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HIGH PRECISION DUAL LINE LEVELING RESEARCH

The aim of this article is to investigate the accuracy of dual line leveling, and develop a methodology for its execution to enhance the precision of elevation determination by accounting for vertical refraction and controlling the non-horizontality of the leveling beam. Methodology. Considering that digital levels can measure distances to the rod and account for the non-horizontality of the beam and vertical refraction during measurements, we propose a method of dual line leveling. The study describes the methodology for performing dual-line trigonometric leveling using the "forward-backward" method. It takes into account vertical refraction along the observation lines. Results. To test the methods for high precision class leveling applying the Holeski method (from the middle) and dual-line leveling ("forward-backward"), we selected a section with a prolonged ascent approximately 1 km in length, consisting of 5 sections. The method was tested using a Trimble DiNi-03 electronic level over two leveling lines. The discrepancies between the elevations obtained from the sections using the "from the middle" and "forward-backward" methods meet the requirements for high precision class leveling. The maximum discrepancy in the sections between the leveling methods was 0.42 mm. And it was 0.06 mm for the entire leveling route, 1,142 meters long. Originality. The paper considers the theoretical justification and experimental studies on the possibility of applying dual line leveling for high precision class observation programs and introducing a correction for vertical refraction on prolonged slopes. It is confirmed that the method of dual line leveling using the "forward-backward" method can be used for high precision class leveling by the "from the middle" method on prolonged slopes and has several advantages over it.

Key words: leveling, vertical refraction, beam non-horizontality, vertical displacements, geodetic monitoring.

Introduction

High-precision elevation determination is used in many sectors of the national economy, especially for high-tech construction, assembly, and operation of structures and equipment, as well as for fundamental scientific research on crustal movements, continental drift, earthquake prediction, and other critical today`s tasks. Monitoring settlements and deformations of engineering structures and assessing the stability of height network reference points [Trevoho et al., 2021; Trevoho et al., 2022] is an essential process that affects the safety of the construction and operation of buildings and structures.

The main strategy of the state to ensure technogenic and environmental safety is the regulation and implementation of measures defined in the resolution of the Verkhovna Rada of Ukraine dated March 5, 1998. Preventing and mitigating possible consequences of emergencies remains a key focus of this policy. The primary actions aimed at preventing technogenic emergencies at potentially dangerous sites include:

- control over the functioning of objects;
- improvement of technological processes regulating the state of objects;
- implementation of technical measures for systematic control aimed at preventing accidents;
- geodetic monitoring of objects and the environment [Hyo Seon Park, 2015].

Implementing geodetic monitoring of large industrial facilities [Ostrovsky, 1997] and the natural environment, as well as introducing reliable observation systems that promptly process measurement results, guarantee quality assessments, and the ability to predict the occurrence of disasters, natural calamities, or emergencies.

The late 20th century saw the intensive development of technologies that mostly became environmentally hazardous for humanity. The structures of aggregates, dams, reactors, and other facilities require monitoring. One of the control components is monitoring the elevations of structural elements in real time. The new requirements for monitoring observations include new methods and techniques for leveling, ensure their accuracy and objectivity, as well as eliminating human bias. Prompt processing of results, and high precision in process prediction are also essential.

Fig. 1 shows the classification of geodetic monitoring methods for vertical displacements.

Leveling with a horizontal sighting beam remains one of the most accurate methods for determining elevation differences, currently used for monitoring areas and objects. High-precision leveling is divided into the following types and classes:

- Ultra-Precise Leveling (Short Beam) $S_{max} = 25$ m.);
- Precise Leveling ($S_{max} = 50$ m.);
- First-Class Leveling (Short Beam) ($S_{max} = 25$ m.);
- First-Class Leveling ($S_{max} = 50$ m.);
- Precise Second-Class Leveling ($S_{max} = 75$ m.).

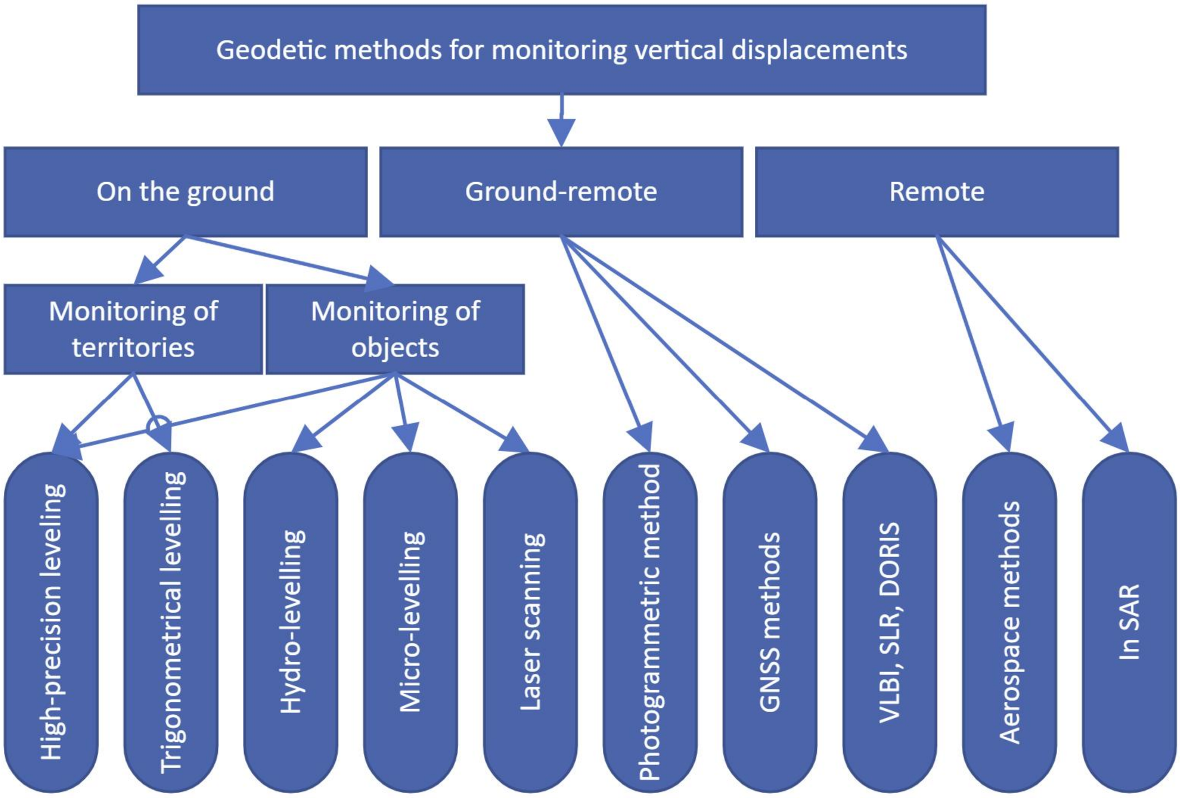

Fig. 1. Classification of Geodetic Monitoring Methods for Vertical Displacements

According to the method of geometric leveling, it is divided into:

- The "from the middle" method.
- The "forward" method.

The instructions [Instruction, 1990] for the "from the middle" method of leveling impose certain restrictions on the methodology of performing high-precision leveling to achieve high accuracy in determining elevations. These restrictions include the maximum distances and inequality of distances to rods, the height of the sighting beam above the ground surface, as well as the time for conducting leveling. The methodological manual [Ostrovsky et al., 1997] provides tables 1-4, developed following the instruction [Instruction, 1990]. These tables include information on sources of errors with numerical characteristics, the maximum errors in determining elevations and differences in elevations, as well as permissible residuals for the mentioned types and classes of leveling. Such requirements and limitations imposed on the process of leveling significantly complicate it and require additional time and effort for its execution. Additionally, careful calibration of the appropriate equipment is necessary for the preparation of such types and classes of leveling.

Vertical refraction and the instability of the leveling beam's horizontality have the greatest impact on the accuracy of high-precision leveling [Pavliv, 1980; Tereshchuk et al., 1998; Periy, 2006; Ostrovsky, 2007; Vashchenko et al., 2009; Periy et al., 2012; Urdzik, 2019].

With the emergence of new digital levels, gross errors are automatically eliminated, and personal errors during leveling are reduced in the measurement results. However, the process of conducting measurements during high-precision leveling remains laborious and requires high-precision equipment, appropriate measurement techniques, metrological support, and qualified performers to achieve the necessary results.

Table 1

Sources of random errors in leveling and their numerical values

No.	Sources of random errors	Marking errors	Leveling				
			I class (mm)	I class s.b. (mm)	Precise (mm)	Ultra-Pr. (mm)	II class (mm)
1.	Alignment of the contact level	m_{al}	0.134	0.063	-	-	0.208
2.	Pointing the bisector to the line	m_{point}	0.123	0.060	0.090	0.050	0.195
3.	The angle of non-horizontality of the beam	$m_з$	0.194	0.051	-	-	0.388
4.	Micrometer reading	m_{read}	0.036	0.036	0.036	0.036	0.036
5.	Displacement of the thread grid	m_{disp}	0.060	0.060	0.010	0.010	0.060
6.	Rod inclination + bull eye level adjustment	$m_{inc} + m_{lev}$	0.149	0.015	0.012	0.012	0.149
7.	Divisions of the rod	$m_{div.r.}$	0.071	0.036	0.036	0.036	0.142
8.	Rod heel	m_{heel}	0.010	0	0	0	0.200
9.	Compensator work	m_{comp}	-		0.090	0.050	-
10.	Other sources	m_{other}	0.090	0.015	0.005	0.005	0.200
11.	Output data	$m_{out.}$	-	-	-	-	0.200
12.	Total error	m_{total}	0.338	0.129	0.147	0.089	0.632

Table 2

Sources of systematic errors of leveling and their numerical values

No.	Sources of errors	Marking errors	Leveling				
			I class (mm)	I class s.b. (mm)	Precise (mm)	Ultra-Pr. (mm)	II class (mm)
1.	Changing the height of transition points	$m_{ch.h.}$	0.0015	0.0015	0.0015	0.0015	0.0020
2.	Vertical refraction	$m_{ref.}$	0.0055	0.0014	0.0055	0.0014	0.0124
3.	One-sided temperature effect on the rod	$m_{tem.}$	0.0025	0.0025	0.0012	0.0012	0.0030
4.	Error comparing rod	$m_{comp.}$	0.005	0.005	0.0050	0.0050	0.0100
5.	Undercompensation	$m_{u/k.}$			0.0030	0.0015	
6.	Total error	$\sigma_{h.}$	0.008	0.0060	0.0080	0.0060	0.0160

We consider the main requirements for first-class leveling [*Instruction*, 1990]:

- First-class leveling is performed in both forward and reverse directions along two pairs of geodetic turning plates, which form two separate lines: the right line, corresponding to the path along the right turning plates, and the left line, along the left turning plates.
- Readings from the rod are taken on two scales (main and auxiliary).
- Equal distances to rods are maintained (tolerance no more than 0.5 m per station), as well as the accumulation of distances to rods inequality (tolerance no more than 1 m per section).
- Height restrictions of the beam above the ground surface are observed (0.8 m).
- - The maximum sighting line length is up to 50 m.

Adhering to the above requirements takes a lot of time to perform fieldwork, and maintaining equal distances to rods often leads to an increase in the number of stations in the course, which in turn affects the final result of the observations and the time taken to complete the work.

As is known, during "midpoint leveling," the level is set up between the terrain points A and B in the middle.

At points A and B – leveling rod are set up, and the leveling instrument's line of sight is brought to a horizontal position. Readings a and b are taken from the rod, which are set vertically at points A and B. The desired height difference h_{AB} is calculated using the formula: $h_{AB} = a - b$. If the height H_A of point A is known, then the height of point B will accordingly be: $H_B = H_A + h_{AB}$.

Table 3

The limiting mean square random and systematic errors of height difference measurements at a single station of leveling

No.	Sources of errors	Limiting errors of height differences		Permissible inconsistencies $f_{h.}$ (mm.)
		$\eta_{h.}$ (mm)	$\sigma_{h.}$ (mm)	
1.	Ultra-Precise Leveling (Short Beam) ($S_{\max}$ = 25 m).	0.063	0.006	$0.09\sqrt{n}$
2.	Precise Leveling ($S_{\max}$ = 50 m).	0.104	0.008	$0.15\sqrt{n}$
3.	First-Class Leveling (Short Beam) ($S_{\max}$ = 25 m).	0.091	0.006	$0.13\sqrt{n}$
4.	First-Class Leveling ($S_{\max}$ = 50 m).	0.239	0.008	$0.3\sqrt{n}$
5.	Precise Second-Class Leveling ($S_{\max}$ = 75 m)	0.632	0.016	$0.8\sqrt{n}$

Table 4

Marginal root mean square random and systematic errors of determining the difference of excesses (vertical displacements) from one leveling station

No.	Sources of errors	Limiting errors of height differences		The average number of stations per 1 km line
		$\Delta\eta_{h.}$ (mm)	$\Delta\sigma_{h.}$ (mm)	
1.	Ultra-Precise Leveling (Short Beam) ($S_{\max}$ = 25 m).	0.089	0.003	n=20
2.	Precise Leveling ($S_{\max}$ = 50 m).	0.147	0.004	n=10
3.	First-Class Leveling (Short Beam) ($S_{\max}$ = 25 m).	0.129	0.003	n=20
4.	First-Class Leveling ($S_{\max}$ = 50 m).	0.338	0.004	n=10
5.	Precise Second-Class Leveling ($S_{\max}$ = 75 m)	0.848	0.008	n=7

The method of "forward" leveling is often used to determine the angle of non-horizontality of the leveling instrument's line of sight during the main verification process. This method is also widely used in engineering and construction for determining the heights of several points from a single setup of the leveling instrument. However, it is not recommended by the instruction [*Instruction*, 1990] for high precision leveling.

We propose the use of two-sided "forward-backward" leveling for performing high-precision leveling.

Objective

The objective of this article is to investigate the accuracy of two-sided leveling, and develop a methodology for its implementation aimed at improving the accuracy of height difference determination by accounting for vertical refraction and controlling the non-horizontality of the leveling beam.

Methodology

Considering that digital levels can measure distances to the rod, account for the non-horizontality of the beam, and include vertical refraction in the results during measurements, we propose the method of two-sided leveling.

The proposed method of two-sided "forward-backward" leveling [Ukrainian Patent No. 41429] (see Fig. 2) involves setting up the level at two stations, which are located within clear visibility of the rod and close to the endpoints of the leveling line.

We propose the following formulas for calculating height differences h_{AB}, the integral coefficient of vertical refraction $\bar{k}$, and the angle $\sum(\bar{i}'' - \bar{r}'')$ of the non-horizontality of the line of sight during two-sided leveling by the "forward-backward" method, taking into account all distances to rods of the leveling:

$$h_{AB} = \bar{h}_{AB} - \Delta h_R - \Delta h_i + \Delta h_k, \quad (1)$$

where $\bar{h}_{AB}$ is the mean value of the height difference from two-sided observations.

$$\bar{h}_{AB} = \frac{(a_{1A} - b_{1B}) + (a_{2A} - b_{2B})}{2}, \quad (2)$$

where a_{1A} and b_{1B} are the readings on the back and front rod from the first leveling station, a_{2A} and b_{2B} are the readings on the back and front rod from the second leveling station.

$$\Delta h_R = \frac{(d_{1A}^2 - d_{1B}^2) - (d_{2B}^2 - d_{2A}^2)}{4R}, \quad (3)$$

where d_{1A} and d_{1B} are the distances to the back and front rod from the first leveling station, d_{2A} and d_{2B} are the distances to the back and front rod from the second leveling station, R is the radius of the curvature of the Earth; Δh_i is the correction for the non-horizontality of the sighting beam caused by the angle i'' and the inequality of distances to rods.

$$\Delta h_i = \frac{i''}{2\rho''}\Big((d_{1A} - d_{1B}) - (d_{2B} - d_{2A})\Big), \quad (4)$$

where ρ'' is the number of seconds in a radian; Δh_k is the correction for vertical refraction determined by the respective coefficients of vertical refraction k_{AB} for the forward direction and k_{BA} for the backward direction of leveling, as well as the inequality of distances to rods.

$$\Delta h_k = \frac{(d_{1A}^2 - d_{1B}^2)k_{AB} - (d_{2B}^2 - d_{2A}^2)k_{BA}}{4R}. \quad (5)$$

The integral coefficient of vertical refraction is determined by the formula:

$$\bar{k} = 1 - \frac{2R}{\rho''}\frac{\Delta h_h \cdot \rho'' - \Big((d_{1A} - d_{1B}) + (d_{2B} - d_{2A})\Big)\cdot i''}{(d_{1A}^2 - d_{1B}^2) + (d_{2B}^2 - d_{2A}^2)}, \quad (6)$$

where Δh_h is the difference in height differences measured in the forward and backward directions:

$$\Delta h_h = (a_{1A} - b_{1B}) - (a_{2A} - b_{2B}). \quad (7)$$

The total angle $\angle\varepsilon = \sum(\bar{i}'' - \bar{r}'')$ of integral vertical refraction $\bar{r}''$ and non-horizontality of the sighting beam $\bar{i}''$, taking into account the distances to rods is determined by the formula:

$$\sum(\bar{i}'' - \bar{r}'') = \frac{\rho''}{2R}\frac{\Delta h_h \cdot 2R - \Big((d_{1A}^2 - d_{1B}^2) + (d_{2B}^2 - d_{2A}^2)\Big)}{(d_{1A} - d_{1B}) + (d_{2B} - d_{2A})}. \quad (8)$$

The value of vertical refraction can be determined during leveling by performing metrological gradient observations directly at the station.

We propose determining the partial coefficients of vertical refraction based on its integral value and equivalent heights:

$$\left.\begin{aligned} k_{AB} &= k_{i\,AB} + \frac{2q}{q+1}\left(\bar{k} - \bar{k}_i\right) \\ k_{BA} &= k_{i\,BA} + \frac{2}{q+1}\left(\bar{k} - \bar{k}_i\right) \end{aligned}\right\}, \quad (9)$$

where $k_{i\ AB}$ and $k_{i\ BA}$ are the values of normal vertical refraction calculated from the measured temperature $T(K°)$ and pressure P (in millibars):

$$k_i = 12.27\frac{P}{T^2}; \qquad (10)$$

where q is the coefficient of refractive correspondence of anomalous values of the coefficients of vertical refraction $k_{àí\ .AB}$ and $k_{àí\ .BA}$ along the observation lines, and during measurements under the same atmospheric conditions—equality of anomalous vertical temperature gradients at the stations, it can be accepted as the ratio of equivalent heights $h_{e.AB}$ and $h_{e.BA}$ of observation directions:

$$q = \frac{k_{àí\ .AB}}{k_{àí\ .BA}} = \frac{h_{e.BA}}{h_{e.AB}}, \qquad (11)$$

where $h_{e.AB}$ and $h_{e.BA}$ are the equivalent heights of the observation directions.

Equivalent heights represent the integral heights of the sighting beam above the reference surface in geometric leveling. It can be assumed to be uniformly inclined. Therefore, equivalent heights can be calculated based on the heights of the leveling instrument or readings on the near a_i and far b rod:

$$h_e = \frac{2}{d^2}\int_0^d h_v(d-l)\partial l = \frac{2a_i + b}{3}. \qquad (12)$$

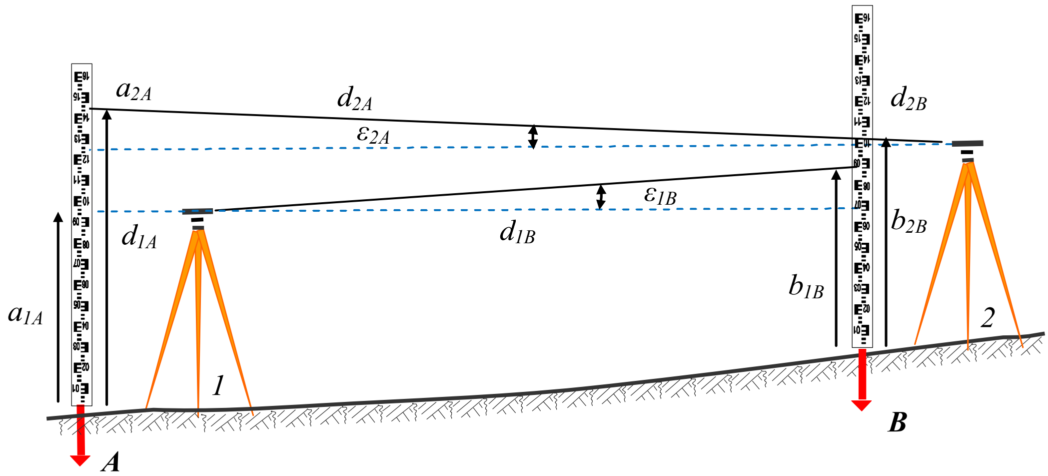

Fig. 2. Two-sided "forward-backward" leveling method

The provided formulas enable the automation of the bilateral "forward-backward" leveling method by programming them into the memory of a digital level to compute height differences and monitor observations for the total angle $\sum(\overline{i''} - \overline{r''})$. Additionally, they facilitate the determination of the integral coefficient of vertical refraction $\overline{k}$ under the condition of a predefined angle i'' in laboratory conditions and inputted into the level's memory. Formula (6) can be significantly simplified by incorporating automatic correction for Earth curvature $\Delta h_R = 0$ (3), the value of the normal coefficient of vertical refraction k_i (10), and the angle of non-horizontality of the sighting beam i'' directly into the rod readings. This became possible with digital levels, which, in addition to automatically reading the position of the horizontal sighting beam on the rod scale, determine its distance and are equipped with sufficiently powerful portable computers for performing certain calculations.

The results

To validate the methods for first-class leveling using the Cholesky method (midpoint) and two-sided leveling ("forward-backward"), a section with a steep ascent approximately 1 km long was selected, consisting of 5 sections (see Fig. 3).

The method was implemented using the DiNi-03 digital level along two survey lines.

Since digital devices currently lack a program for conducting bilateral leveling ("forward-backward") with simultaneous measurements along two survey lines, we opted for the programmed Cholesky method (the method of observing two survey lines from one station).

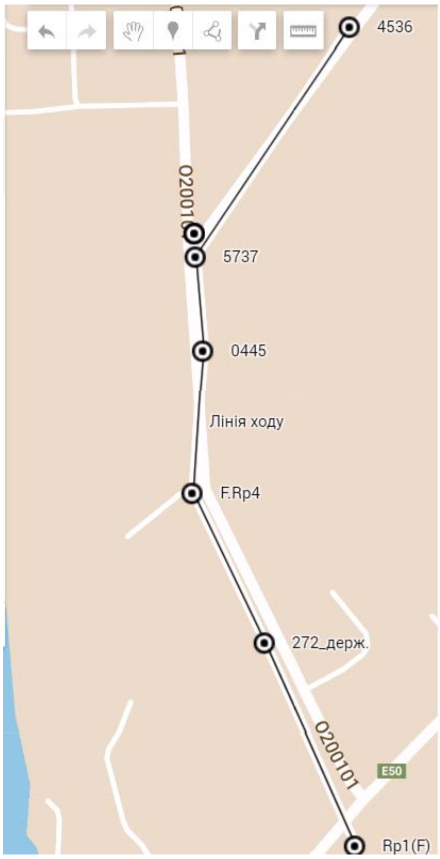

Fig. 3. Leveling scheme

This method is applied during observations on two parallel leveling lines, where height differences are measured simultaneously, satisfying the requirements of the [*Instruction*, 1990] for first-class leveling (leveling is carried out along two pairs of turning plates in both forward and reverse directions). The Cholesky method (as a two-point method) is predominantly used in China (see Fig. 4).

For the Cholesky method, there are two observation variants:

- ChSp - Cholesky Simple (reading rods clockwise);
- ChAd - Cholesky Advanced (reading rods clockwise and counterclockwise).

In both variants of the observation method, the requirement for equality of distances to rods is applied. However, in leveling using the "forward-backward" method, we exclude the function of distances to rods equality control to simplify the research, and we choose the ChSp method (Cholesky Simple - reading rods clockwise).

At each leveling station, we strive to maximize the difference in distances to rods and exclude the function of accounting for corrections for Earth curvature in the instrument.

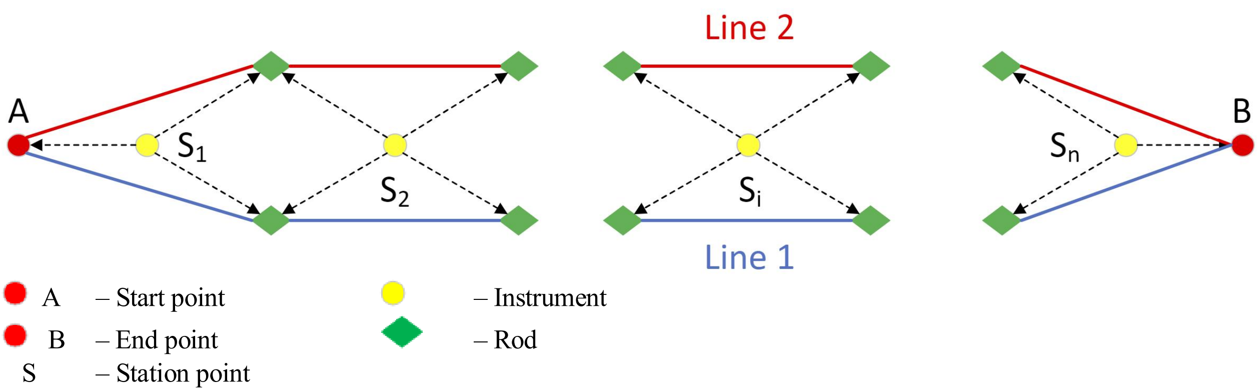

Fig. 4. Cholesky Method Levelling Scheme

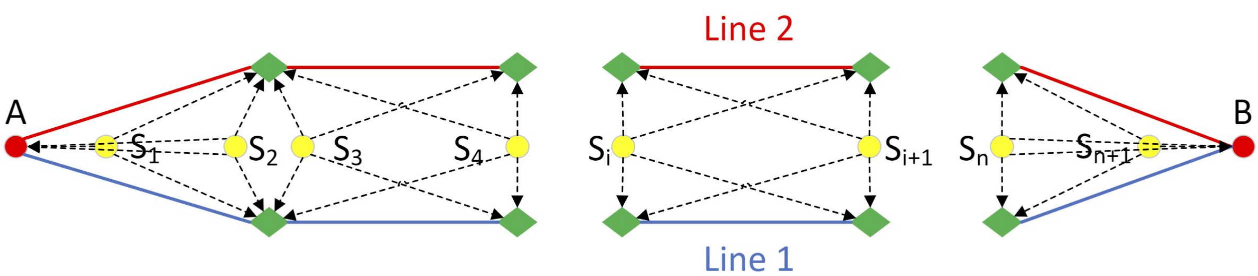

Fig. 5. Scheme of two-sided leveling using the "Forward-Backward" method.

The leveling was performed using two pairs of geodetic turning plates, forming two leveling lines with simultaneous measurements "forward and backward" clockwise. Specifically:

Forward:

- Measurements were taken on 2 near turning plates (right and left).
- Measurements were taken on 2 far-turning plates (left, right).
- Transition with the instrument along the direction of travel to the second leveling station.

Backward:

- Measurements were taken on 2 far turning plates (left, and right) to minimize the effect of refocusing.
- Measurements were taken on 2 near-turning plates (right and left).

Table 5 shows the average results of the "forward-backward" leveling method for sections of the traverse. The discrepancy between the average values of the lines is calculated, and for comparison, the allowable discrepancies according to the [*Instruction*, 1990] for first-class leveling are indicated.

After obtaining the average values of the lines in the sections, corrections for refraction and earth curvature were applied to the obtained results. The obtained results are presented in Table 6.

Analyzing columns 6 and 8 of Table 6, it can be concluded that the discrepancies between the determined height differences obtained directly and in reverse along the sections of the leveling traverse after applying corrections for refraction and earth curvature are small and in terms of accuracy comply with the requirements for first-class leveling according to the [*Instruction*, 1990].

Table 5

Average values of the results of the two leveling lines using the 'forward-backward' method (field measurements)

Section name	Leveling line direction	Leveling line name	The length of the line in m.	Number of tripods in a section	Height difference in m.	Difference in lines, in mm.	Additional difference in lines in mm.	The average value of the excess (forward-backward) in the city.	Difference of lines (forward-backward) in mm.	Additional lines difference (forward-backward) in mm.
Rp1F – Rp272	forward	left	295	12	13.57410	0.43	1.05	13.57389	-2.21	1.58
		rights			13.57367					
	backward	left		12	13.57590	-0.40	1.05	13.57610		
		rights			13.57630					
Rp272 – FRp4	forward	left	217	8	6.72707	-0.63	0.93	6.72675	-0.35	1.39
		rights			6.72644					
	backward	left		8	6.72677	-0.67	0.93	6.72693		
		rights			6.72744					
FRp4- Rp 0445	forward	left	180	6	5.68930	0.68	0.85	5.68964	0.01	1.27
		rights			5.68998					
	backward	left		6	5.68979	0.33	0.85	5.68962		
		rights			5.68946					
Rp 445- Rp 5737	forward	left	120	6	7.77744	0.43	0.74	7.77722	1.27	1.11
		rights			7.77701					
	backward	left		6	7.77611	0.32	0.74	7.77595		
		rights			7.77579					
Rp5737- Rp 536	forward	left	330	16	16.08243	0.36	1.22	16.08225	0.49	1.83
		rights			16.08207					
	backward	left		16	16.08221	0.91	1.22	16.08176		
		rights			16.08130					

Table 6

The average values of the results of the two leveling lines using the 'forward-backward' method (including corrections for refraction and earth curvature)

Section name	Leveling line direction	The length of the line in m.	Number of tripods in a section	Before the introduction of corrections for refraction and curvature of the Earth		After making corrections for refraction and curvature of the Earth		Additional line difference (forward-backward) in mm.	The average value of the height differences (forward- backward) in m. after corrections for refraction and curvature of the earth
				The average value of the height differences of lines (forward-backward) in m.	Difference of lines (forward-reverse) in mm.	The average value of the height differences of lines (forward-backward) in m.	Difference of lines (forward-backward) in mm.		
Rp 1F-Rp 272	forward	295	12	13.57389	-2.21	13.57560	0.58	1.58	13.57531
	backward		12	13.57610		13.57502			
Rp 272-FRp 4	forward	217	8	6.72675	-0.35	6.72695	0.01	1.39	6.72695
	backward		8	6.72693		6.72694			
FRp 4-Rp 0445	forward	180	6	5.68964	0.01	5.68951	0.17	1.27	5.68960
	backward		6	5.68962		5.68968			
Rp 445-Rp 5737	forward	120	6	7.77722	1.27	7.77679	0.37	1.11	7.77660
	backward		6	7.77595		7.77642			
Rp5737-Rp 536	forward	330	16	16.08225	0.49	16.08232	0.61	1.83	16.08202
	backward		16	16.08176		16.08171			

Since we did not have reference height differences for the given sections and could not evaluate the results of our experiment, it was decided to conduct leveling according to the first-class program using the method from the middle and compare them with the results obtained from the 'forward-backward' method. The leveling results from the method from the middle are presented in Table 7.

The corrections for the curvature of the Earth are small, namely 0.00-0.01 mm, so the total angle $\sum(\overline{i''}-\overline{r''})$ consists of averaged angular values of $\overline{i''}$ - the angle of non-horizontality.

The comparison of the obtained height differences determined by the method from the middle and the forward-backward method is reflected in Table 8. Analyzing the results of the last column of Table 8, it can be concluded that the differences between the determined height differences obtained for the sections of the leveling route (by the method from the middle and the forward-backward method) are small and in terms of accuracy comply with the requirements for first-class leveling according to the instruction [*Instruction*, 1990]. The maximum discrepancy in the sections between the leveling methods was 0.42 mm, and over the entire leveling route with a length of 1142 meters, it was 0.06 mm. It should be noted that for conducting the experiment using the forward-backward method, we chose the ChSp method – Cholesky Simple (reading the rods clockwise). To increase the accuracy of our method, the ChAd method – Cholesky Advanced (reading the rods clockwise and counterclockwise) can be used.

Since the research was carried out on two leveling lines, we obtained two values of the total angle $\sum(\overline{i''}-\overline{r''})$. Graphs showing the change in angle along the sections of our leveling route are constructed based on the differences in the total angles $\sum(\overline{i''}-\overline{r''})$ (see Fig. 6-10).

Table 7

The average values of the results of the two leveling lines by the method "from the middle" (field measurements)

Section name	Leveling line direction	Leveling line name	The length of the lines in m.	The number of tripods in the section	The amount of height differences in m.	The amount of height differences in mm.	Additional difference in lines in mm.	The average value of the height differences of lines (forward - backward) in m.	Difference of lines (forward-backward) in mm.	Permissible deviations of lines (forward- backward) in mm.	The average value of the height differences in the section, in m.
Rp1F – Rp272	forward	left	295	10	13.57524	0.28	1.08	13.57510	0.31	1.58	13.57495
		rights			13.57496						
	backward	left		10	13.57480	0.02		13.57479			
		rights			13.57478						
Rp272 – FRp4	forward	left	217	6	6.72652	0.38	0.93	6.72671	0.09	1.39	6.72667
		rights			6.72691						
	backward	left		6	6.72634	-0.56		6.72662			
		rights			6.72690						
FRp4- Rp 0445	forward	left	180	4	5.69041	0.55	0.82	5.69014	0.24	1.24	5.69002
		rights			5.69086						
	backward	left		4	5.68969	0.41		5.68990			
		rights			5.69010						
Rp 445- Rp 5737	forward	left	120	4	7.77690	0.19	0.69	7.77681	0.01	0.69	7.77680
		rights			7.77671						
	backward	left		4	7.77680	0.00		7.77680			
		rights			7.77680						
Rp5737-Rp 536	forward	left	330	10	16.08222	0.59	1.15	16.08193	0.10	1.72	16.08198
		rights			16.08163						
	backward	left		10	16.08214	0.22		16.08203			
		rights			16.08192						

Table 8

Comparison of the obtained height differences determined by the method from the middle and the "forward-backward" method

Section name	Line length, in m.	The average value of the height differences by the method from the middle, in	Number of tripods in the section, pcs.	The average value of the height differences by the forward-backward method, in m.	Number of tripods in the section, pcs.	The average value of the height differences by the forward-backward method, in m.
Rp1F –Rp272	295	13.57495	10	13.57531	12	-0.36
Rp272 –FRp4	217	6.72667	6	6.72695	8	-0.28
FRp4- Rp 0445	180	5.69002	4	5.68960	6	0.42
Rp445-Rp 5737	120	7.77680	4	7.77660	6	0.20
Rp5737-Rp 536	330	16.08198	10	16.08202	16	-0.04
Total	1142	49.85042	34	49.85048	48	-0.06

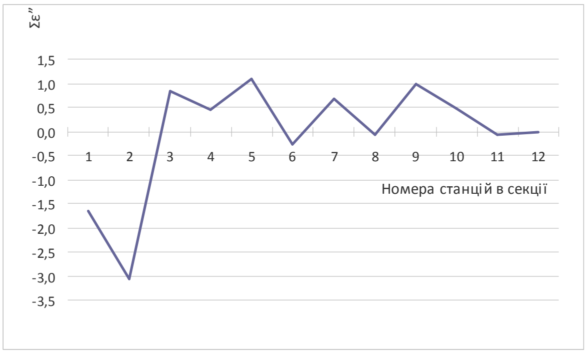

Fig. 6. The difference of angles ε"
in the section Rp 1F - Rp 272

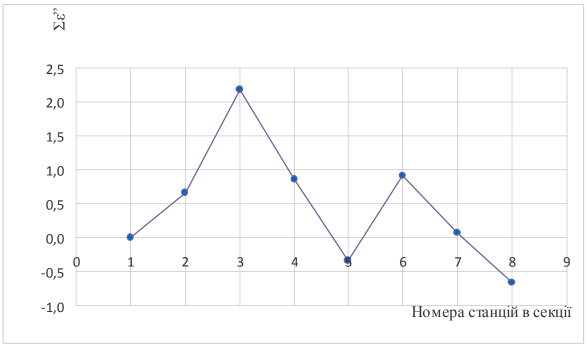

Fig. 7. The difference of angles ε"
in the section Rp 272 - F Rp 4

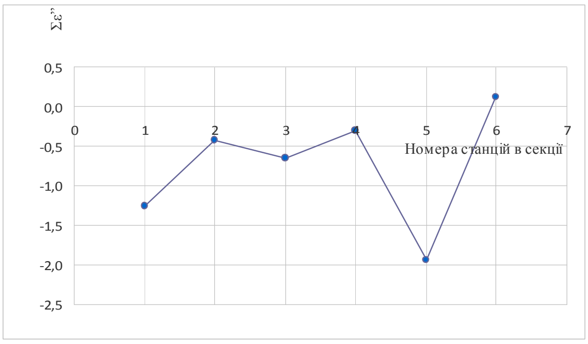

Fig. 8. The difference of angles ε"
in the section F Rp 4 - Rp 0445

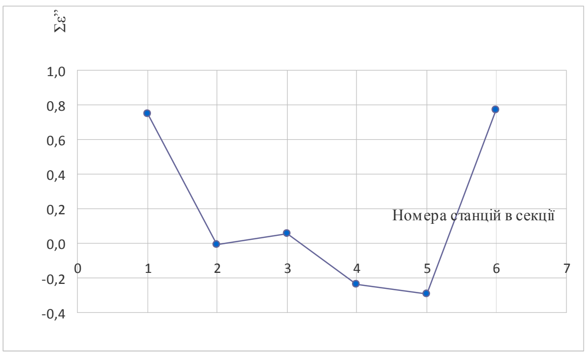

Fig. 9. The difference of angles ε"
in the section Rp 0445-Rp 5737

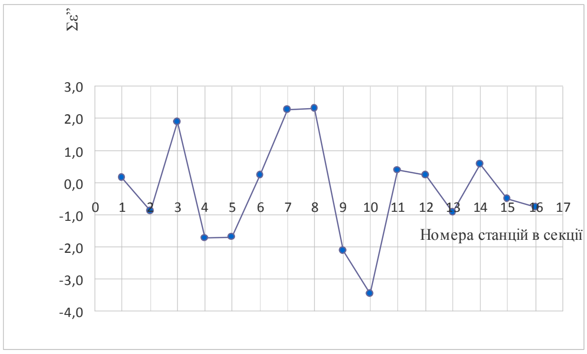

Fig. 10. The difference of angles ε"
in the section Rp 5737-Rp 4536

As can be seen from the figures, the total angle ε" is not stable during the observation period and undergoes large changes due to the effect of vertical refraction, since the compensator of the leveler guarantees the establishment of the sighting beam in a horizontal position with an accuracy of 0.5". So, its average value of the difference between the angles of two lines on each section different, namely:

1. section	Rp 1F - Rp 272	-0,1″
2. section	Rp 272 - F Rp 4	+0,5I
3. section	F Rp 4 - Rp 0445	-0,7I
4. section	Rp 0445-Rp 5737	+0,2I
5. section	Rp 5737-Rp 4536	-0,3I

Conclusions

The method of two-way leveling, "forward-backward," can be used for Class I leveling on extensive slopes and has several advantages over the method of leveling "from the middle," namely:

- Convenience in selecting and marking the leveling route and choosing instrument installation location (deviation from the equality of arms regulated by the instruction [*Instruction*, 1990]);
- Leveling control along the observation line for the angle of horizonality of the ray;
- Practical determination of the main condition of the level on each observation line;
- Increased accuracy of leveling due to double measurements of elevations;
- Possibility of replacing double leveling traverses, reducing the time for laying out the route for the return leveling;
- Possibility of introducing a correction for vertical refraction on extensive slopes;

- Investigation of the sighting ray and the angle of vertical refraction $\overline{r}''$.

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ДОСЛІДЖЕННЯ ВИСОКОТОЧНОГО ДВОСТОРОННЬОГО
ГЕОМЕТРИЧНОГО НІВЕЛЮВАННЯ

Метою статті є дослідження точності двостороннього геометричного нівелювання, розроблення методики його виконання з ціллю підвищення точності визначення перевищень шляхом врахування вертикальної рефракції та контролю негоризонтальності променя нівелювання. Методика. Враховуючи те, що в цифрових нівелірах є можливість вимірювати віддалі до рейок, враховувати негоризонтальність променя, та вертикальної рефракції в результати під час вимірювань, нами запропоновано спосіб двостороннього геометричного нівелювання. Описана запропонована методика виконання двостороннього тригонометричного нівелювання способом “вперед-назад” із врахуванням вертикальної рефракції по лініям спостереження. Результати. Для апробації способів для нівелювання 1 класу методом Холескі (із середини) та двостороннього геометричного нівелювання (“вперед-назад”) була вибрана ділянка ходу з затяжним підйомом довжиною біля 1 км, яка складалася з 5-ти секцій Апробацію способу було виконано електронним нівеліром Trimble DiNi-03 по двох лініях ходу. Розбіжність між перевищеннями отриманими по секціях нівелірного ходу способами “із середини” та “вперед-назад” відповідають вимогам для нівелювання I класу. Величина максимального розходження в секціях між способами нівелювання склала 0.42 мм, а на весь хід нівелювання, довжиною 1142 метри, – 0.06 мм. Наукова новизна. Розглянуто теоретичне обґрунтування та проведено експериментальні дослідження можливості застосування двостороннього геометричного нівелювання за програмою спостережень I класу та введення поправки за вертикальну рефракцію на затяжних схилах.Підтверджено, що метод двостороннього геометричного нівелювання способом “вперед-назад” можна використовувати для нівелювання I класу способом “із середини” на затяжних схилах і має ряд переваг перед ним.

Ключові слова: геометричне нівелювання, вертикальна рефракція, не горизонтальність променя, вертикальні зміщення, геодезичний моніторинг.

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