of special expensive equipment, and an order of magnitude lower costs for technological equipment.

One of the aspects of the versatility of the method studied in this article is the ability to cut gears of any profile with a disc cutter. The significance of this feature is that nowadays, various sectors of the domestic industry use Western equipment whose drives and gears are made according to standards other than metric. This applies to products manufactured, in particular, in the United States and includes equipment and machinery in the mining, fuel and energy, shipbuilding, automotive, and military industries. In these conditions, the restoration and repair of gears of such drives, gearboxes and transmissions using the radial-circular method eliminates the need for special cutting tools and can be carried out with simple tools even outside specialized production facilities, in particular, in the field in mobile auto repair shops.

Conclusions

Based on behavioral research, the following has been established.

1. In the process of cutting gears using the radial-circular method with a disc cutter installed eccentrically, teeth of a sinusoidal profile are formed. However, following the law of periodic radial movement of the cutter, it is possible to produce teeth of any profile, including those not manufactured according to ISO standards. This creates conditions for repairing and rebuilding gearboxes that use non-modular gears, such as pitch gears. Thus, this method is suitable for the repair of equipment, in particular, manufactured in the United States in non-specialized production facilities.
2. The bulk of the cutting work is performed by the top blades, which eliminate the allowance mainly in the inlet part of the flute (in the area of increasing eccentricity). Only the outlet blade is loaded at the outlet part of the gap. The thickness of the cuts on the exit blade and the apex blade at the base of the gear tooth pedicle is minimal. In such conditions, the teeth located on the cutter in the area of maximum eccentricity, in particular, near the initial tooth tips, will wear out.
3. The cutting force and torque on the circular cutter variate within a very wide range, with the force and torque increasing to their maximum values from values close to zero. This indicates a high level of unevenness in the cutting process.
4. Variations in force and torque lead to elastic circular deformations of the disc cutter, which increases the actual cutting path and negatively affects its service life.
5. The cutting time of one gear by the radial-circular method is two times less than the time of its cutting by a worm cutter due to a significantly higher axial feed. In turn, this increase in feed is possible due to an increase in the number of teeth of the disc cutter and the possibility of reducing the feed per tooth.
6. Taking into account the low cost of the cutting tool – a thin disc cutter compared to complex and expensive worm cutters or cup cutters, as well as the ability to use the same cutter for the manufacture of gears in a wide range of modules, this method, along with its versatility, requires significantly less cost for its practical use.

References

[1] E. M. Blagut, "Sposib narizky zubchastykh kolis synusoidalnoi peredachi Blahuta" ["Method for cutting gears of the Blagut sinusoidal transmission"], UA Patent 73550, Publication bulletin No. 8, 2005 (in Ukrainian).

[2] I. E. Hrytsay, "Sinusoidal Gears and Alternative Method of Tooth Generation" in *Advances in Gear Theory and Gear Cutting Tool Design*, S. P. Radzevich, M. Storchak, Eds. Cham, Switzerland: Springer Publ., 2022, pp. 233–254.

[3] I. E. Hrytsay, S. I. Gromnyuk, "Pidvyshchennia efektyvnosti protsesu narizannia zubchastykh kolis na osnovi radialno-obertovoho metodu v umovakh obkochuvannia" ["Increasing the efficiency of the process of cutting gears on the basis of the radial-rotating method in the conditions of generating"], *Zbirnyk naukovykh prats Donbaskoi derzhavnoi mashynobudivnoi akademii* [*Collection of scientific works of Donbass State Machine-Building Academy],* Vol. 32, pp. 226–229, 2013 (in Ukrainian).

[4] S. I. Hromniuk, I. E. Hrytsaj, "The study of the cutting force and its components in radial-circular cutting of the gears", *Optymizatsiia vyrobnychykh protsesiv i tekhn. kontrol u mashynobuduvanni ta pryladobuduvanni [Optimization of production processes and technical control in mechanical engineering and instrumentation],* Issue 786, pp. 61–67, 2014.

[5] M. Inui, Y. Huang, H. Onozuka, N. Umezu, "Geometric simulation of power skiving of internal gear using solid model with triple-dexel representation", *Procedia Manufacturing,* Vol. 48, pp. 520–527, 2020.

[6] B. Vargas, M. Zapf, J. Klose, F. Zanger, V. Schulze, "Numerical modelling of cutting forces in gear skiving", *Procedia CIRP,* Vol. 82, pp. 455–460, 2019.

[7] T. Bergs, A. Georgoussis, C. Löpenhaus, "Development of a numerical simulation method for gear skiving", *Procedia CIRP,* Vol. 88, pp. 352–357, 2020.

[8] P. McCloskey, A. Katz, L. Berglind, K. Erkorkmaz, E. Ozturk, F. Ismail, "Chip geometry and cutting forces in gear power skiving", *CIRP Annals,* Vol. 68, Issue 1, pp. 109–112, 2019.

[9] T. Nishikawa, S. Shimada, G. Kobayashi, Z. Ren, N. Sugita, "Using Power Skiving to Increase the Efficiency and Precision of Internal Gear Cutting", *Komatsu Tech. Rep.,* Vol. 64, pp. 1–7, 2018.

[10] C. Janßen, J. Brimmers, T. Bergs, "Validation of the plane-based penetration calculation for gear skiving", *Procedia CIRP*, Vol. 99, pp. 220–225, 2021.

[11] M. Inui, Y. Huang, H. Onozuka, N. Umezu, "Geometric simulation of power skiving of internal gear using solid model with triple-dexel representation", *Procedia Manufacturing,* Vol. 48, pp. 520–527, 2020.

[12] C. Janßen, J. Brimmers, T. Bergs, "Validation of the plane-based penetration calculation for gear skiving", *Procedia CIRP,* Vol. 99, pp. 220–225, 2021.

[13] H. Onozuka, F. Tayama, Y. Huang, M. Inui, "Cutting force model for power skiving of internal gear", *Journal of Manufacturing Processes,* Vol. 56, pp. 1277–1285, 2020.

[14] N. Tapoglou, "Calculation of non-deformed chip and gear geometry in power skiving using a CAD-based simulation", *The International Journal of Advanced Manufacturing Technology,* Vol. 100, pp. 1779–1785, 2019.

[15] T. Bergs, A. Georgoussis, C. Löpenhaus, "Development of a numerical simulation method for gear skiving", *Procedia CIRP,* vol. 88, pp. 352-357, 2020.

[16] P. McCloskey, A. Katz, L. Berglind, K. Erkorkmaz, E. Ozturk, F. Ismail, "Chip geometry and cutting forces in gear power skiving", *CIRP Annals,* Vol. 68, No. 1, pp. 109–112, 2019.

[17] A. Antoniadis, "Gear skiving – CAD simulation approach", *Computer-Aided Design*, Vol. 44, Issue 7, pp. 611–616, 2012.

[18] A. Antoniadis, N. Vidakis, N. Bilalis, "A simulation model of gear skiving", *Journal of Materials Processing Technology*, Vol. 146, Issue 2, pp. 213–220, 2004.

[19] M. Inui, Y. Huang, H. Onozuka, N. Umezu, "Geometric simulation of power skiving of internal gear using solid model with triple-dexel representation", *Procedia Manufacturing,* Vol. 48, pp. 520–527, 2020.

[20] B. Vargas, M. Zapf, J. Klose, F. Zanger, V. Schulze, "Numerical modelling of cutting forces in gear skiving", *Procedia CIRP,* Vol. 82, pp. 455–460, 2019.

[21] H. Onozuka, F. Tayama, Y. Huang, M. Inui, "Cutting force model for power skiving of internal gear", *Journal of Manufacturing Processes*, Vol. 56, pp. 1277–1285, 2020.

[22] H. Onozuka, F. Tayama, Y. Huang, M. Inui, "Cutting force model for power skiving of internal gear", *Journal of Manufacturing Processes*, Vol. 56, part B, pp. 1277–1285, 2020.

[23] I. Hrytsay, V. Stupnytskyy, V. Topchii, "Improved Method of Gear Hobbing Computer Aided Simulation", *Archive of mechanical engineering*, Vol. 66, No. 4, pp. 475–494, 2019.

[24] I. Hrytsay, V. Topchij, A. Kuk, "Computer 3D modelling of chip geometry and force field in the hobbing process", *Ukrainian Journal of Mechanical Engineering and Materials Science*, Vol. 4, No. 2, pp. 68–9, 2018.