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A meshfree TSDT-based approach for modeling a skew plate

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Taking into account new frontiers in materials technology to improve construction materials, we investigate the bending analysis of a Functionally Graded Material (FGM) skew plate using a meshless approach based on Third-order Shear Deformation Theory (TSDT). We assume that the material distribution is functionally graded across the thickness of the skew plate. The proposed approach uses both mixture rule theory and the meshless method. The mixture rule theory is used to estimate the effective material properties of the skew plate.

Keywords: *meshfree method; third-order shear deformation theory; functionally graded material; skew plate.*

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1. Introduction

Integration of skew plate geometry has become very important due to its performance and usefulness for optimization purposes. These structures are frequently used in construction for reinforcement or optimization purposes. The skew plate structure is used in the construction of bridges, buildings and ship hulls, etc. Because of the importance of these structures, they have penetrated the high-tech sector, where they have become commonplace in the aeronautics, automotive and military industries. Due to the importance of these structures, they have penetrated the high technologies area where they have become frequent in the aeronautical, automobile and military industries. Buragohain and Patodi (1978) treated the large deflection skew plate problem where they obtained a set of nonlinear algebraic equations using the Newton–Raphson iterative method with increments to solve it [1]. They demonstrated their formulation for the isotropic skew plate of constant thickness simply and clamped supported, subjected to uniformly distributed transverse load to be independent of boundary conditions. Chia (1980) used an analytical method to study the small deflection elastic behavior of isotropic and anisotropic skew plates [2]. Daripa and Singha (2009) analyzed the impact of corner stresses on the behavior of composite skew plates [3]. Das et al. (2008) studied the dynamic problem of skew plates simply supported and clamped boundary conditions using a variational method [4]. For non-linear vibration problems for laminated skew plates, the Differential Quadrature Method (DQM) is used in [5]. Duan and Mahendran (2003) used the oblique coordinate systems to study a new non-linear quadrilateral hybrid/mixed shell with five degrees of freedom at each node and analyze the behavior of skew plate for the large deflection under concentrated and distributed load [6]. Liew and Han (1997) using Reissner/Mindlin theory studied the bending analysis of the thick skew plate simply supported based on the first-order shear deformation [7]. They introduced the geometric transformation of the governing differential equations and boundary conditions of the plate from the physical domain into a unit square computational domain. Subsequently, they derived a set of linear algebraic equations from the transformed differential equations via DQM, and the approximate solutions of the problem are obtained by solving the set of algebraic equations. Liew et al. (2004) analyzed the buckling of thin plates under non-uniform loading using the mesh-free radial basis function method [8]. They obtained initial (i.e., pre-buckling) stresses by discretizing the variational form of the static system of equations and calculated the static buckling loads of the plates by solving the resultant eigenvalue equation. Malekzadeh and Fiouz (2007) based on the thin plate theory to study two different differential quadrature approaches and analyzed the large deformation of the thin orthotropic skew plate using the first order shear deformation plate theory with rotationally restrained edges [5]. They have modeled the geometrical nonlinearity of the plate by using Green's strain and Von Karman's assumption for the two approaches. Rajamohan and Raamachandran (1997) have solved a fundamental solution, avoiding numerical integration since domain integrals for oblique coordinates, for the analysis of skew plates under transverse loading using the boundary element method [9].

In this work, an alternative based on meshless methods [10–15] to simulate the bending analysis of a FGM skew plate. This proposed approach is based on Third-order Shear Deformation Theory (TSDT) [16, 17] in the formulation and WLS for variables approximation. Material distribution is assumed to be functionally graded across the entire thickness of the skew plate. In the proposed meshless approach, mixing rule theory is used to estimate the effective material properties for the skew plate.

2. Mechanical and kinematic characteristics

The TSDT [16, 17] is a structural theory that is based on similar assumptions as the classical plate theory and the FSDT (First-order Shear Deformation Theory). However, it relaxes the assumption of straightness and normality of a transverse normal after deformation by expanding the in-plane displacements (u, v) as cubic functions of the thickness coordinate. In the TSDT, the displacement field is expressed as follows:

$$\begin{cases} u(x,y,z) = u_0(x,y) + z\theta_x(x,y) - \dfrac{4}{3h^2}z^3\left(\theta_x(x,y) + \dfrac{\partial w_0(x,y)}{\partial x}\right), \\ v(x,y,z) = v_0(x,y) + z\theta_y(x,y) - \dfrac{4}{3h^2}z^3\left(\theta_y(x,y) + \dfrac{\partial w_0(x,y)}{\partial y}\right), \\ w(x,y,z) = w_0(x,y). \end{cases}$$

The TSDT accounts for transverse shear deformation effects more accurately than the classical plate theory and the FSDT. By including cubic terms in the displacement field, it allows for a better representation of the transverse normal and shear strains throughout the thickness of the plate or shell. The relationship between deformation and displacement for small deformation are defined as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.$$

The deformations can be written as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z\begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^3\begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} + z^2\begin{Bmatrix} \gamma^2_{xz} \\ \gamma^2_{yz} \end{Bmatrix},$$

where:

$$\begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} = -\frac{4}{3h^2}\begin{Bmatrix} \frac{\partial \theta_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial \theta_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_0}{\partial x} + \theta_x \\ \frac{\partial w_0}{\partial y} + \theta_y \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma^2_{xz} \\ \gamma^2_{yz} \end{Bmatrix} = -\frac{4}{h^2}\begin{Bmatrix} \frac{\partial w_0}{\partial x} + \theta_x \\ \frac{\partial w_0}{\partial y} + \theta_y \end{Bmatrix}.$$

3. Constitutive relations of FG plates

The constitutive relation corresponding to Hook's relation for FG plates is given as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} C_{11}(z) & C_{12}(z) & 0 & 0 & 0 \\ C_{21}(z) & C_{22}(z) & 0 & 0 & 0 \\ 0 & 0 & C_{66}(z) & 0 & 0 \\ 0 & 0 & 0 & C_{55}(z) & 0 \\ 0 & 0 & 0 & 0 & C_{44}(z) \end{Bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix},$$

where:

$$C_{11}(z) = C_{22}(z) = -\frac{E(z)}{1-\nu(z)^2},$$

$$C_{12}(z) = \nu(z)\, C_{11}(z),$$

$$C_{44}(z) = C_{55}(z) = C_{66}(z) = -\frac{E(z)}{2(1+\nu(z))}.$$

Many researchers use a power law function "P-FGM" to describe the material properties of materials with graduated functionality. Once the local volume fraction $V(z)$ is defined, the material properties of "P-FGM" plate [16, 17] can be determined by the rule of mixtures:

- Young's Modulus:

$$E(z) = (E_c - E_m) \times V(z) + E_m;$$

- Poisson's ratio:

$$\nu(z) = (\nu_c - \nu_m) \times V(z) + \nu_m;$$

where E_c and v_c define the properties of the ceramic material and E_m and v_m define the properties of the metal material. The expression of volume fraction of the P-FGM is given by a power law function:

$$V(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^N,$$

where h is the thickness of the plate and N $(0 \leqslant N \leqslant \infty)$ is an exponent of the volume fraction which represents the variation of the material through the thickness of the layer in FGM.

4. Governing equations

We obtain the TSDT equilibrium equations from the virtual displacement principle:

$$\delta u - \delta v = 0.$$

The virtual strain energy δu, and the virtual work done applied force δv are given by:

$$\delta u = \int_{\Omega_0} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}),$$

$$\delta v = -\int_{\Omega_0} q\, \delta\omega\, dx\, dy,$$

where Ω_0 and q are respectively the middle-line of plane and concentrated load applied on the surface. We obtained the equation below by integrating the deformation on equilibrium equation:

$$\int_{\Omega_0} \int_{-\frac{h}{2}}^{\frac{h}{2}} \Bigg[\sigma_{xx}\left(\delta\varepsilon^0_{xx} + z\delta\varepsilon^1_{xx} - \frac{4}{3h^2}z^3\delta\varepsilon^3_{xx}\right) + \sigma_{yy}\left(\delta\varepsilon^0_{yy} + z\delta\varepsilon^1_{yy} - \frac{4}{3h^2}z^3\delta\varepsilon^3_{yy}\right)$$
$$+ \tau_{xy}\left(\delta\gamma^0_{xy} + z\delta\gamma^1_{xy} - \frac{4}{3h^2}z^3\delta\gamma^3_{xy}\right) + \tau_{xz}\left(\delta\gamma^0_{xz} + z\delta\gamma^1_{xz} - \frac{4}{3h^2}z^3\delta\gamma^3_{xz}\right)\Bigg] dx\, dy$$
$$+ \int_{\Omega_0} q\, \delta\omega\, dx\, dy = 0,$$

where

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z \\ z^3 \end{Bmatrix} dz, \qquad \begin{Bmatrix} R_{\alpha} \\ Q_{\alpha} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha z} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz.$$

So, the equilibrium equations are given as follows:

$$\delta u\colon \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0,$$

$$\delta v\colon \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0,$$

$$\delta w\colon \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{4}{h^2}\left(\frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y}\right) + q = 0,$$

$$\delta \theta_x\colon \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - \frac{4}{3h^2}\left(\frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y}\right) - Q_x + \frac{4}{h^2}R_x = 0,$$

$$\delta \theta_y\colon \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - \frac{4}{3h^2}\left(\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yy}}{\partial y}\right) - Q_y + \frac{4}{h^2}R_y = 0.$$

Taking into account the law of behavior and the kinematics, we find:

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{12}\frac{\partial^2 v_0}{\partial x \partial y} + A_{66}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x \partial y}\right) + B_{11}\frac{\partial^2 \theta_x}{\partial x^2} + B_{12}\frac{\partial^2 \theta_y}{\partial x \partial y} - \frac{4}{3h^2}E_{11}\left(\frac{\partial^2 \theta_x}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right)$$
$$+ B_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y}\right) - \frac{4}{3h^2}E_{12}\left(\frac{\partial^2 \theta_y}{\partial x \partial y} + \frac{\partial^3 w}{\partial x \partial y^2}\right) - \frac{4}{3h^2}E_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y} + 2\frac{\partial^3 w}{\partial x \partial y^2}\right) = 0, \quad (1)$$

$$A_{12}\frac{\partial^2 u_0}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial x \partial y}\right) + B_{12}\frac{\partial^2 \theta_y}{\partial x \partial y} + B_{22}\frac{\partial^2 \theta_y}{\partial y^2} + B_{66}\left(\frac{\partial^2 \theta_y}{\partial x^2} + \frac{\partial^2 \theta_x}{\partial x \partial y}\right)$$
$$- \frac{4}{3h^2}E_{12}\left(\frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^2 \theta_x}{\partial x \partial y}\right) - \frac{4}{3h^2}E_{22}\left(\frac{\partial^2 \theta_y}{\partial y^2} + \frac{\partial^3 w}{\partial y^3}\right) - \frac{4}{3h^2}E_{66}\left(\frac{\partial^2 \theta_y}{\partial x^2} + \frac{\partial^2 \theta_x}{\partial x \partial y} + 2\frac{\partial^3 w}{\partial x^2 \partial y}\right) = 0, \quad (2)$$

$$A_{44}\left(\frac{\partial \theta_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) + A_{55}\left(\frac{\partial \theta_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \frac{4}{h^2}D_{44}\left(\frac{\partial \theta_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) - \frac{4}{h^2}D_{55}\left(\frac{\partial \theta_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + q$$
$$- \frac{4}{h^2}\left(D_{44}\left(\frac{\partial \theta_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) + \left(\frac{\partial \theta_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \frac{4}{h^2}F_{44}\left(\frac{\partial \theta_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) - \frac{4}{h^2}F_{55}\left(\frac{\partial \theta_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)\right) = 0, \quad (3)$$

$$- A_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right) + B_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{12}\frac{\partial^2 u_0}{\partial x \partial y} + B_{66}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x \partial y}\right) + D_{11}\frac{\partial^2 \theta_x}{\partial x^2} + D_{12}\frac{\partial^2 \theta_y}{\partial x \partial y}$$
$$+ \frac{4}{h^2}D_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right) + D_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y}\right) - \frac{4}{3h^2}F_{11}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^2 \theta_x}{\partial x^2}\right) - \frac{4}{3h^2}F_{12}\left(\frac{\partial^3 w}{\partial x \partial x^2} + \frac{\partial^2 \theta_y}{\partial x \partial x}\right)$$
$$- \frac{4}{3h^2}F_{66}\left(\frac{\partial^2 \theta_y}{\partial x \partial y} + \frac{\partial^2 \theta_x}{\partial y^2} + 2\frac{\partial^3 w}{\partial x \partial y^2}\right) + \frac{4}{h^2}\left(D_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right) - \frac{4}{h^2}F_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right)\right)$$
$$- \frac{4}{3h^2}\left(E_{11}\frac{\partial^2 u_0}{\partial x^2} + E_{12}\frac{\partial^2 u_0}{\partial x \partial y} + E_{66}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x \partial y}\right) + D_{11}\frac{\partial^2 \theta_x}{\partial x^2} + D_{12}\frac{\partial^2 \theta_y}{\partial x \partial y} + F_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y}\right)\right.$$
$$\left. - \frac{4}{3h^2}H_{11}\left(\frac{\partial^2 \theta_x}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) - \frac{4}{3h^2}H_{12}\left(\frac{\partial^2 \theta_y}{\partial x \partial y} + \frac{\partial^3 w}{\partial x \partial y^2}\right) - \frac{4}{3h^2}H_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y} + 2\frac{\partial^3 w}{\partial x \partial y^2}\right)\right) = 0, \quad (4)$$

$$- A_{44}\left(\theta_y + \frac{\partial w}{\partial y}\right) + B_{22}\frac{\partial^2 u_0}{\partial y^2} + B_{12}\frac{\partial^2 u_0}{\partial x \partial y} + B_{66}\left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial^2 v_0}{\partial x^2}\right) + D_{22}\frac{\partial^2 \theta_y}{\partial y^2} + D_{12}\frac{\partial^2 \theta_x}{\partial x \partial y}$$
$$+ \frac{4}{h^2}D_{44}\left(\theta_y + \frac{\partial w}{\partial y}\right) + D_{66}\left(\frac{\partial^2 \theta_y}{\partial x^2} + \frac{\partial^2 \theta_x}{\partial x \partial y}\right) - \frac{4}{3h^2}F_{22}\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^2 \theta_y}{\partial y^2}\right) - \frac{4}{3h^2}F_{12}\left(\frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^2 \theta_x}{\partial x \partial y}\right)$$

$$
\begin{aligned}
&-\frac{4}{3h^2}F_{66}\left(\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{\partial^2\theta_y}{\partial x^2}+2\frac{\partial^3 w}{\partial x^2\partial y}\right)+\frac{4}{h^2}\left(D_{44}\left(\theta_y+\frac{\partial w}{\partial y}\right)-\frac{4}{h^2}F_{44}\left(\theta_y+\frac{\partial w}{\partial y}\right)\right)\\
&-\frac{4}{3h^2}\left(E_{22}\frac{\partial^2 u_0}{\partial y^2}+E_{12}\frac{\partial^2 u_0}{\partial x\partial y}+E_{66}\left(\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 u_0}{\partial x\partial y}\right)+F_{22}\frac{\partial^2\theta_y}{\partial y^2}+F_{12}\frac{\partial^2\theta_x}{\partial x\partial y}+F_{66}\left(\frac{\partial^2\theta_y}{\partial x^2}+\frac{\partial^2\theta_x}{\partial x\partial y}\right)\right.\\
&\left.-\frac{4}{3h^2}H_{22}\left(\frac{\partial^2\theta_y}{\partial y^2}+\frac{\partial^3 w}{\partial y^3}\right)-\frac{4}{3h^2}H_{12}\left(\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{\partial^3 w}{\partial x^2\partial y}\right)-\frac{4}{3h^2}H_{66}\left(\frac{\partial^2\theta_y}{\partial x^2}+\frac{\partial^2\theta_x}{\partial x\partial y}+2\frac{\partial^3 w}{\partial x^2\partial y}\right)\right)=0. \quad (5)
\end{aligned}
$$

5. Weighted least squares approximation constructed over the local support domain

To determine the shape functions of the Weighted Least Squares (WLS) approximation [18], it is necessary to assign to each point of interest X a set of neighboring points included in the local support domain. The WLS approximation at a point of interest X is given by:

$$
u(X)=\sum_{j=1}^{m}P_j(X)a_j=\langle P(X)\rangle\{a\}. \quad (6)
$$

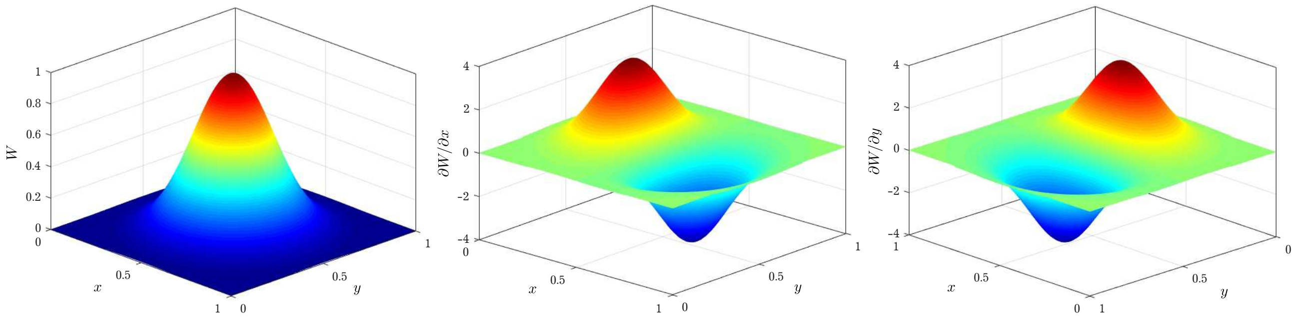
The coefficient vector $\{a\}$ is given by:

$$
\{a\}^T=\langle a_0\ a_1\ a_2\ldots\ a_m\rangle.
$$

We can prove the polynomial $P(x)$ using Pascal's triangle and choosing a complete base. We minimize the discrete weighted norm $J(a)$ representing the square of the distance between the approximation at any point X and the set of nodal values:

$$
\begin{aligned}
J&=\sum_{i=1}^{n}W(q_i)[u(X)-u_i]^2\\
&=\sum_{i=1}^{n}W(q_i)[P^T(X_i)a-u_i]^2,
\end{aligned}
$$

where $W(q_i)$ is the weight function, u_i are the nodal values of the unknown field, n is the number of points in the domain of influence of the point X. The chosen weight function $W(q_i)$ is Gaussian, and this function and its derivatives are illustrated in Figure 1.



a (W) ***b*** $(\partial W/\partial x)$ ***c*** $(\partial W/\partial y)$

Fig. 1. Weight function and its derivatives.

Minimizing $J(a)$ allows us to write in relation to the coefficient $\{a\}$:

$$
\frac{\partial J}{\partial a}=0,
$$

$$
\begin{cases}
\sum\limits_{i=1}^{n}W(q_i)2P_1(X_i)[P^T(X_i)a-u_i]=0,\\
\sum\limits_{i=1}^{n}W(q_i)2P_2(X_i)[P^T(X_i)a-u_i]=0,\\
\ldots,\\
\sum\limits_{i=1}^{n}W(q_i)2P_m(X_i)[P^T(X_i)a-u_i]=0,
\end{cases}
$$

$$\begin{cases} \sum_{i=1}^{n} W(q_i) P_1(X_i) P^T(X_i) a = \sum_{i=1}^{n} W(q_i) P_1(X_i) u_i, \\ \sum_{i=1}^{n} W(q_i) P_2(X_i) P^T(X_i) a = \sum_{i=1}^{n} W(q_i) P_2(X_i) u_i, \\ \dots, \\ \sum_{i=1}^{n} W(q_i) P_m(X_i) P^T(X_i) a = \sum_{i=1}^{n} W(q_i) P_m(X_i) u_i, \end{cases}$$

The above equations can be written in compact form:

$$[A]\{a\} = [B]\{U_n\},$$

where $\{U_n\}$ is the vector that assembles the nodal unknowns of the domain of influence. $[A]$ is the matrix of moments whose size is equal to $(m \times m)$ and given by:

$$[A] = \sum_{i=1}^{n} W(q_i) P(X_i) P^T(X_i),$$

$[B]$ is given as follows:

$$[B] = \sum_{i=1}^{n} W(q_i) P(X_i).$$

The coefficients of the vector $\{a\}$ are determined from:

$$\{a\} = [A]^{-1}[B]\{U_n\}.$$

Thanks to equation (6) and the integration of the constant coefficients calculated therein, the WLS approximation is given by the following relationship:

$$\begin{aligned} u(X) &= \langle P(X) \rangle [A]^{-1}[B]\{U_n\} \\ &= \sum_{i=1}^{n} \phi_i(X) u_i \\ &= \langle \phi(X) \rangle \{U_n\}, \end{aligned}$$

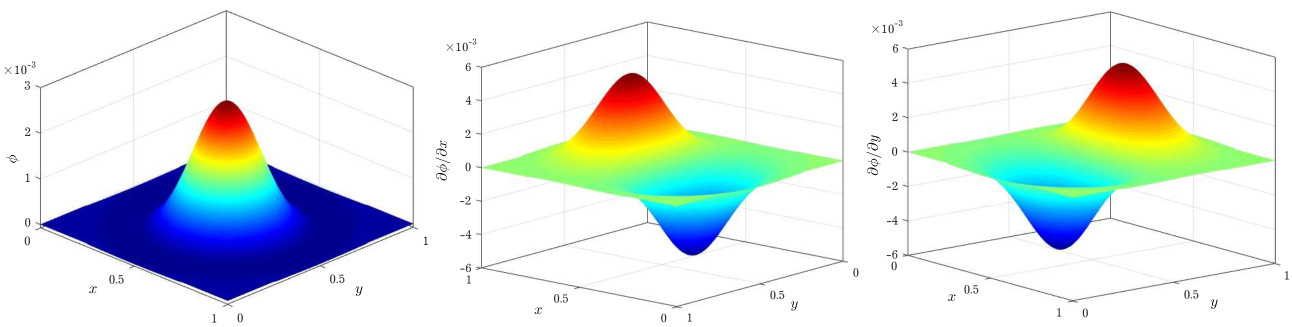
where the vector of shape functions can be written as follows:

$$\langle \phi(X) \rangle = \langle P(X) \rangle [A]^{-1}[B].$$

The formulas for the first and second derivatives of the shape function with respect to spatial coordinates are as follows:

$$\begin{cases} \langle \phi(X) \rangle = \langle P(X) \rangle [A]^{-1}[B], \\ \langle \phi_{,k}(X) \rangle = \langle P_{,k}(X) \rangle [A]^{-1}[B], \\ \langle \phi_{,kl}(X) \rangle = \langle P_{,kl}(X) \rangle [A]^{-1}[B]. \end{cases} \tag{7}$$

The shape function and its derivatives are given in Figure 2.



a (ϕ) ***b*** $(\partial\phi/\partial x)$ ***c*** $(\partial\phi/\partial y)$

Fig. 2. Shape function and its derivatives.

6. TSDT-based meshless approach

By replacing the approximations of the shape functions (7) in the local equilibrium equations (1)–(5), we obtain:

$$[K^e]\{U^e\} = \{F^e\},$$

where the components of the matrix $[K]$ are as follows:

$$K_{11}^e = A_{11}\frac{\partial^2\phi_i}{\partial x^2} + A_{66}\frac{\partial^2\phi_i}{\partial y^2},$$

$$K_{12}^e = A_{12}\frac{\partial^2\phi_i}{\partial x\partial y} + A_{66}\frac{\partial^2\phi_i}{\partial x\partial y} + B_{66}\frac{\partial^2\phi_i}{\partial x\partial y},$$

$$K_{13}^e = -\frac{4}{3h^2}E_{11}\frac{\partial^3\phi_i}{\partial x^3} - \frac{4}{3h^2}E_{12}\frac{\partial^3\phi_i}{\partial x\partial y^2} - \frac{8}{3h^2}E_{66}\frac{\partial^3\phi_i}{\partial x\partial y^2} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial x^2},$$

$$K_{14}^e = B_{11}\frac{\partial^2\phi_i}{\partial x^2} - \frac{4}{3h^2}E_{11}\frac{\partial^2\phi_i}{\partial x^2} + B_{66}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{3h^2}E_{12}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial y^2},$$

$$K_{15}^e = B_{12}\frac{\partial^2\phi_i}{\partial x^2} + B_{66}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{12}\frac{\partial^3\phi_i}{\partial x\partial y^2} - \frac{4}{3h^2}E_{12}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial y^2},$$

$$K_{21}^e = A_{12}\frac{\partial^2\phi_i}{\partial x\partial y} + A_{66}\frac{\partial^2\phi_i}{\partial x\partial y},$$

$$K_{22}^e = A_{22}\frac{\partial^2\phi_i}{\partial y^2} + A_{66}\frac{\partial^2\phi_i}{\partial x^2},$$

$$K_{23}^e = \frac{4}{3h^2}E_{12}\frac{\partial^3\phi_i}{\partial x^2\partial y} - \frac{4}{3h^2}E_{22}\frac{\partial^3\phi_i}{\partial y^3} - \frac{8}{3h^2}E_{66}\frac{\partial^3\phi_i}{\partial x^2\partial y},$$

$$K_{24}^e = B_{12}\frac{\partial^2\phi_i}{\partial x\partial y} + B_{66}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{12}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial x\partial y},$$

$$K_{25}^e = B_{22}\frac{\partial^2\phi_i}{\partial y^2} + B_{66}\frac{\partial^2\phi_i}{\partial x^2} - \frac{4}{3h^2}E_{22}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial x^2},$$

$$K_{31}^e = 0,$$

$$K_{32}^e = 0,$$

$$K_{33}^e = A_{44}\frac{\partial^2\phi_i}{\partial y^2} + A_{55}\frac{\partial^2\phi_i}{\partial x^2} - \frac{4}{h^2}D_{44}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{h^2}D_{55}\frac{\partial^2\phi_i}{\partial x^2}$$
$$- \frac{4}{h^2}\left(D_{44}\frac{\partial^2\phi_i}{\partial y^2} + D_{55}\frac{\partial^2\phi_i}{\partial x^2} - \frac{4}{h^2}F_{44}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{h^2}F_{55}\frac{\partial^2\phi_i}{\partial x^2}\right),$$

$$K_{34}^e = A_{55}\frac{\partial\phi_i}{\partial x} - \frac{4}{h^2}D_{55}\frac{\partial\phi_i}{\partial x} - \frac{4}{h^2}\left(D_{55}\frac{\partial\phi_i}{\partial x} - \frac{4}{h^2}F_{55}\frac{\partial\phi_i}{\partial x}\right),$$

$$K_{35}^e = A_{44}\frac{\partial\phi_i}{\partial y} - \frac{4}{h^2}D_{44}\frac{\partial\phi_i}{\partial y} - \frac{4}{h^2}\left(D_{44}\frac{\partial\phi_i}{\partial y} - \frac{4}{h^2}F_{44}\frac{\partial\phi_i}{\partial y}\right),$$

$$K_{41}^e = -B_{11}\frac{\partial^2\phi_i}{\partial y^2} + B_{12}\frac{\partial^2\phi_i}{\partial x\partial y} + B_{66}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{3h^2}\left(E_{11}\frac{\partial^2\phi_i}{\partial x^2} + E_{12}\frac{\partial^2\phi_i}{\partial x\partial y} + E_{66}\frac{\partial^2\phi_i}{\partial y^2}\right),$$

$$K_{42}^e = B_{66}\frac{\partial^2\phi_i}{\partial x\partial y} - \frac{4}{3h^2}E_{66}\frac{\partial^2\phi_i}{\partial x\partial y},$$

$$K_{43}^e = -A_{55}\frac{\partial\phi_i}{\partial x} + D_{55}\frac{\partial\phi_i}{\partial x} - \frac{4}{3h^2}F_{11}\frac{\partial^3\phi_i}{\partial x^3} - \frac{4}{3h^2}F_{12}\frac{\partial^3\phi_i}{\partial x\partial y^2} - \frac{8}{3h^2}F_{66}\frac{\partial^3\phi_i}{\partial x\partial y^2}$$
$$+ \frac{4}{h^2}\left(D_{55}\frac{\partial\phi_i}{\partial x} - \frac{4}{h^2}F_{55}\frac{\partial\phi_i}{\partial x}\right) - \frac{4}{3h^2}\left(-\frac{4}{3h^2}H_{11}\frac{\partial^3\phi_i}{\partial x^3} - \frac{4}{3h^2}H_{12}\frac{\partial^3\phi_i}{\partial x\partial y^2} - \frac{8}{3h^2}H_{66}\frac{\partial^3\phi_i}{\partial x\partial y^2}\right),$$

$$K_{44}^e = -A_{55}\phi_i + D_{11}\frac{\partial^2\phi_i}{\partial x^2} + \frac{8}{h^2}D_{55}\phi_i + D_{66}\frac{\partial^2\phi_i}{\partial y^2} - \frac{4}{3h^2}F_{11}\frac{\partial^2\phi_i}{\partial x^2}$$

$$- \frac{4}{3h^2} F_{66} \frac{\partial^2 \phi_i}{\partial y^2} - \frac{16}{h^4} F_{55} \phi_i - \frac{4}{3h^2} F_{66} \frac{\partial^2 \phi_i}{\partial y^2} + \frac{16}{9h^4} H_{11} \frac{\partial^2 \phi_i}{\partial x^2} + \frac{16}{9h^4} H_{66} \frac{\partial^2 \phi_i}{\partial y^2},$$

$$K_{45}^e = D_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + D_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} F_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} F_{66} \frac{\partial^2 \phi_i}{\partial x \partial y}$$
$$- \frac{4}{3h^2} \left(F_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + F_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} H_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} H_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} \right),$$

$$K_{51}^e = B_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + B_{22} \frac{\partial^2 \phi_i}{\partial y^2} + B_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} \left(E_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + E_{22} \frac{\partial^2 \phi_i}{\partial y^2} + E_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} \right),$$

$$K_{52}^e = B_{66} \frac{\partial^2 \phi_i}{\partial x^2} - \frac{4}{3h^2} E_{66} \frac{\partial^2 \phi_i}{\partial x^2},$$

$$K_{53}^e = -A_{44} \frac{\partial \phi_i}{\partial y} + \frac{4}{h^2} D_{44} \frac{\partial \phi_i}{\partial y} - \frac{4}{3h^2} F_{12} \frac{\partial^3 \phi_i}{\partial x^2 \partial y} - \frac{4}{3h^2} F_{22} \frac{\partial^3 \phi_i}{\partial y^3} - \frac{8}{3h^2} F_{66} \frac{\partial^3 \phi_i}{\partial x^2 \partial y}$$
$$+ \frac{4}{h^2} \left(D_{44} \frac{\partial \phi_i}{\partial y} - \frac{4}{h^2} F_{44} \frac{\partial \phi_i}{\partial y} \right) - \frac{4}{3h^2} \left(- \frac{4}{3h^2} H_{12} \frac{\partial^3 \phi_i}{\partial x^2 \partial y} - \frac{4}{3h^2} H_{22} \frac{\partial^3 \phi_i}{\partial y^3} - \frac{8}{3h^2} H_{66} \frac{\partial^3 \phi_i}{\partial x^2 \partial y} \right),$$

$$K_{54}^e = D_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + D_{66} - \frac{4}{3h^2} F_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} F_{66} \frac{\partial^2 \phi_i}{\partial x \partial y}$$
$$- \frac{4}{3h^2} \left(F_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} + F_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} H_{12} \frac{\partial^2 \phi_i}{\partial x \partial y} - \frac{4}{3h^2} H_{66} \frac{\partial^2 \phi_i}{\partial x \partial y} \right),$$

$$K_{55}^e = -A_{44} \phi_i + D_{22} \frac{\partial^2 \phi_i}{\partial y^2} + \frac{4}{h^2} D_{44} \phi_i + D_{66} \frac{\partial^2 \phi_i}{\partial x^2} - \frac{4}{3h^2} F_{22} \frac{\partial^2 \phi_i}{\partial y^2} - \frac{4}{3h^2} F_{66} \frac{\partial^2 \phi_i}{\partial x^2}$$
$$+ \frac{4}{h^2} \left(D_{44} \phi_i - \frac{4}{h^2} D_{44} \phi_i \right) - \frac{4}{3h^2} \left(F_{22} \frac{\partial^2 \phi_i}{\partial y^2} + F_{66} \frac{\partial^2 \phi_i}{\partial x^2} - \frac{4}{3h^2} H_{22} \frac{\partial^2 \phi_i}{\partial y^2} - \frac{4}{3h^2} H_{66} \frac{\partial^2 \phi_i}{\partial y^2} \right),$$

and $\{F^e\}$ is given by:

$$\{F^e\} = \begin{Bmatrix} 0 \\ 0 \\ q \\ 0 \\ 0 \end{Bmatrix}.$$

After assembly, the problem is written as:

$$[K]\{U\} = \{F\}.$$

7. Numerical results and discussions

7.1. Bending analysis of a homogeneous skew plate

We consider a homogeneous square plate with side $a = 20\,\text{cm}$. The deformed plate after deflection is shown in Figure 3.

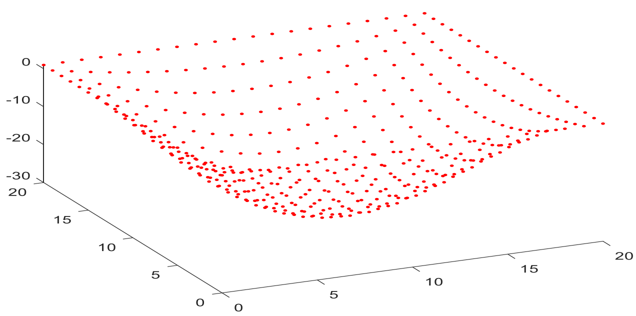


Fig. 3. Deflection for a square plate.

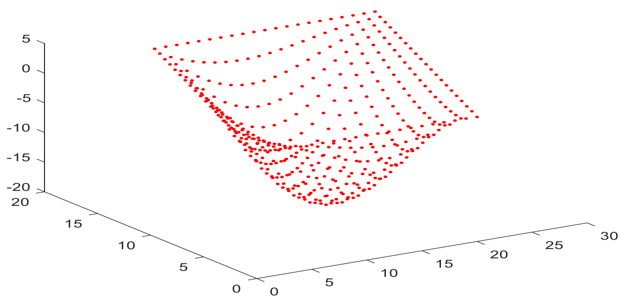


Fig. 4. Deflection for a plate with a skew angle of 30°.

The results of the proposed model are compared with those of Ref. [19], as shown in Table 1.

As a second example, we consider the same plate with a skew angle of 30°. The deformed configuration after deflection is shown in Figure 4.

Table 1. Non-dimensional displacement for a square plate.

Model	$\frac{wD\,10^3}{q_0 a^4}$
Present study	4.06
FDM (Morley 1962) Ref. [19]	4.06

Table 2. Non-dimensional displacement for a plate with a skew angle of 30°.

Model	$\frac{wD\,10^3}{q_0 a^4}$
Present study	2.56
FDM (Morley 1962) Ref. [19]	2.54

The results of the proposed model are compared with those of [19], as shown in Table 2.

7.2. Bending analysis of a FGM plate

In this part, the results of the bending analysis of the simply supported perfect and porous FG-plate are presented. All results of bending analysis are in the dimensionless form as:

$$\bar{w} = \frac{10E_c h^3}{q_0 a^4} w\left(\frac{a}{2}, \frac{b}{2}\right).$$

Table 3. Comparison of the nondimensional stress and displacements of Al/Al_2O_3 square plates ($a/h = 10$).

Exponent N	Model	$\bar{w}$
1	Tounsi et al. [20]	0.5888
	Present	0.586
2	Tounsi et al. [20]	0.7573
	Present	0.754
4	Tounsi et al. [20]	0.882
	Present	0.88

Table 3 shows the dimensionless center deflections ($\bar{w}$) of Al Al/Al_2O_3 P-FG plates under sinusoidal loads, the obtained results are compared with those given by Tounsi et al. [20] using an analytical solution.

8. Conclusion

The present research proposes a meshless method based on the WLS approximation to simulate the bending behavior of an FG skew plate. We obtained excellent results in comparison with Morley (1962) [19] and Tounsi (2020) [20]. This study opens up very important perspectives for further verification of FGM skew plates. Work is in progress to address the different boundary conditions and non-linear behavior of FGM skew plates.

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Безсітковий підхід на основі теорії зсувної деформації третього порядку для моделювання косої пластини

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Беручи до уваги нові межі технології матеріалів для вдосконалення конструкційних матеріалів, досліджуємо аналіз вигину косої пластини з функціонально-градієнтного матеріалу (FGM) за допомогою безсіткового підходу на основі теорії зсувної деформації третього порядку (TSDT). Припускаємо, що розподіл матеріалу функціонально градуйований за товщиною косої пластини. Запропонований підхід використовує як теорію суміші, так і безсітковий метод. Теорія суміші використовується для оцінки ефективних властивостей матеріалу для косої пластини.

Ключові слова: *безсітковий метод; теорія зсувної деформації третього порядку; функціонально-градієнтний матеріал; коса пластина.*