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COMPARATIVE ANALYSIS OF METHODS FOR CUTTING SPUR GEARS BASED ON TECHNICAL AND ECONOMIC PARAMETERS

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Abstract. The results of the simulation and study of the process of cutting gears by the traditional method of hobbing and the new radial-circular method (RCM), which is carried out by a thin disk cutter, but in the conditions of continuous profile generation, as it is in gear cutting with a hob. It has been shown that due to the peculiarities of this process, its kinematics, and the structure of the disk cutter, RCM is superior to hobbing in all the parameters studied: volumetric productivity, chip thickness ratio, and cutting forces, machining time and machining costs. The higher efficiency of the radial-circular method has thus been demonstrated and the feasibility of its practical application has been confirmed.

Keywords: spur gear, radial-circular method, gear cutting, thin disc cutter, chip, cutting force.

Introduction

Gears are an integral part of modern machines, being one of their main components and being incorporated in drives, gearboxes, and transmissions. They are characterized by increased complexity and high requirements for the accuracy and quality of their surfaces, considerable labor intensity in manufacturing, and large annual production volumes. Given the importance and indispensability of gears, the problem of optimizing their manufacturing processes remains relevant to modern engineering and is at the center of the attention of scientists and manufacturers. Considering the importance of solving this problem and the need to increase the efficiency of manufacturing gears and transmissions, new and improved traditional processes for cutting gear surfaces have been developed in recent years, and research is underway in the field of mathematical modeling and optimization of their manufacture, hardening, and design of tools and equipment.

Despite the development of new gear-cutting processes and their rapid adoption in various engineering industries, such as power skiving and hard skidding, hobbing remains the primary method of producing these parts. Despite its widespread use, it has some disadvantages that cannot be eliminated. These include the unevenness of the hobbing process, significant cutting forces and energy consumption, significant dynamic loads on the technological system of the machine tool, and limited accuracy of the toothed surfaces. At the same time, there is another method of gear cutting developed at the Department of Mechanical Engineering Technology of Lviv Polytechnic University. It is based on the use of the simplest cutting tool, a thin disk cutter, and has the greatest versatility of all known methods. The use of this method, called Radial-Circular Gear Cutting (RCM), makes it possible to cut all known types of gears and gear pairs with one tool on one gear-cutting machine, which requires a minimum of modernization.

Implementing this method can significantly reduce the cost and production value of these parts and increase the efficiency of gear machining processes.

This study aims to compare the processes of hobbing and the radial-circular method by the main technical and economic parameters.

Analysis of primary sources

Traditionally, considerable attention has been paid to the study of gear cutting as the most massive gear manufacturing process. Numerous publications present the results of modeling and experimental studies of various processes and phenomena that accompany cutting and shaping with a hob, characterizing the state of the tool and machining efficiency: cutting parameters, cutting forces, thermal processes, wear and tool life [1–26]. As for RCM, this technology has not yet found its practical application and is being studied by researchers at the Lviv Polytechnic [27–29]. At the same time, there are some works devoted to sinusoidal gears with external and internal gearing and sinusoidal gears that can be manufactured by the RCM disc cutter [30–31]. The work on this subject [32–33] provides data on the advantages of sinusoidal gears and describes the cutting of such gears with hobs having a sinusoidal initial contour of the base rack. Given the novelty of the proposed gear-cutting method, a comparative analysis of these two technologies is of scientific and practical importance.

Results of the study

The comparison of the two processes is as follows:

– Determination of cutting parameters and related parameters: volumetric productivity and intensity of shearing plastic deformation of the cut layer.

– Calculation of forces and torques, consumed cutting power.

– Determination of the roughness of the tooth profiles.

– Comparison of processes in terms of time taken to form the tooth surface.

– Comparison of gear machining costs.

To ensure that the calculations and process modeling were consistent, the same initial data were used: involute gear module is 5 mm, the eccentricity of the equivalent sine gear is 3.64 mm; axial feed rate is 1.75 mm/rev of the workpiece; the cutting to full profile height; number of teeth: gear Z_{gear} = 33, hob – q = 9 rails (end teeth), disc cutter Z_{mill} = 9 teeth; outer diameter of tools 110 mm; gear width 35 mm; tool material – high-speed steel R6M5; workpiece material – steel 40X, tensile strength $\sigma_{в}$ = 660 MPa, shear strength limit (according to the third strength theory) $[\tau]$ = 350 MPa; cutting speed 32 m/min. For gears with a module of up to 5 mm, it is possible to cut the entire height of the profile in one pass.

Modeling of cut parameters

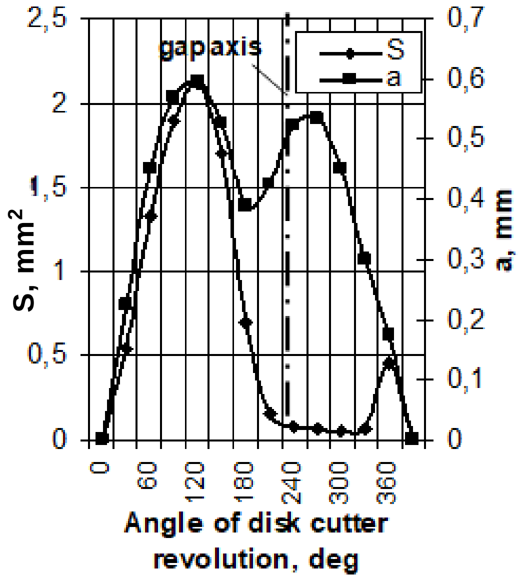
The data on the hobbing process for the above initial data are taken from the source. Fig. 1 shows graphs of the cross-sectional area of the cuts and the thickness of the cut layers of a hob (a) and a disc cutter in the RCM (b).

The processes under consideration have different kinematics that affect the distribution of cuts between the teeth of the tools. The teeth of a hob are arranged on a helical surface and perform cutting by the rotation of the cutter (the main cutting movement) and their constructive movement along the axis of the cutter in a helical line when the cutter rotates synchronously with the rotation of the gear. The number of active teeth of the hob that perform cutting and profile shaping is determined by the ratio:

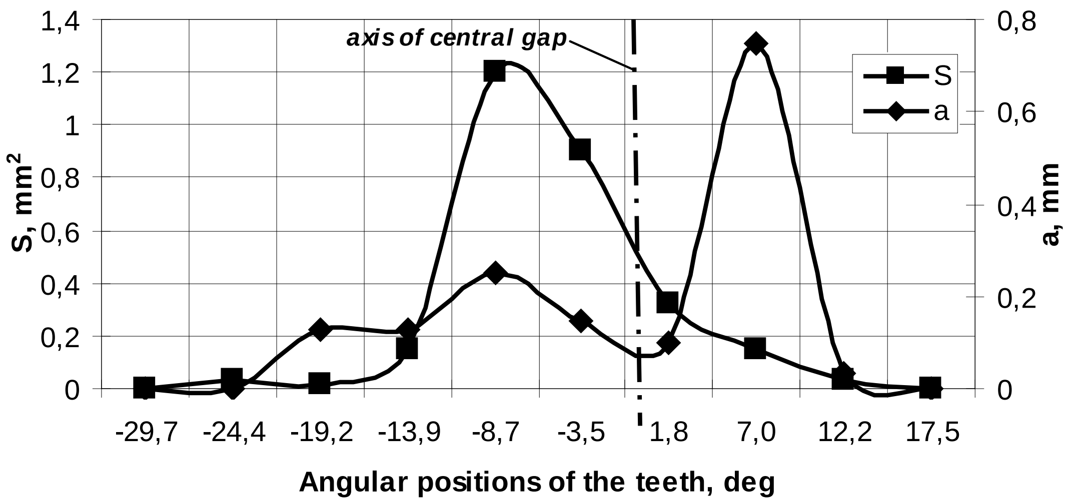
$$Z_{hob} = q \cdot \frac{\alpha_{cont}}{\tau_{ang}} \tag{1}$$

where τ_{ang} is the end angular pitch of the gear; α_{cont} is the contact angle in the plan of the cutter with the gear to be machined; q is the number of cutter rails (hob columns); $\tau_{ang} = \frac{360^0}{Z_{gear}}$. For this case, τ_{ang} = 10.9°;

α_{cont} = 36.65°*;* q = 9; end overlap coefficient is 3.3; the number of teeth that eliminate the allowance from one gap between the gear teeth is 30.



a



b

Fig. 1. Cross-sectional area and thickness of cuts in the process of hobbing (a) and in the radial-circular method (b)

Thus, the allowance removed from the gap by the hob is distributed among 30 teeth, and by the disc cutter – among 9 teeth, as shown in the graphs in Fig. 1.

The volume of material W that each tool removes from the single gap in one working cycle corresponding to the axial feed can be determined as the product of the cut area and the length of contact between the corresponding tooth and the workpiece. The average contact length for a hob is 23.6 mm, and for a disc cutter, it is 20.1 mm. According to these data, the average amount of allowance per tooth of a hob W = 7.035 mm^3; for a disc cutter W = 8.97 mm^3; the total amount of allowance removed from one gap in one working cycle, a multiple of the axial feed rate, is: for a hob W = 211.1 mm^3, for a disc cutter 80.7 mm^3.

Volumetric performance of the methods

According to the initial data, at a cutting speed of 32 m/min, the tool rotation frequency is 92.6 min^{-1}, and the time for one revolution of both tools is 1.54 s.

In one cycle of axial feed, the hob performs several revolutions equal to the number of cutter teeth in the engagement at the gear end, i. e., the end overlap ratio, i. e., 3.3. The cutting time for a hob with 3.3 revolutions is 5.1 seconds. The cutting time of a disc cutter corresponds to the time of its one revolution, i. e., 1.54 seconds.

The productivity of allowance removal from a single tooth gap can be set as a fraction of the volume of allowance material in a single gap divided by the time it is removed. Then for a disc cutter, this parameter is 52.4 mm^3/s, and for a hob – 41.4 mm^3/s. From these data, it follows that the volumetric productivity of the radial-circular gear-cutting process is 1.27 times higher than hobbing. This result, obtained for certain initial conditions, is valid for other data.

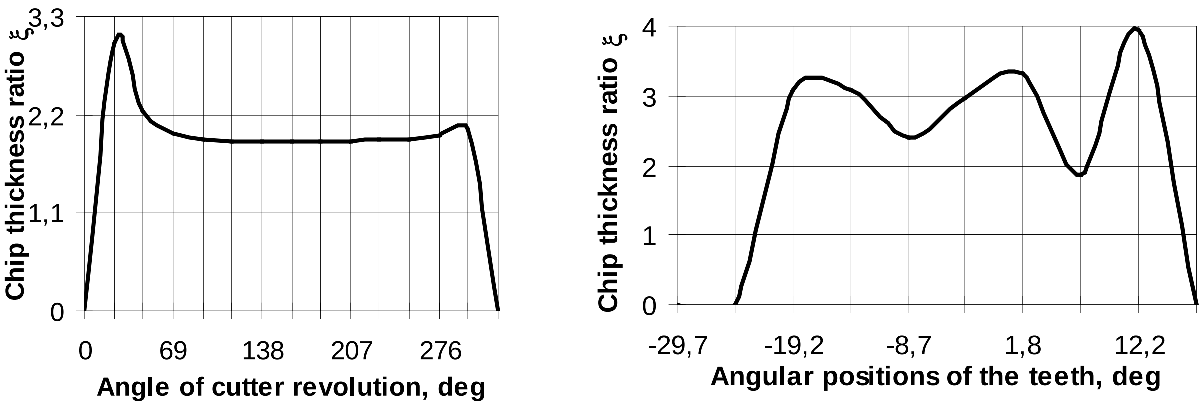
Intensity of the cutting process

According to the basic principles of cutting theory, the thickness of the cut layers determines the shear intensity and the value of the chip thickness ratio ξ. This dependence for the conditions corresponding to this article was determined using the *Deform 2D* system [] and is expressed by the following Eq. (2):

$$y = -2.2107x^5 + 12.514x^4 - 26.863x^3 + 27.167x^2 - 13.179x + 4.3815, \qquad (2)$$

where $y = \xi$; $x = a$.

Using this relationship, the chip thickness ratio for the compared methods was found based on the values of the slice thicknesses (Fig. 1), the graphs are shown in Fig. 2. The average chip thickness ratio for the hob is 2.95, and for the disc cutter, it is 2.04. The increase in the parameter ξ for the hob is explained by the fact that with a larger number of active teeth and a decrease in the thickness of the cuts, the intensity chip thickness ratio increases.



a b

Fig. 2. Chip thickness ratio during gear cutting with a hob (a) and during cutting using the RCM method (b)

Cutting forces

The cutting force is fully determined by its main component P_o, which coincides with the cutting speed vector in the direction. This force can be represented by a function of the specific cutting force *p*, the value of which is equal to the ratio of the force to the cross-sectional area of the cuts normal to the speed vector, i. e:

$$p = \frac{P_o}{S} \tag{3}$$

In cutting theory, the specific force is also determined by the Eq. (4):

$$p = \xi \cdot [\tau] \tag{4}$$

where $[\tau]$ is the shear strength limit of the workpiece material. Then we get the following dependence for the force P_o:

$$P_o = \xi \cdot S \cdot [\tau] \tag{5}$$

Using this relationship and the values of its components, Fig. 3 shows the cutting force that occurs during gear cutting and in RCM.

The graphs in Fig. 3 characterize the force P_o on the teeth following the spiral line of the hob and this force on the teeth of the disc cutter at its angle of rotation of 360°. However, both gear-cutting methods are characterized by the fact that more than one tooth can be in contact with the workpiece at the same time. For a disc cutter, these are the teeth along the length of the angle and arc of contact with the workpiece, i. e. in the face of the tool. For a gear hob, in addition to this overlap, there is also the number of teeth in contact with the cutter rail in the end plane of the gear being cut. Under these conditions, the number of teeth on the contact length at the end of both tools does not exceed one, i.e. the end coefficient for them is less than or equal to one. At the end of the gear, as shown above, there are 3.3 teeth in contact with the gear. Using the force diagrams in Fig. 3, we can obtain the force diagrams for the above conditions with the overlapping contours of the part shown in Fig. 4.

As can be seen from the data obtained, the cutting force of the disc cutter is significantly higher. However, the number of teeth taken as a starting value (Z = 9) is not typical for a disc cutter with the corresponding outer diameter (110 mm). Commercial disc cutters of this diameter have some teeth of 18 or more. Unlike a hob, the number of teeth and tooth shape of a disc cutter does not directly correlate with the resulting tooth profile. Taking this into account, the cuts' and cutting force parameters were determined for Z = 18, and the force graphs are shown in Figure 4. For these data, the single tooth cutting force is reduced by 1.7 times compared to the hob.

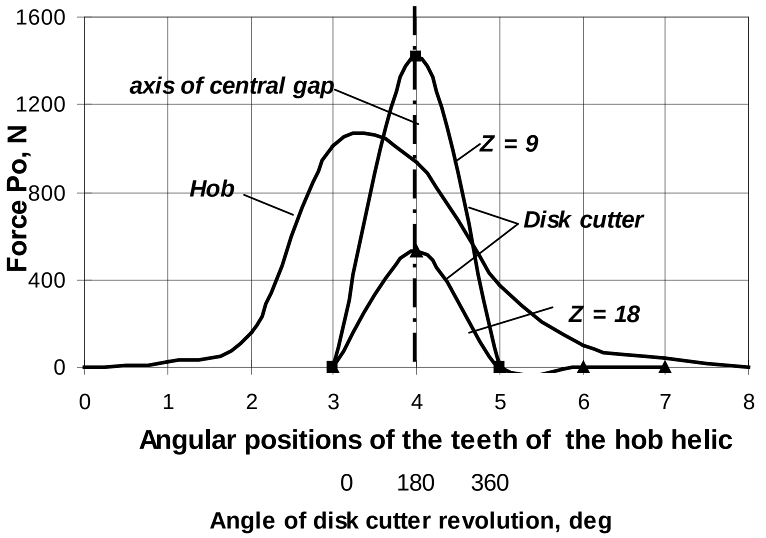
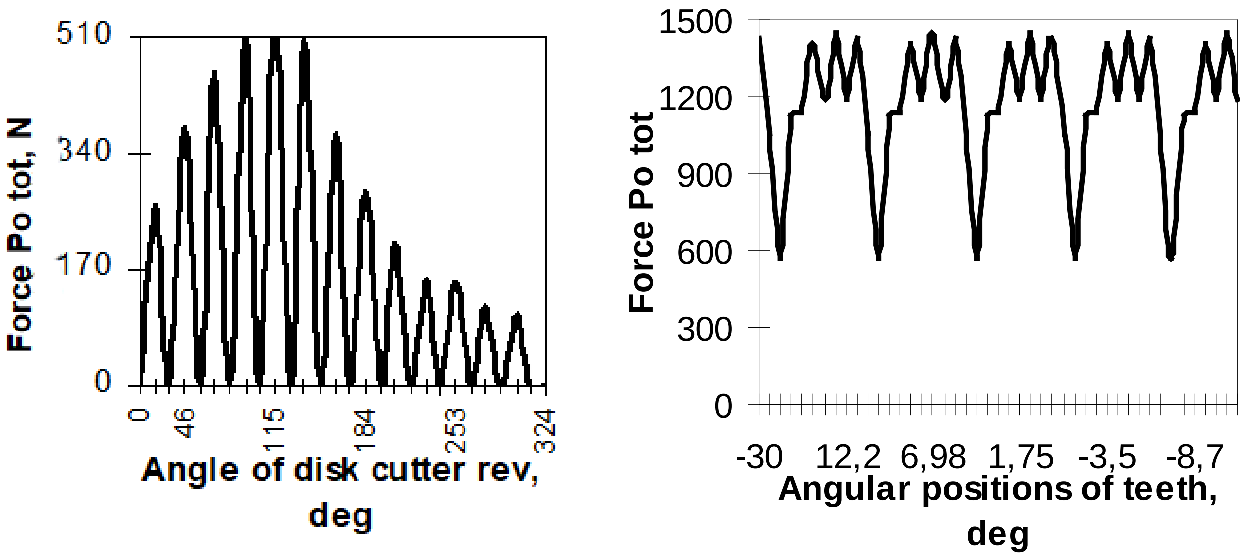


Fig. 3. Cutting forces on the reaming of a hob (on a helical surface) and on the angle of rotation of a disc cutter at the number of teeth of a disc cutter 9 and 18

As the number of teeth of the disc cutter increases, the contact angle and the coefficient of end overlap increase, and consequently the total cutting force, but for the Z = 18 disc cutter this force is much lower than for the hob, both in terms of average value and its variation during continuous machining (Fig. 4).



a b

Fig. 4. Diagrams of the total force $P_{o\ tot}$ change with the angle of rotation of a disc cutter with Z = 18 (a) and on the helical surface of a hob (b)

Higher cutting forces cause larger elastic deformations in the elastic system of the gear-cutting machine tool and cause larger machining errors. In addition, the change in the oscillation frequency of this force during hob operation hurts machining accuracy. For a disc cutter, this frequency is equal to the tooth frequency, i. e. a multiple of the number of teeth (9). It will be 833 revolutions per minute or 14 Hz. The frequency of change of the total force Ro for a hob is 3.3 times lower, i.e. 252 revolutions per minute, or 4.2 Hz. It is known that vibrations at lower frequencies carry more energy, so their negative impact on the quality of gears is greater for a hob.

The roughness of the cut profiles in two methods of tooth processing

In cutting and forming processes carried out under continuous cutting conditions, the profile of the gear is formed as a result of the intersection of successive discrete positions of the side blades of a tool tooth sliding along a conditional ideal profile. This results in methodological errors in the profiles, known as faceting. The height of these micro-irregularities is the main component of the roughness parameter.

The height of the micro-irregularities in a facet depends on the number of cuts that form the profile and determines the angle of profiling, or on a single cut ψ. This is the main difference between the methods being compared. In RCM, a gap is formed between the teeth, i. e. the left and right profiles of adjacent teeth in the axial feed cycle are formed during one revolution of the cutter, in our case the number of profiling cuts is 9.

A gap is formed in one cycle of axial feed of the hob in the number of revolutions of the cutter equal to the coefficient of overlap in the plane of the gear face. For our initial data, this coefficient is 3.3 and the number of profile cuts is 30. Consequently, for the initial data used here, the roughness of the lateral profiles of the gear teeth will be greater in the RCM method (Fig. 5). At the same time, if we take the number of teeth of the disc cutter to be equal to the number of active teeth on the helical line of the hob, i.e. 30, then the level of microroughness will be the same for both variants.

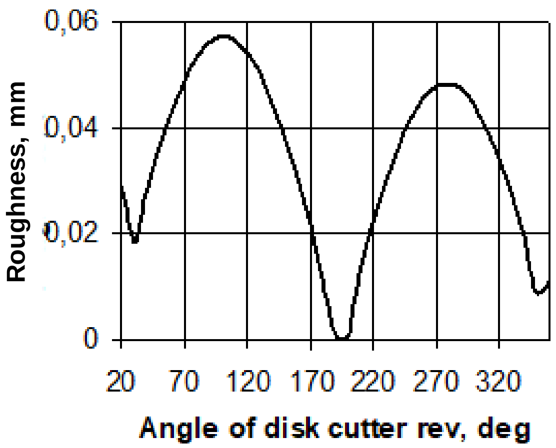
Processing performance. Machine time

Let's calculate the machining time for both methods. At a cutting speed of 32 m/min, the speed of the tools is 92.6 revolutions per minute and the time for one revolution of the cutters is 0.65 seconds. The machining time is given by the Eq. (6):

$$t_o = \frac{L_{cut}}{f_{ax} \cdot n_{cut}} \quad (6)$$

where L_{cut} is the total cutting path, mm; f_{ax} is the axial feed rate, mm/rev; n_{cut} is the tool speed, rev/min.

Given the same initial data, the machine time for the compared methods will be the same and will be 0.56 minutes, or 34 seconds.



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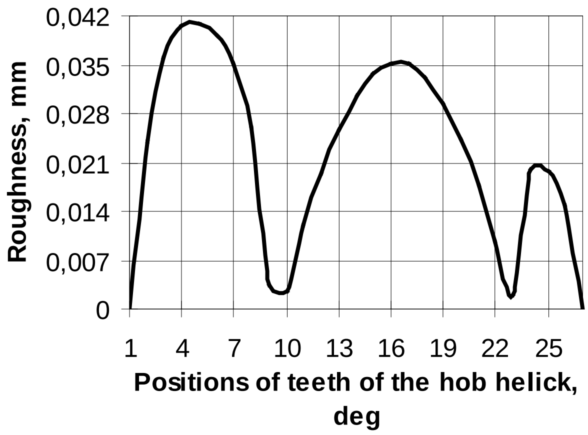

b

Fig. 5. Height of micro-irregularities in the tooth profiles of a gear cut by a disc cutter (a) and a hob (b)

In practice, however, the axial feed rate of a hob is limited. The reason for this is the unevenness of the cutting process, which results in a significant fluctuation in cutting force per revolution of the cutter and significant shock loads on the tool and machine. To reduce the force and torque peaks shown in Fig. 5, the value of the axial feed is taken in the range of 0.12–15 mm per revolution of the workpiece. If we take f_a = 0.15, then the time to cut a gear with a hob will be 2.52 minutes, i.e. the productivity of gear machining by the radial-circular method will be 4.5 times higher.

Cost and expenses

Reducing machine time is the main factor in reducing costs in both processes. This parameter determines the salaries of the main and auxiliary workers, the cost of various resources: energy, the cost of cutting tools, their replacement and maintenance, the number of equipment units, and the cost of their depreciation. As it can be seen, the RCM has a significant advantage on this parameter.

Another efficiency factor is the cost of the cutting tool. Taking into account the complexity of the tools (Fig. 6), the price of a disc cutter is an order of magnitude or lower than that of a hob. On the scale of current tool prices, this ratio is 1 to 30. This means that the economic efficiency of the radial-circular method will be tens of times higher.

Conclusions

The results of the comparison of the two methods of gear cutting show that the radial-circular method has the following advantages in all respects.

1. For the same initial conditions, the volumetric productivity of RCM for eliminating the allowance is 1.3 times higher.

The shear strain intensity and chip thickness ratio in the RCM are lower.

2. With the same number of teeth, the cutting force of a disc cutter is greater. At the same time, the accepted number of teeth on a disc cutter, identical to a hob, is not typical of mass-produced disc cutters. Since, unlike a hob, the tooth shape of a disc cutter using the radial-circular method does not follow the tooth profile of the gear to be cut, the number of teeth of a disc cutter can be two or more times higher and the cutting force is then significantly reduced.

a b

Fig. 6. Comparison of hob (a) and disc cutters (b) in terms of design complexity

3. With the same number of teeth on a disc cutter and active teeth on the helical surface of a hob, the micro-unevenness of the tooth profiles will be the same.

The comparable axial feed rate (1.75 mm / workpiece circumference) for hobs is unacceptably high due to the unevenness of the cutting process. If the actual axial feeds per tool tooth were used in both methods, the cutting time of the gear by RCM would be 4.5 times shorter.

The radial-circular method is tens of times cheaper and more cost-effective than hobbing due to the significant reduction in machining time and the much lower cost of the cutting tool.

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