ADVANCES IN CYBER-PHYSICAL SYSTEMS

Vol. 9, No. 1, 2024

THE THEORY OF STRUCTURING MULTIFUNCTIONAL ELEMENTS OF COMPLEX SYSTEMS

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https://doi.org/10.23939/acps2024.01.075

Submitted on 31.03.2024

Abstract**: This article considers the problem of formalization of elements and binary connections of multifunctional data, which are components of cyber-physical systems. The concept of a free and active element of a complex system and its classes relatively interact with environment resources and consumer information messages. Six attributes of binary relationship elements such as information, material, energy, optics, management, and general have been classified. The table shows four classes of active interaction between elements of complex systems of signs. The concept of an element of a complex system has been defined. The basics of the theory of solving the task of structuring multifunctional data have been outlined, which made it possible to improve the system characteristics of the components of cyber-physical systems due to the reduction of structural, hardware, and time complexity.**

Index Terms**: System Element, Communication, Interaction, Environment, Data Structuring.**

I. INTRODUCTION

An important issue of the theory of complex systems is the decomposition of their components, the systematization of informational, material, and interactive control relationships [1-8].

The fundamental provisions of the complex systems theory are closely interrelated with the methodology and technology of building multifunctional data movement models. One of the promising areas for solving this problem is the theory and formalization of data movement matrix models [2, 9].

II. LITERATURE REVIEW AND PROBLEM STATEMENT

The concept of multifunctional data covers a wide class of information message types in the environment of physical, logical and virtual data. This includes analog and digital data at the output of sensors and analog-to-digital converters, digital data at the output of compression encoders, encryption and protection of information from errors, output data of special processors and controllers of digital information processing, modulated and manipulated signals of data transmission systems, physical and logical data databases and knowledge bases, alphanumeric data, graphic information.

The concept of structuring multifunctional data in a broad aspect covers the theory of systems and the interaction of their components, the theory of cyber-physical systems and the architecture of computerized systems and is connected with the processes of development and improvement of information systems.

III. SCOPE OF WORK AND OBJECTIVES

This work is aimed at formalizing elements and binary relations of multifunctional data in complex systems. To achieve the goal, the attributes of the elements of binary relations are defined and the concept of a free and active element of a complex system is proved.

IV. PRESENTATION OF THE MAIN PROVISIONS

An advantage of the current state of the theory is the developed information technology for formalizing the construction of data movement-derived models in single-level and multi-level distributed computer systems. At the same time, this technology is based on the components of colored Petri nets of such types as: ⊙- a source of information, ○ - information processing point, ⊗ - information approval point, as well as on a two-dimensional matrix of incidents (Fig. 1). The components with binary information links, which are described by four parameters: a – the beginning of the operation; b – the duration of the operation; c – the type of the operation; $P-V$ – the cost of the operation (P - profits, V - expenses) are located in the nodes of the matrix model.

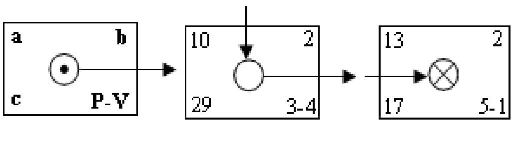

Fig. 1. Components of colored Petri nets

The functional disadvantage of matrix modeling of information flows in complex systems is the lack of decomposition of each attribute to the level of free and active system elements. This situation complicates the possibility of a deeper formalized description of the complex system components and a better optimization design of this class of systems. That is, an element of complex systems of this technology is considered a cybernetic attribute of the "black box" type [9, 10].

There is no need to learn the internal structure and components of the system, which can be quite complex for operational analysis if the "black box" model of the cybernetic system is used (Fig. 2.).

Fig. 2. "Black box" model of an element

The advantage of such a system element (SE) model is the possibility of the appropriate determination of its characteristics by feeding functional or test signals to the inputs and analyzing its initial response to external perturbations.

At the same time, the significant disadvantages of such a model are as follows: it is impossible to improve the architecture of the SE, appropriately divide it into separate components, as well as to optimize its functions.

Data structuring theory (DST) should be based on the fundamental provisions of the theory of complex systems. One of the peculiarities of the concept of a “system” is divisibility [11]. That is, the system is structurally divided into a system element, a subsystem, and a system.

System element (SE) is an elementary component of a subsystem and a system of any complexity, which can be denoted by a symbol $\boxed{e}$.

The element consists of two components: a nucleus – e and a shell, in biology □ – a membrane, through which the nucleus of the element interacts with similar to it or functionally different from it.

Such an element of complex system theory is called free if it lacks information and energy relationships, as well as material or managerial ones with other elements of complex systems and the external environment in general.

The external environment is considered to be the two "spaces" or "conglomerates" or "ensembles" that correspond to the resources I and consumers or users I of multifunctional information data (Fig. 3).

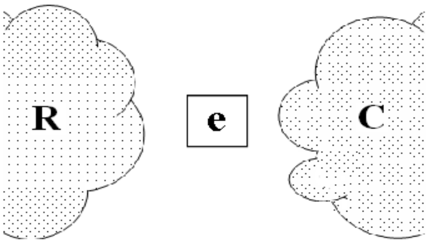

Fig.3 Interaction of a system element
with the environments of resources I and users I

Thus, without claiming the absolute completeness of the classification of the external interactions of SE, based on the formulated concept of SE, the following six forms of interaction of SE with R and C external environments, according to the attributes of symbolism, can be distinguished (Table 1).

Table 1

Attributes of SE relationships

Symbol of the interaction of SE attribute	Content, form, essence, and principle of interaction
⟶	informational
——▷	material
⤳	energy, electromagnetic
·········➤	optical and energy
← – – – – →	managerial
↭	any of the relationships is a generalized attribute of SE

Using the attribute of the SE relationship, a stricter concept of an active SE can be formulated: it is a SE, which has no more than one invariant relationship with the external environment (Fig. 4).

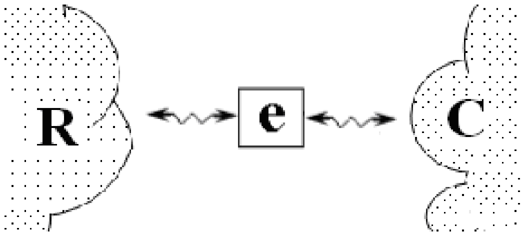

Fig. 4 Active SE

Thus, it is possible to systematize the set of existing Ses, i.e., elements whose cores interact with no more than one attribute of external RC environments (Table 2).

Table 2

Binary interactions of active Ses

№	Interaction pairs	Attributes of binary interactions of Ses $\mathbf{R}\leftrightsquigarrow\mathbf{e}\leftrightsquigarrow\mathbf{C}$
1	2	3
1	$\mathbf{e}\longrightarrow\mathbf{C}$	$\boxed{e}\rightarrow$, $\boxed{e}\!-\!\triangleright$, $\boxed{e}\dashrightarrow$, $\boxed{e}\cdots\!\!\rightarrow$, $\boxed{e}\leadsto$
2	$\mathbf{R}\longleftarrow\mathbf{e}$	$\leftarrow\boxed{e}$, $\triangleleft\!-\boxed{e}$, $\leftarrow\!\cdots\boxed{e}$, $\dashleftarrow\boxed{e}$, $\leftsquigarrow\boxed{e}$
3	$\mathbf{e}\longleftarrow\mathbf{C}$	$\boxed{e}\leftarrow$, $\boxed{e}\triangleleft\!-$, $\boxed{e}\dashleftarrow$, $\boxed{e}\leftarrow\!\cdots$, $\boxed{e}\leftsquigarrow$
4	$\mathbf{R}\longrightarrow\mathbf{e}$	$\rightarrow\boxed{e}$, $-\!\triangleright\boxed{e}$, $\dashrightarrow\boxed{e}$, $\cdots\!\!\rightarrow\boxed{e}$, $\leadsto\boxed{e}$

Continuation of Table 2

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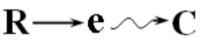
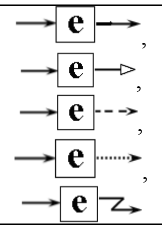
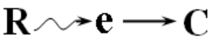
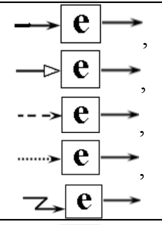
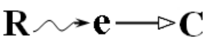
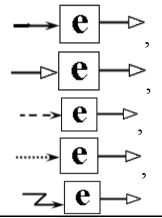
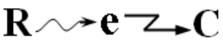
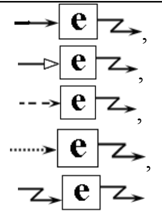
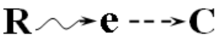
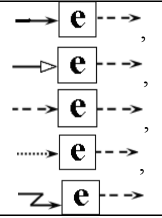
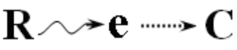
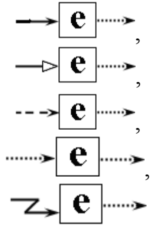
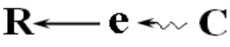
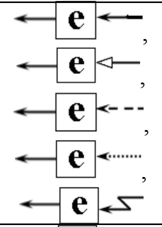
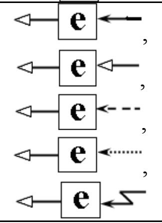
5		
6		
7		
8		
9		
10		
11		
12		

Continuation of Table 2

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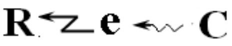
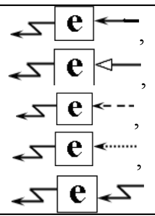
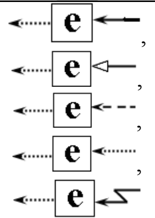
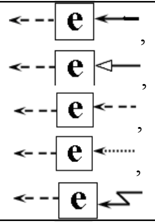
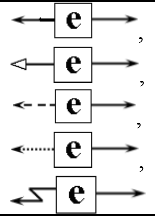
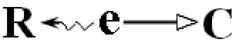
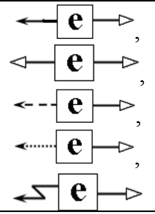
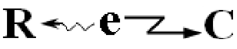
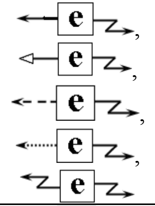
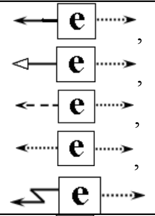
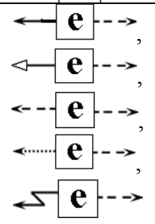
13		
14		
15		
16		
17		
18		
19		
20		

As is seen in Table 2, in the above classification of SEs, classes are distinguished according to functional features:

- functional incomplete SEs: generating (1, 2) and absorbing (3, 4);
- transit C (5-10), R (11-15) SEs in environments R and C;
- invariant fully-functional SEs (16-20).

Given systematization of the SEs determines the complexity of the structuring processes and their formalized descriptions in different fields of knowledge.

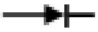
Such areas of knowledge of DST are undoubtedly the theories of signals, information, random processes, coding, models of information sources and data movement, graphs, numbers, mathematics, as well as audio and video images, sensors, and circuitry [12].

Based on the functionally defined concept of a SE, its implementation in certain areas of knowledge can be singled out:

- quantum of light;

● – dot – a symbol of black and white graphic images that form symbols: numbers, letters, signs, lines, functions, matrices, schemes, structures, etc .;

○ – pixel – a symbol of a color image, which consists of three color components (blue, green, red);

 – logic gate, an elementary component of microelectronics, PLD and computers;

 – neuron (receptor) of biological systems;

 – components of chemical compounds (atom, molecule);

A, T, G, C – nucleotides – components of DNA;

- gene component of DNA;

DNA – macromolecule of living organisms;

– components of electrical circuits (I – current, U – voltage, C – capacity, L –inductance, G – current and voltage source);

– unit of time (µs, MLS, s);

p, t, M – pressure, temperature, mass;

- mathematical symbols – digit, function, matrix, differential, etc.;
- geometric symbols – line, circle, triangle, square, ellipse, cube, etc.

The list of applicants for the formal concept of SE in various fields of knowledge can be encyclopedically extended, but the following should be postulated: each of the named and unnamed applicants for the formal concept of SE can be unambiguously described in terms of computer science and systems engineering based on a single integrated assessment of its information capacity, that is, its entropy and multifunctional description in the form of structured data.

For example, the estimate of the structural capacity of graphical attributes, by which almost 98% of all information knowledge is presented in the form of alphanumeric and graphical data streams, including color and holographic images, can be quantified according to the theory of structural complexity [12, 13].

The coefficient of structural complexity (1):

$$k_c = \sum_{i=1}^{n} \alpha_i P_i \,, \qquad (1)$$

includes α_i - expert estimates of the complexity coefficient of elements and components of structured data, which are informative parameters of structures` attributes.

Also taking into account the functional and informative characteristics of structured data $\sum_{j=1}^{m} f_i$, we will obtain a quantitative assessment of the structural complexity of the representation of the elements of a complex system:

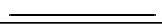
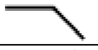
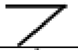
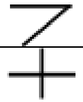
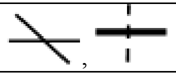
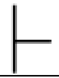
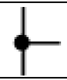
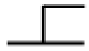
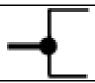
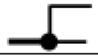
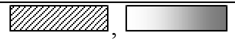
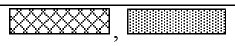
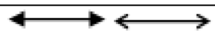
$$k_e = K \cdot \Big(\sum_{j=1}^{m} f_j \Big/ \sum_{i=1}^{n} \alpha_i P_i\Big) \Rightarrow \max \,, \qquad (2)$$

where K - level identifier ($K = n$,… – respectively for n-level images).

Table 3 shows the symbols of structured data components and their complexity coefficients α_i .

Table 3

Systematization of expert assessments for structural complexity of graphical structure components

Letter / Value	Symbols	α_i
1	2	3
l / Line		1
		1,5
		1,1
		1,2
		1,2
		1,7
P / Bend		2
		2,2
		2,2
x / Intersection		3
		3,1
d / Touching		2
		2,2
r / Branching		4
		6,2
		4,2
h / Method of filling		2
		2
Z / Directed line		2
		3
		2,4

Continuation of Table 2

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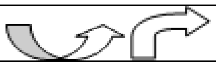
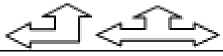
		2,5
		3,4
		3,5
b / Letter	Aa…Яя, …, *Aa…Яя*, Aa…Яя, …, *Aa…Яя*,	8-10
	Aa…Zz, …, *Aa...Zz* Aa…Zz, …, *Aa...Zz*	8-10
c / Digit	1, 2,…0….	4
i / Index	1, 2,…0, a, A	4
s / Symbol	©, ®, π, ψ, ω, &, %, @, §, Θ, №, ∑, ∫, ∞	4
	☺, ☼ ♫, µ, $, *, €, ∏, ♀, ♂, ♠, ♫…	4
n / Sign	+, -, <, >, =, ±, ≡, ≈..	2
	≠, ≤, ≥, (, “, {, !, ?...	2

Thus, the maximum increase in k_e value is a comparative characteristic of different implementations of the device structures and multifunctional data (MFD). For example, at a given structural complexity k_c, the functionality of the structures is expanded, or at a given functionality, structural complexity is reduced. This can lead to changes in target characteristics by reducing hardware, time, or computational complexities.

Functional and informative characteristics of the device structure can be represented by the following estimates:

1. Functional completeness of the device inputs-outputs, which is determined by the total estimate

$$f_j = \sum_{i=1}^{n} \beta_i \cdot f_{input} + \sum_{i=1}^{m} \lambda_i \cdot f_{output}\ , \qquad (3)$$

where f_j - functional information characteristics of the device structure, β, λ - coefficients of input-output functions informativity, m, n - the number of inputs-outputs, f_{input}, f_{output} - input-output functions (e.g., input/output channel, input/output n/m-bit buses, synchronization input, crystal selection c/s, power supply $+/-$);

2. The degree of availability of certain functions in the formalized structures of data movement derivative models. An example of the calculation is carried out according to the expression:

$$f_j = \sum_{j=1}^{m} f_i\ . \qquad (4)$$

3. Evaluation of structural complexity is based on entropy characteristics of input calculation and output information flows.

The entropy estimates are calculated according to coding (R. Hartley), probabilistic (K. Shannon) and correlation (Y.Nykolaychuk) information entropy measures [11, 14-16]:

- R. Hartley: $H = n \cdot \hat{E}\left[\log_2 S\right] = n \cdot]\log_2 S[\ ,$ (5)
- K. Shannon: $H = -k \sum_{j=0}^{S} p_j \log p_j\ ,$ (6)
- Y. Nykolaychuk:

$$I_x = n \cdot \hat{\mathrm{E}}\left[\frac{1}{2}\log_2 \frac{1}{m} \times \sum_{j=1}^{m}\left(D_x^2 - R_{xx}^2(j)\right)\right], \qquad (7)$$

where H – the amount of information; S – the number of independent equally probable states of the information source (IS); n – the number of samples; $\hat{E}[\bullet]$ – an integer-valued function with rounding to a larger whole; k – a positive coefficient that takes into account the base of the logarithm; p_j – the probability of the s_j-th state of the discrete IS; S – the number of independent IS states; $\overset{\circ}{x}_i = x_i - M_x$ – centered values of the data array; $D_x = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - M_x\right)^2$ – dispersion x_i; $M_x = \frac{1}{n}\sum_{i=1}^{n} x_i$ – mathematical expectation; $R_{xx}(j) = \frac{1}{n}\sum_{i=1}^{n} \overset{\circ}{x}_i \cdot \overset{\circ}{x}_{i+j}$ – autocorrelation function (ACF); m – the number of points of the function $R_{xx}(j)$ on the correlation interval;

Thus, the estimation of the entropy functional complexity looks as follows:

$$f_j = I_{input} + I_X + I_{output}\ , \qquad (8)$$

where $I_{input}, I_X, I_{output}$ - quantification of entropy at the input, during the transformation and at the output of the structure, respectively.

Based on the proposed coefficient of structural complexity, a method for estimating the structural complexity of the attributes of multi-level DCS data movement models, as well as alphanumeric and functional symbols, which are displayed on monitors and hard copies of physical data, is developed [12].

V. EXAMPLES OF ESTIMATING THE STRUCTURAL COMPLEXITY OF ELEMENTARY UNITS OF COMPUTER TECHNOLOGY

The above systematization of structural elements formed the basis of the methodology for estimating the structural complexity of logical elements and structures of microelectronic devices in computer technology.

The logical element is the most elementary component of computer technology. Therefore, the estimation of the structural complexity of any component of DCSs (ADC, adders, multipliers, entropy measurement devices, processors, special-purpose processors, memory, etc.) is based on determining the structural complexity of the logical elements, taking into account the relationships between them.

Table 4

Informative characteristics of the logical element structural complexity of computer technology

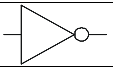
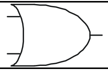
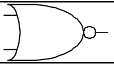
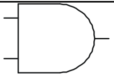
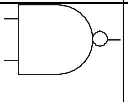
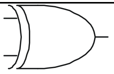
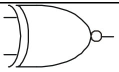
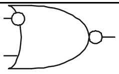
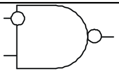
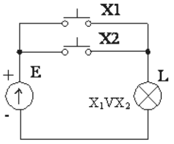
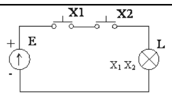
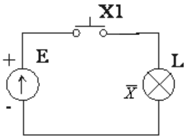
№	Function, type of a component	Symbolic graphic notation	Criterion parameters for structural complexity $l, P, x, d, r, h, z, b, c, i, n$	Total estimate of structural complexity
1	Denial		2,3,-,3,-,-.-,-,-,-,-	14,8
2	Disjunction		1,2,-,3,-,-,-,-,-,-.-	11,6
3	Denial of disjunction		2,2,-,4,-,-,-,-,-,-,-	14,8
4	Conjunction		1,2,-,3,-,-,-,-,-,-,-	11,2
5	Denial of conjunction		2,2,-,4,-,-,-,-,-,-,-	14,4
6	Equivalency		2,2,-,3,-,-,-,-,-,-,-	12,8
7	Denial of equivalency		3,2,-,4,-,-,-,-,-,-,-	16
8	Implication		3,2,-,6,-,-,-,-,-,-	20
9	Prohibition		3,2,-,6,-,-,-,-,-,-	19,6

Table 5

Symbolic notation of graphic images in electrical automatics

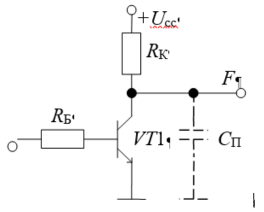
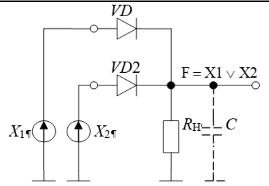
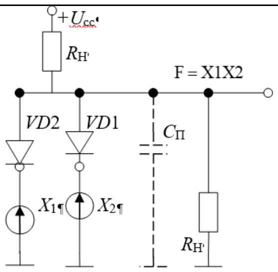
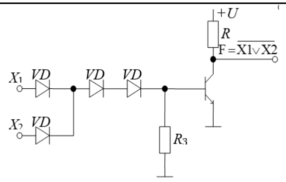
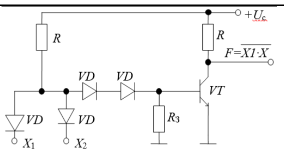
№	Function, type of a component	Symbolic graphic notation	Total estimate of structural complexity
1	Disjunction		130
2	Conjunction		111
3	Denial		70

Based on criterion (1) and the data in Table 2, the structural complexity of the microelectronic components of the processors is estimated (Table 4-7).

The following tables show examples of estimating the structural complexity and informativity of various graphic images in electrical automatics, and crystals of logical and microelectronic components.

Table 6

Nonlinear diode transistor elements

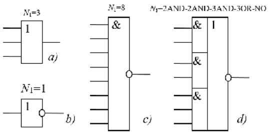
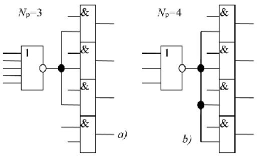
№	**Function, type of a component**	**Symbolic graphic notation**	**Total estimate of structural complexity**
1	Element NO		180
2	Element OR		251
3	Element AND		315
4	Element OR-NO		368
5	Element AND-NO		393

The possibility of structuring information and transition from one structure to another provides a basis for efficient analysis and evaluation, processing and regrouping.

The developed foundations of the DST can be the basis for the development of modern methods and tools for identification and optimization of representation, transformation and use of multifunctional information messages brought to the state of the structured data.

Table 7

Symbolic notation of logical element crystals

№	Function, type of a component	Symbolic graphic notation	Total estimate of structural complexity
1	Examples of coefficient N_1 values		a) – 28 b) – 23 c) – 42 d) – 74
2	Majority elements		a) – 177 b) – 232

VI. CONCLUSION

The basics of solving the problem of the theory of multifunctional data structuring were outlined, and the concept of an element of a complex system and examples of their implementation in various fields of knowledge and real-time systems were defined and formulated. The following examples of estimating the structural complexity and informativity of graphic images in electrical automatics, and crystals of logical and microelectronic components were given. The presented theory made it possible to determine the quantitative assessment of the structural complexity of multifunctional data, which enabled improvement of the system characteristics of the components of computerized systems and their effective usage of in cyber-physical systems.

References

[1] Nykolaichuk Ya.M, Pitukh I.R., Vozna N.Ya. (2008) Theory models of motion data distributed computer systems: Monograph. Ternopil: Terno-graph, 216 p.

[2] Nykolaichuk Ya.M., Vozna N.Ya, Pitukh I.R. (2013) Structuring the movement of data in computer systems. Ternopil: Terno-graf, 284 p.

[3] Palani, S. (2022). Multi-rate Digital signal processing. *Principles of Digital Signal Processing: 2nd Edition* (pp. 623-674). Cham: Springer International Publishing. DOI:10.1007/978-3-030-96322-4_6

[4] Hu, Z., Tereykovskiy, I. A., Tereykovska, L. O., & Pogorelov, V. V. (2017). Determination of structural parameters of multilayer perceptron designed to estimate parameters of technical systems. *International Journal of Intelligent Systems and Applications*, *9*(10), 57. DOI: 10.5815/ijisa.2017.10.07

[5] Dean, W. (2019). Computational complexity theory and the philosophy of mathematics. *Philosophia mathematica*, *27*(3), 381-439. DOI: 10.1093/philmat/nkz021

[6] Suomela J. Structural information and communication complexity. *23rd International Colloquium, SIROCCO 2016, Helsinki, Finland, July 19-21, 2016, Revised Selected Papers, Springer, Cham.* 408 p. DOI:10.1007/978-3-319-48314-6

[7] Kari, J., Manea, F., & Petre, I. (2017). Computable transformations of structures *Unveiling Dynamics and Complexity. Springer International Publishing AG. Lecture Notes in Computer Science,* vol 10307. Springer, Cham. DOI:10.1007/978-3-319-58741-7_9

[8] Jun, S., & Kochan, O. (2015). Common mode noise rejection in measuring channels. *Instruments and Experimental Techniques*, *58*, 86-89. DOI:10.1134/S0020441215010091

[9] Nykolaichuk Ya. Specialized Computer Systems Design / Tutorial / Ya Nykolaichuk., N. Vozna, I. Pitukh, - Ternopil: Terno-graph, 2010. – 392 p.

[10] Kučera, E., Haffner, O., Drahoš, P., Cigánek, J., Štefanovič, J., & Kozák, Š. (2020). New software tool for modelling and control of discrete-event and hybrid systems using Petri nets. *Computing and Informatics*, *39*(3), 568-586. DOI: 10.31577/cai_2020_3_568

[11] Nykolaichuk Ya.M. (2008). Theory of information sources. Ternopol: TNEU, 536 p.

[12] Vozna N.Ya. (2018) Structuring of multifunctional data: theory, methods and tools: Monograph. Ternopil: TNEU, 378 p.

[13] Hlukhov, V., & Hlukhova, A. (2013, September). Galois field elements multipliers structural complexity evaluation. In *Proceedings of the 6-th International Conference ACSN-2013. September* , 16-18.

[14] Zheng, Z. (2022). *Modern Cryptography Volume 1: A Classical Introduction to Informational and Mathematical Principle* (p. 359). Springer Nature. DOI:10.1007/978-981-19-0920-7_3.

[15] Jones, K. (2010). The regularized fast Hartley transform. *Netherlands: Springer*. DOI:10.1007/978-3-030-68245-3_3.

[16] Voronych, A., Nykolaychuk, L., Grynchyshyn, T., Hryha, V., Melnychuk, S., & Nykolaychuk, Y. (2020, September). Development of theory, scope and tools for entropy signals and data processing. *2020 10th International Conference on Advanced Computer Information Technologies (ACIT)* (pp. 260-264). IEEE. DOI: 10.1109/ACIT49673.2020.9208912

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