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Mathematical modeling for assessing the seismic response of buildings in Al Hoceima

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The purpose of this paper is to apply structural dynamics principles to conduct a comprehensive mathematical analysis of how buildings respond to earthquakes, with a particular focus on the dynamic behavior of a reinforced concrete structure in the city of Al Hoceima, located in the north of Morocco, which is subjected to frequent seismic activity. Considering various parameters such as building configurations, materials, soil characteristics, and seismic conditions, we will build a mathematical model for response of structures to earthquakes and conduct numerical experiments using ETABS software on a typical building in this region with a G+2 house elevation.

Keywords: *ODE; seismology; earthquakes; global dynamics; earthquake problems; geological problems.*

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1. Introduction

The seismic risk in northern Morocco is a matter of paramount importance, given the region's susceptibility to significant seismic activity. Located at the convergence of the African and Eurasian lithospheric plates, northern Morocco is subjected to intense tectonic movements that can trigger devastating earthquakes. This area, particularly near the Strait of Gibraltar, is characterized by intense Plio-Quaternary seismic activity resulting from the ongoing convergence of plates. This tectonic dynamic, coupled with the region's population density and increasing urbanization, underscores the critical importance of understanding and effectively managing seismic risk to mitigate human and material losses.

The city of Al Hoceima is particularly notable for its unique vulnerability to seismic hazards. Its location near the Mid-Atlantic Ridge and its placement in a subduction zone exposes Al Hoceima to a heightened likelihood of experiencing earthquakes. Seismology is essential for comprehending and preparing for this peril through the analysis of historical seismic events and the prediction of potential future risks. Previous catastrophic earthquakes in Al Hoceima underscore the necessity of increasing public awareness of seismic hazards and implementing measures to mitigate risks to both the population and infrastructure [1–4].

The Al Hoceima earthquake of February 24, 2004, in the vicinity of the northern coast of Morocco, was a consequential event. This strike-slip earthquake had a moment magnitude of 6.3 and reached an intensity of IX (Violent) on the Mercalli intensity scale. The earthquake had severe consequences, resulting in around 630 fatalities, 930 injuries, and displacing up to 15 000 people in the Al Hoceima, Imzourene and Beni Abdallah regions.

In this research, a mathematical model is developed to investigate the seismic response of buildings in Al Hoceima. The objective is to evaluate the structural behavior of buildings in response to seismic events, with a focus on typical structures that are characteristic of the urban environment. The model predicts displacements that may present considerable risks for buildings built before 2004. It is worth noting that seismic building codes have been enforced in Morocco since 2004, with the aim to enhance the structural robustness of buildings against seismic hazards [5–9].

2. Geodynamic, geological and seismological setting

2.1. Geodynamic

Morocco is located on a major boundary between two tectonic plates: the Eurasian plate and the African plate. These two lithospheric plates are moving towards each other at a rate of 5 mm/year in the region. Thanks to its unique geographical location, the town of Al Hoceima lies at the heart of the convergence of the African and Eurasian tectonic plates, making it an area prone to seismic activity and an important study site for tectonic geology [10].

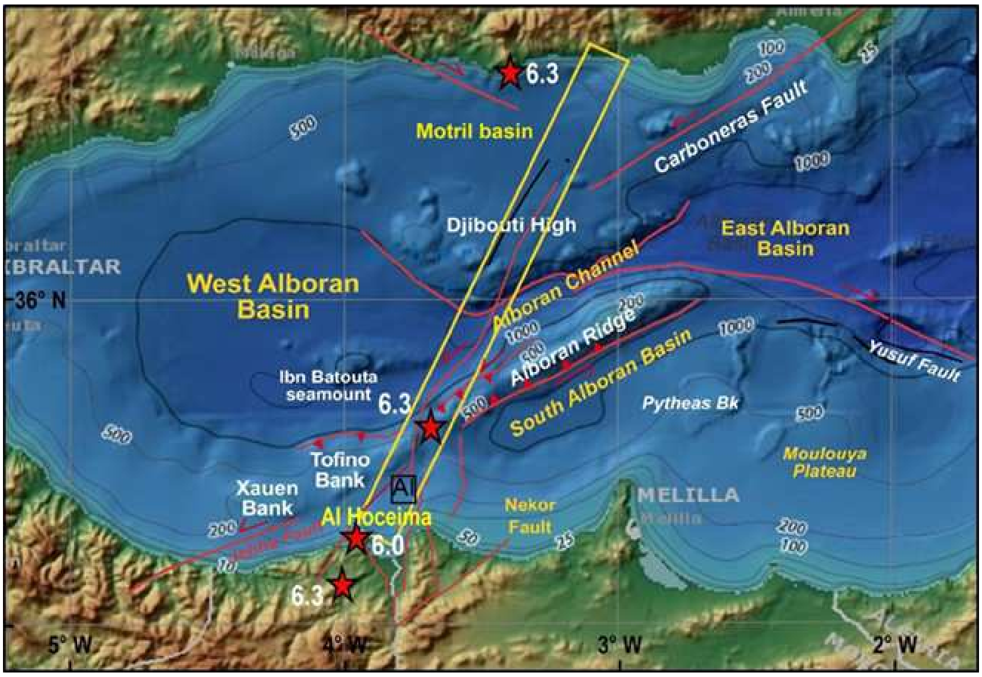

Fig. 1. Major structures in the Al Hoceima region superimposed on NOAA bathymetric and geomorphic maps [10].

The Al Hoceima region is located in a geologically active area (Figure 1), within the Alboran Basin, formed during the early Miocene. This basin emerged due to intense tectonic movements, resulting in the collapse and reduction of the depth of the Alboran block to approximately 15 kilometers. These phenomena were caused by orogenic uplift processes closely linked to the collision between the African and Iberian plates.

2.2. Seismological and seismotectonic setting

The context of seismology and seismotectonics in Al Hoceima is marked by the dynamic convergence of tectonic plates, historically documented seismic activity and an ongoing commitment to understanding and reducing seismic risk. This region is of particular importance due to its geographical location at the meeting point of the African and Eurasian plates, generating a series of unique challenges and opportunities.

Plate convergence: the constant convergence of the African and Eurasian plates creates active tectonics in Al Hoceima. The forces at play cause geological deformation, increased stress and the formation of faults, predisposing the region to seismic events. Seismic heritage: Al Hoceima has been the scene of some notable historic earthquakes, including the major one in 2004.

This documented seismic history underlines the need for constant vigilance and ongoing research efforts to understand the seismic mechanisms specific to the region. Research efforts: scientists, geologists and seismologists are engaged in deep studies to map active faults, monitor seismic activity in real time and assess potential risks. These research efforts contribute to our knowledge of local seismic phenomena and to the development of preparedness and response strategies.

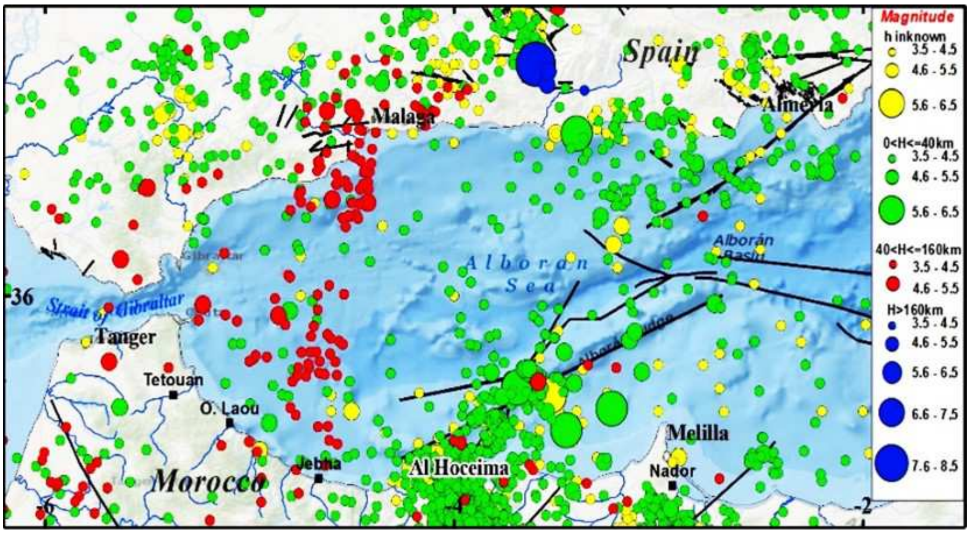

Fig. 2. Seismicity in the Al Hoceima Sea from 1901 to March 31, 2016 (Mw $\geqslant 3.5$) [10].

Safety and resilience: the safety of the people of Al Hoceima is an absolute priority. The lessons learned from seismological research are incorporated into the earthquake-resistant design of infrastructure, public awareness of safety measures in the event of an earthquake, and emergency planning. This comprehensive approach aims to strengthen the community's resilience to future seismic events.

3. Description of the structure

3.1. Geometry

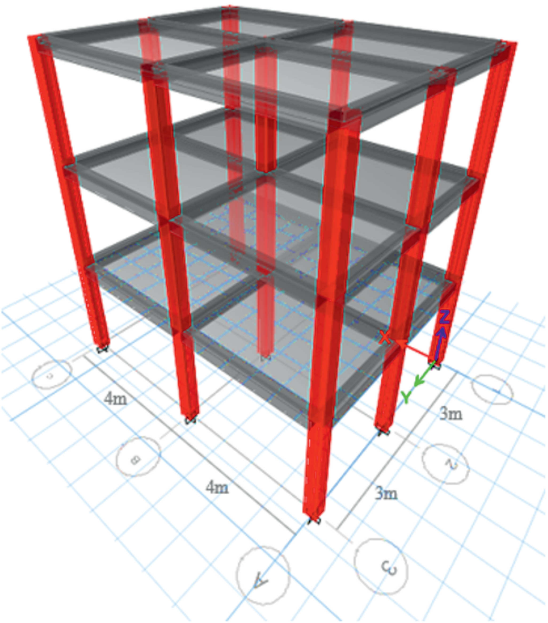

Fig. 3. General view of the 3D model.

The building we will consider in our study will be a two-story reinforced concrete building. The height of the building is 9 m with 3 m on each floor. The thickness of the slabs is 15 cm, and the dimension of the beams are 30×30 cm. The columns are 30×30 cm. The dead loads (G) of the standard floor at 800 kg/m^2. The live loads (Q) of the current floor are 250 kg/m^2.

3.2. Seismic data

The various parameters introduced in ETABS are displayed in Table 1.

Table 1. Seismic data.

Total weight W (KN)	1224
Priority factor I	1
Horizontal ground acceleration A	0.16
Behavior factor R	2
Amplification factor D	2.5
Shear force V (KN)	293.76

4. Mathematical modeling of the problem

Multiple studies have effectively utilized ordinary differential equations (ODEs) to simulate the behavior of buildings during seismic events. For instance, scholars have employed ODEs to analyze the dynamic characteristics of diverse types of structures, including high-rise buildings, bridges, and heritage structures, under different seismic conditions. These models have played a crucial role in identifying key factors influencing building response, such as structural stiffness, damping properties, and distribution of mass.

Besides ordinary differential equations (ODEs), cutting-edge computational techniques like finite element analysis and numerical simulations are frequently integrated with mathematical modeling to improve the precision and reliability of forecasts. Through the synthesis of these methodologies, scientists are able to construct comprehensive models that effectively depict the intricate relationships between structures and seismic forces, resulting in more efficient earthquake risk mitigation strategies.

Mathematical modeling of building response to earthquakes utilizing Ordinary Differential Equations (ODEs) provides a valuable tool for comprehending the dynamic characteristics of structures subjected to seismic forces. Ongoing research and progress in modeling methodologies allow engineers and researchers to further enhance their understanding of earthquake dynamics and devise novel strategies to enhance the resilience of essential infrastructure systems [11, 12].

For the modal analysis, the determination of the eigenmodes must be done with the free system ($F = 0$) undamped ($C = 0$). This amounts to solving the following system:

$$M\ddot{x}(t) + Kx(t) = 0,$$

where M is mass matrix, K is stiffness matrix.

We are looking for a solution in the form:

$$x(t) = \varphi\, \mathrm{e}^{\mathrm{i}\omega t}, \quad \big(K - \omega^2 M\big) = 0,$$

where $\omega^2 = \lambda$ is eigenvalue.

The characteristic equation of this system:

$$|K - \lambda M| = 0, \qquad \left| \begin{pmatrix} K_1 + K_2 & -K_2 & 0 \\ -K_2 & K_2 + K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{pmatrix} - \lambda \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \right| = 0,$$

where K_1, K_2, and K_3 are the stiffness of the three posts of levels 1, 2, and 3:

$$E = 32164 \text{ MPa}, \quad I = 6.7 \times 10^{-4} \text{ m}^4, \quad K = K_1 = K_2 = K_3 = 86.19 \text{ MN/m}.$$

And m_1, m_2, and m_3 are the masses of the three levels 1, 2, and 3 (we neglect the weight of the beams in front the slab):

$$m = m_1 = m_2 = m_3 = G + \psi \cdot Q = 408 \text{ KN},$$

$$\left| \begin{pmatrix} 2K & -K & 0 \\ -K & 2K & -K \\ 0 & -K & K \end{pmatrix} - \lambda \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \right| = 0.$$

We find the following solutions:

$$\begin{cases} \lambda_1 = \omega_1^2 = 685.924, \\ \lambda_2 = \omega_2^2 = 328.485, \\ \lambda_3 = \omega_3^2 = 41.8407, \end{cases} \Leftrightarrow \begin{cases} f_1 = 4.16 \text{ Hz}, \\ f_2 = 2.88 \text{ Hz}, \\ f_3 = 1.03 \text{ Hz}, \end{cases} \Leftrightarrow \begin{cases} T_1 = 0.24 \text{ s}, \\ T_2 = 0.34 \text{ s}, \\ T_3 = 0.97 \text{ s} \end{cases}.$$

According to this system the eigenmodes are

$$(K - \lambda_i M)\, \phi_i = 0.$$

$$\text{Mode 1}: \quad \phi_1 = \begin{pmatrix} 0.33 \\ 0.6 \\ 0.74 \end{pmatrix}; \qquad \text{Mode 2}: \quad \phi_2 = \begin{pmatrix} -0.74 \\ -0.33 \\ 0.6 \end{pmatrix}; \qquad \text{Mode 3}: \quad \phi_3 = \begin{pmatrix} 0.6 \\ -0.74 \\ 0.33 \end{pmatrix}.$$

Mode normalization. Generalized mass matrix:

$$\tilde{M}_i = \phi_i^t M \phi_i.$$

The normalized eigenmodes matrix:

$$\psi_i = \frac{\phi_i}{\sqrt{\widetilde{M}_i}} = \begin{pmatrix} 0.805 & -0.72 & 1.45 \\ 1.5 & -0.32 & -1.79 \\ 1.85 & 0.59 & 0.79 \end{pmatrix}.$$

Modal participation vector:

$$L=\psi_i^t M r=\begin{pmatrix}1.69\\-0.18\\0.18\end{pmatrix}\quad\text{for}\quad r=\begin{pmatrix}1\\1\\1\end{pmatrix}.$$

Spectral response. In accordance with RPS2000, the design spectrum representing the seismic action is as follows:

$$\frac{s_a}{g}=\begin{cases}1.25A\left[1+\dfrac{T}{T_1}\left(2.5\eta\dfrac{Q}{R}-1\right)\right], & 0\leqslant T\leqslant T_1;\\ 2.5\eta(1.25A)\dfrac{Q}{R}, & T_1\leqslant T\leqslant T_2;\\ 2.5\eta(1.25A)\dfrac{Q}{R}\left(\dfrac{T_2}{T}\right)^{2/3} & T_2\leqslant T\leqslant 3\text{ s};\\ 2.5\eta(1.25A)\dfrac{Q}{R}\left(\dfrac{T_2}{3}\right)^{2/3}\left(\dfrac{3}{T}\right)^{5/3} & T\geqslant 3\text{ s},\end{cases}$$

where $A=0.16$ is acceleration coefficient of the zone, $\xi=5\%$ is critical damping coefficient, $R=2$ is behavior coefficient of the structure, $Q=1$ is quality factor.

Table 2. Spectral responses in each mode.

Mode	Period	$S_{\mathrm{a}i}$
1	0.24	$4.7\ \mathrm{m/s^2}$
2	0.34	$4.7\ \mathrm{m/s^2}$
3	0.97	$2.6\ \mathrm{m/s^2}$

Displacements calculation.

Mode 1: $X_1=\Psi_1 L_1\dfrac{S_{\mathrm{a}1}}{\omega_1^2}=\begin{pmatrix}9.3\\17.37\\21.42\end{pmatrix}$ (mm); Mode 2: $X_2=\Psi_1 L_1\dfrac{S_{\mathrm{a}2}}{\omega_1^2}=\begin{pmatrix}1.85\\0.82\\-2.29\end{pmatrix}$ (mm);

Mode 3: $X_3=\Psi_1 L_1\dfrac{S_{\mathrm{a}3}}{\omega_1^2}=\begin{pmatrix}16.5\\-20\\8.83\end{pmatrix}$ (mm).

5. Conclusion

Earthquakes are geological phenomena that possess the potential to induce substantial destruction to architectural edifices, frequently resulting in fatalities and economic losses. To enhance comprehension and forecast the reaction of structures to seismic events, mathematical modeling methodologies are employed. A prevalent technique involves the utilization of Ordinary Differential Equations (ODEs) to elucidate the movement and characteristics of building constructs subjected to seismic activity. Through the formulation of these equations, scientists are able to replicate and assess the dynamic reaction of edifices to seismic disturbances, offering beneficial perspectives into plausible susceptibility and methodologies for augmenting seismic resilience.

The field of structural dynamics stands as a pivotal aspect of civil engineering, delving deep into the mathematical intricacies governing a building's response to seismic activity. Through intricate mathematical methodologies like differential equations and numerical techniques, this discipline aims to anticipate and evaluate how structures will withstand seismic forces, encompassing vibrations, deformations, and stresses. By factoring in diverse parameters such as building configurations, materials, soil traits, and seismic nuances, this approach facilitates a precise forecast of the dynamic behavior of a specific reinforced concrete structure (G+2) situated in Al Hoceima. Ultimately, this meticulous mathematical modeling serves as a robust foundation, empowering engineers to design buildings better equipped to endure and withstand earthquakes.

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Математичне моделювання для оцінки сейсмічної реакції будівель в Аль-Хосеймі

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Метою цієї роботи є застосування принципів структурної динаміки для проведення комплексного математичного аналізу того, як будівлі реагують на землетруси, з особливим акцентом на динамічній поведінці залізобетонної конструкції в місті Аль-Хосейма, розташованому на півночі Марокко, яке піддається частій сейсмічній активності. Враховуючи різні параметри, такі як конфігурація будівлі, матеріали, характеристики ґрунту та сейсмічні умови, побудуємо математичну модель реакції конструкцій на землетруси та проведемо чисельні експерименти за допомогою програмного забезпечення ETABS на типовій будівлі в цьому регіоні з висотою будинку G+2.

Ключові слова: *ODE; сейсмологія; землетруси; глобальна динаміка; проблеми землетрусів; геологічні проблеми.*