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# THE IMPLEMENTATION OF THE METHOD OF REDUCED MATRIX D-TREES IN THE UDF MAOPCS ENVIRONMENT

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**Abstract:** The article explores the content of the MatrixDtrees function, which extends the functionality of the UDF MAOPCs and is designed to generate symbolic transfer functions of linear parametric circuits. This function represents a software implementation of the Transformed Matrix D-trees method. This method is an extension of the symbolic d-trees method, developed for constant parameters circuits, to parametric circuits. The extension involves the transition from algebraic operations with numbers and symbols in the d-trees method to matrix algebraic operations, taking into account the non-commutativity of matrix multiplication. The Transformed Matrix D-trees method significantly reduces computational time for modeling parametric circuits by factoring out similarities in complex symbolic expressions generated during the analysis process.

The MatrixDtrees function allows for the steady-state analysis in highly complex parametric circuits. By such circuits, we mean circuits that contain dozens or hundreds of nodes and elements (including parametric ones).

The article includes the results of analyzing a parametric long line model, which is represented by a combination of lumped parameters and consists of many cascaded elementary sections. Each of these sections is a combination of parametric inductance and constant capacitance.

The paper presents the results of an experiment to determine the output voltage of a long line model containing 1025 nodes, 1024 constant capacitances, and 1024 parametric inductances. The results are comparable to calculations of the same long line model using the MicroCap program. The relative deviation between the calculation results for both programs was less than 1%. The calculation time for the Transformed Matrix D-trees method using the MatrixDtrees function was 18 minutes, whereas for the MicroCap program, it was 36 hours.

**Keywords:** LPTV, MAOPCs, parametric long line model, frequency symbolic method, Transformed Matrix D-trees method.

## 1. Introduction

Despite its convenience, the frequency symbolic method [1,2,3] for forming transfer functions of linear parametric circuits in a steady-state regime (like most symbolic methods) is known for its drawback. As the complexity of the analyzed circuits increases, the calculation time for forming their parametric transfer functions significantly increases.

This article proposes a software solution to overcome this drawback based on applying the idea of the subcircuit method to the frequency symbolic method of analyzing linear parametric circuits. We propose the use of the Transformed Matrix D-trees method presented in our previous works [4]. This method is an extension of the d-trees method, which was designed for the symbolic analysis of linear circuits with constant parameters [5,6]. According to the Transformed Matrix D-trees method, the circuit is divided into initial subcircuits, followed by their pairwise merging (if there are $l$ initial subcircuits, there will be $(l-1)$ mergers). It ensures a substantial decrease in the computer simulation time required for circuits. The mathematical essence of this method lies in algorithmically factoring out similarities in complex symbolic expressions that contain a large number of terms with the products of a small number of parameters of circuit elements. Since the number of terms in such symbolic expressions is significant, this factoring results in a substantial reduction in computer simulation time.

From the above, it follows the necessity to facilitate a quick analysis of Linear Parameter Time-Varying (LPTV) circuits using the Transformed Matrix D-trees method in the form of a user-defined function (UDF). We implemented the necessary conditions in the form of a user-defined function MatrixDtrees and also incorporated it into the UDF MAOPCs [7]. In particular, the MatrixDtrees function was applied to solve the task of analyzing the long line model, consisting of a cascaded connection of a large number of elementary sections [8]. It is worth noting that each elementary section in such a model contains a time-varying kinetic inductance [9] and a constant capacitance. Hence, the number of parametric inductances present in such a long line model is significantly greater than in typical parametric electrical circuits [10].

Therefore, the relevance lies in the software implementation of the function that significantly reduces the calculation time when analyzing LPTV circuits with hundreds or even thousands of parametric elements, and its integration into the UDF MAOPCs system.

I express sincere gratitude to Shapovalov Yu. and Bachyk D. for their assistance in writing this work.

**2. User-defined function MatrixDtrees and the algorithm of the Transformed Matrix D-trees method.**

The algorithm for analyzing LPTV circuits using the Transformed Matrix D-trees method, implemented in the user-defined function MatrixDtrees, is as follows:

Step 1. Declare the input parameters [11].

Step 2. Form sets of Transformed Matrix D-trees for initial subcircuits.

Step 3. Following a specific algorithm [1], a sequence of pairwise merging of subcircuits is selected (if there are $l$ initial subcircuits, there will be $(l-1)$ mergers).

Step 4. According to the corresponding formulas [4], the two selected subcircuits in the chosen pair are merged into one, for which the set of Transformed Matrix D-trees is formed.

Step 5. Through $(l-1)$ such mergers, the Transformed Matrix D-trees for the entire linear parametric circuit are obtained, and the required transfer function $W(s,t)$ is formed, where $s$ is a complex variable, and $t$ is time.

The function is called by the command

$$TF = MatrixDtrees(l, Mode, \mathbf{DT}); \qquad (1)$$

where the user specifies: $l$ - the number of initial subcircuits; $Mode$ - the method of merging these subcircuits; $\mathbf{DT}$ - a two-dimensional block matrix, where each block represents a separate Transformed Matrix D-tree of the corresponding initial subcircuit of the circuit. The user-defined function MatrixDtrees returns $TF$ - the symbolic expression of the transfer function.

**3. Modeling a parametric long line using the user-defined function MatrixDtrees.**

*Computer experiment.* A discrete model of a parametric long line from Fig. 1a, consisting of a cascading connection of 1024 elementary links from Fig. 1b, was chosen for the experiment.

Let a complex current source be applied at the input of the specified model of a long parametric line:

$$i(t) = 0.0001 \cdot e^{j(3\cdot 10^8 \cdot t - \frac{\pi}{4})}. \qquad (2)$$

Task 1. Using the UDF MatrixDtrees, determine the real part of the transfer function $W(s,t)$ for the given value $s = \sqrt{-1} \cdot 2 \cdot \pi \cdot 3e^8$: $Re[W(t)] = Re[U_{out}(t) / i(t)]$ with a single symbolic variable $t$.

Task 2. Utilizing the obtained transfer function, calculate the steady-state output voltage $U_{out}(t) = \mathrm{Re}[W(t) \cdot i(t)]$ of the line with a modulation depth $m = 0.002$ for the inverse inductance $g(t)$ over the time range $t = 4e^{-5} - 4.0009e^{-5}$ s.

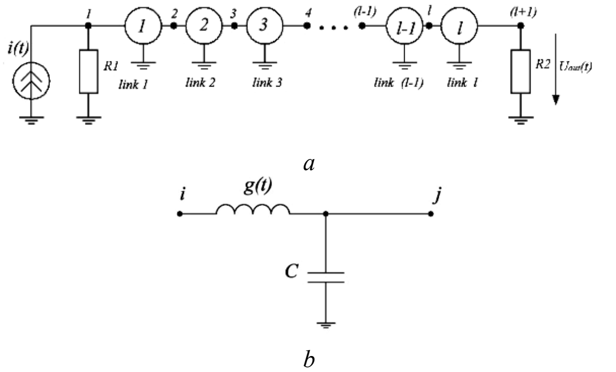


*Fig. 1. a) Model of a parametric long line containing $l = 1024$ elementary parametric links; b) Structure of the elementary link; $C = const$, the time-varying inverse inductance $g(t)$, $g(t) = 1 / L(t) = g_0 \cdot (1 + m \cdot cos(\Omega \cdot t))$ where $L(t)$ is the time-varying inductance, $g_0$ is the initial value, $m$ is the modulation depth, $\Omega$ is the pumping frequency. $R1$, $R2$ are the resistors of the input signal source and the load, respectively. $i(t)$ - input signal.*

Task 3. Assess the relative deviation between the outcomes obtained through the UDF MatrixDtrees and the MicroCap program [12].

*Software implementation.* We select elementary links from Figure 1b as initial subcircuits. According to the algorithm described in Section 2, the program for analyzing the model of a parametric long line will take the form depicted in Figures 2-4.

```
% Step 1
% Number of subcircuits
l = 1024;
% Merging method
Mode = 'pyramid';
% Step 2
% Initial subcircuit matrix D-trees
for idx = 1:l
    DT{1, idx } = MxC;
    DT{2, idx } = diag(ones(3,1)) +
(MxG)\(MxC);
    DT{3, idx } = diag(ones(3,1));
    DT{4, idx } = diag(ones(3,1));
    DT{5, idx } = diag(ones(3,1));
    DT{6, idx } = inv(MxG);
end
```

*Fig. 2. A fragment of the program code in the MAOPCs UDF environment, where* **MxC** *is the capacitor matrix, and* **MxG** *is the inverse inductance matrix [11].*

Using the UDF MatrixDtrees with the corresponding formulas [4], we combine the subcircuits and determine the transfer function of the circuit $W(t) = TF$.

```
% Step 3, 4, 5
TF = MatrixDtrees(l, Mode, DT);
```

*Fig. 3. A fragment of the source code in the MAOPCs UDF environment, where l is the number of subcircuits in the circuit; Mode is the mode of combining subcircuits;* **DT** *is a two-dimensional block matrix, TF is the transfer function.*

Using the obtained transfer function, we create a symbolic expression for the output voltage:

```
% Current source
i = 0.0001*(exp(1i*(3*10^8*t-(pi/4))));
% Output voltage function
U = real(i * TF);
```

*Fig. 4. A fragment of the source code in the MAOPCs UDF environment, where i is the current source (2), U is the output voltage function, TF is the transfer function.*

Substituting time points into $U_{out}(t) = U$, we obtain instantaneous values of the output voltage.

*Results.* A comparison of the results obtained using the UDF MAOPCs and the Micro-Cap program is presented in the table and Figure 5.

**The output voltage obtained by the two programs**

| t, μs | $U_{out}(t)$, mV, by Micro-Cap program | $U_{out}(t)$, mV, by MAOPCs | Relative deviation of results, % |
|---|---|---|---|
| 40.000645 | 0.6151 | 0.6157 | 0.103 |
| 40.001489 | 3.4477 | 3.4464 | 0.037 |
| 40.002333 | -0.7535 | -0.7541 | 0.091 |
| 40.003177 | -3.4177 | -3.4163 | 0.040 |
| 40.004021 | 0.8907 | 0.8913 | 0.070 |
| 40.004865 | 3.3821 | 3.3806 | 0.043 |
| 40.005709 | -1.0266 | -1.0270 | 0.046 |
| 40.006553 | -3.3409 | -3.3395 | 0.041 |
| 40.007397 | 1.1606 | 1.1611 | 0.046 |
| 40.008241 | 3.2944 | 3.2930 | 0.044 |

The table reveals that the relative deviations between the results obtained by both programs fall within the range of 0.040% to 0.103%.

In Fig. 5, the dependencies of the output signal $U_{out}(t)$ (red) and the relative deviation (blue) from time $t$ are illustrated. As we can see in Fig. 5, the instantaneous values of the output voltage in the time range $t = 4e^{-5} - 4.0009e^{-5}$ s. coincide for both programs. This accomplishment is especially remarkable given that the Micro-Cap program required 36 hours to generate the graph, while the MAOPCs system, employing a single harmonic component approximation of the Fourier series for the transfer function, produced the time-dependent function $U_{out}(t)$ in just 1082 seconds, approximately 18 minutes. It is important to note that the relative deviation increases for instantaneous voltage values close to zero, as illustrated in Fig. 5.

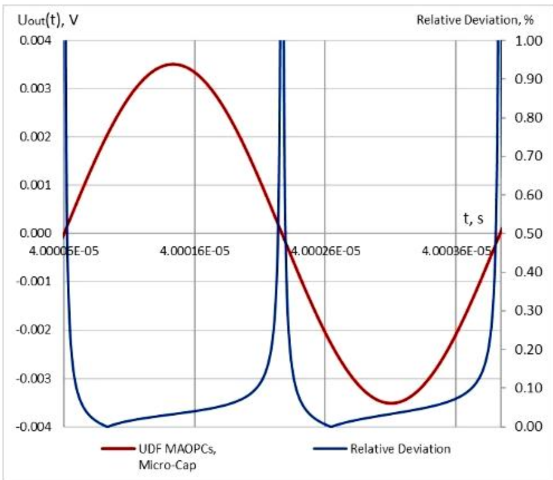


*Fig. 5. The overall view of the output signal (red) from the MAOPCs system and the MicroCap program coincides. The relative deviation of results from both programs (blue).*

**Conclusion**

In this work:

1. the software implementation of the method of reduced matrix D-trees in the form of the UDF MatrixD-trees was considered;
2. based on this function, the model of a parametric long line was analyzed:
   - the agreement of the obtained results with the Micro-Cap program reached 99%;
   - the computer time required to generate the symbolic transfer function and construct the output signal graph using the MatrixDtrees function was thousands of times less than that of the Micro-Cap program;
3. the MatrixDtrees function is incorporated into the UDF MAOPCs, significantly expanding the capabilities of this system in terms of the complexity of circuits permissible for analysis. This function is used to analyze complex parametric circuits and design systems containing hundreds and thousands of nodes and elements, including those with time-varying parameters.

Overall, it is worth noting that we are not aware of any other symbolic methods and programs that could analyze such a number (1024) of parametric elements in such a relatively short period (18 minutes) with such a high level of agreement.

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## РЕАЛІЗАЦІЯ МЕТОДУ РЕДУКОВАНИХ МАТРИЧНИХ D-ДЕРЕВ В СЕРЕДОВИЩІ UDF MAOPCS

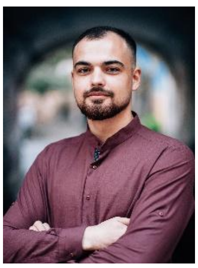
Романюк Роман

У статті розглянуто зміст функції MatrixDtrees, яка розширює функціонал системи UDF MAOPCs і призначена для формування символьних передавальних функцій лінійних параметричних кіл. Ця функція представляє собою програмну реалізацію методу приведених матричних D-дерев (Transformed Matrix D-trees). Цей метод є поширенням символьного методу d-дерев, розробленого для кіл з постійними параметрами, на параметричні кола. Поширення полягає у переході від алгебраїчних дій з числами та символами у методі d-дерев до матричних алгебраїчних дій з врахуванням некомутативності матричних множень. Метод приведених матричних D-дерев суттєво зменшує комп'ютерний час моделювання параметричних кіл, шляхом винесення подібних у громіздких символьних виразах, що формуються у процесі аналізу.

Функція MatrixDtrees дозволяє аналізувати усталені режими параметричних кіл підвищеної складності. Такими колами вважаємо кола, які містять десятки і сотні вузлів і елементів (в тому числі параметричних).

Стаття включає в себе результати аналізу моделі параметричної довгої лінії, що моделюється зосередженими параметрами і складається з багатьох каскадно з'єднаних елементарних ланок. Кожна така ланка представляє собою з'єднання параметричної індуктивності та постійної ємності.

У роботі наведені результати експерименту з визначення вихідної напруги лінії, що містить 1025 вузлів, 1024 постійних ємностей і 1024 параметричних індуктивностей. Результати порівняні з розрахунками цієї ж моделі довгої лінії за програмою MicroCap. Відносне відхилення між результатами обчислень за обома програмами склало менше 1%. Комп'ютерний час обчислень для методу приведених матричних D-дерев за функцією MatrixDtrees склав 18 хв., а для програми MicroCap - 36 год.



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