

The methods of optimization and regulation of the convective drying process of materials in drying installations

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(Received 21 October 2023; Revised 14 June 2024; Accepted 26 June 2024)

In this work, based on fundamental principles well-established in the field of drying technology, optimization for the process of material drying involves controlling the mechanism of moisture transfer by influencing diffusion and thermo-diffusion processes. Based on the Kirpichov criterion, a quantitative measure of moisture transfer dynamics is ensured, while Nusselt numbers help control temperature gradient and efficient moisture removal. The article proposes the use of empirical relationships between Nusselt numbers and problem parameters such as moisture content, temperature, and airflow velocity. Optimizing drying parameters based on the proposed equations can contribute to improving drying quality, reducing process time, and lowering energy consumption. The proposed methods of moisture and temperature gradient control within the material are sustainable and allow us to achieve uniform drying without causing excessive stresses or deformation.

Keywords: *sustainable drying technology; moisture transfer; thermo-diffusion; Nusselt numbers; Kirpichov criterion; optimization.*

2010 MSC: 76T10, 80A20, 80A22, 80A32, 80A35

DOI: 10.23939/mmc2024.02.546

1. Introduction

Mathematical modelling of heat and mass transfer processes in multi-component bodies with capillary-porous structure, including grains, under various drying regimes, is one of the most effective and cost-saving methods for optimizing the drying of agricultural and food industry products [1].

Understanding the mechanisms of heat and mass transfer during drying can help engineers develop more precise and resource-efficient drying technologies. By minimizing energy and material waste, these technologies can promote sustainability by conserving resources and reducing environmental impact.

The peculiarities of the flow of technological drying regimes are determined by the direct or indirect influence of the local microscopic structure of the material. Such influence is associated with the presence of fast paths for moisture migration, its local phase transformations, and the accompanying stress-strain state that arises during the flow of these processes [2]. The driving force of phase transitions is the difference in potentials of interfacial interaction. The process of phase transformation can occur with complete constancy of external conditions – constant temperature and pressure – and manifests itself as a sudden change in substance properties. Although phase transitions occur in the form of non-equilibrium processes, their theory focuses on equilibrium state changes when constructed.

Due to the complexity of the simultaneous solving the problem, a single-parameter model is used, according to which the longitudinal scattering of liquid particles in the phase is likened to the process of diffusion with a conditional coefficient of longitudinal diffusion, which ensures sufficient accuracy of calculations. The coefficient of longitudinal diffusion depends on the hydrodynamics of the flow. This method of reflecting the process in mass transfer equipment is used under the assumption that the phase flows are well mixed in the transverse direction, and the concentration changes only in the longitudinal direction. Such a single-parameter model is applied, in particular, in modelling the grain drying process.

The technique and technology of heating and cooling grain during drying and hydrothermal treatment are based on the theory of heat and mass transfer [3–7]. In these studies, the fundamental laws of energy and substance transfer in colloidal capillary-porous materials have been identified. The designs of heat exchange equipment provide for various options for thermal treatment of grain: in a dense or loose layer, in a fluidized state [8,9].

For a complete description of moisture transfer in grain, it is necessary to know the thermodynamic characteristics of the material (isothermal moisture capacity), kinetic coefficients of moisture transfer (moisture diffusion coefficients, moisture conductivity), temperature coefficients of moisture transfer potential (thermal conductivity), and similarity criteria. A significant number of studies are dedicated to researching the thermo-physical properties of grain (see, for example, [10]).

In drying tasks, the issue of energy efficiency and sustainability of the process and ensuring the required quality of the dried product remains relevant, which requires studying the peculiarities of mathematical modelling of heat and mass transfer processes in multiphase, multi-component bodies, taking into account the sizes and nature of the phases. The presented work provides insights into various aspects of drying processes and suggests methods for optimizing them based on a deeper understanding of the underlying principles and mechanisms involved.

2. Theoretical background

Optimal operating parameters – ambient temperature T_s , air humidity in the chamber φ , and airflow velocity v – should be chosen taking into account the technological changes of the drying material. Moisture removal under heating is determined by the maximum allowable temperature for the given material, as well as the permissible gradients of its change over time. It is known that temperature and humidity control in dryers is achieved by regulating the dry-bulb temperature T_c and wet-bulb temperature T_m using drying agent thermometers. The drying process in steam chamber drying installations occurs according to the laws of change in dry-bulb T_c and wet-bulb T_m temperatures.

During the isothermal heating stage, the surface temperature and the temperature of the central surface of the material do not change, the temperature gradient inside the material is zero, and the difference between the air temperature and the surface temperature of the material is a constant value. Then, with a constant heat transfer coefficient, the drying intensity will be constant. This period is called the constant drying rate period and is characterized by a constant material temperature.

During this period, moisture evaporates from the surface of the material as it comes from the internal layers, so the surface remains initially wet. The material temperature is close to the wet-bulb temperature. The drying rate reaches its maximum. This stage continues until certain moisture content is reached, from which the surface temperature of the material increases over time, and the drying rate decreases. The linear portion of the drying curve transitions to a curve that asymptotically approaches equilibrium moisture content. The temperature of the central surface also increases, but its temperature curve lags slightly behind the surface curve. Thus, a temperature gradient within the material arises, which gradually decreases, and upon reaching equilibrium moisture content, becomes zero. In the equilibrium state, moisture content reduction does not occur, and the material temperature equals the air temperature.

When the temperature of all layers is uniform, it is possible to reduce the wet-bulb temperature by removing moisture from the chamber with the help of an exhaust fan, increasing the dry-bulb temperature, and reducing wetting by steam. The reduction of the wet-bulb temperature and the increase in the dry-bulb temperature occur until the psychrometric difference $T_c - T_m$, which corresponds to a certain material moisture content, equals the specified value. This two-stage mode should be set depending on the thickness, initial and final moisture content, and airflow velocity inside the chamber.

With uniform temperature distribution, shrinkage does not cause dangerous stresses. The development of shrinkage is associated with the appearance of internal stresses. To prevent material from cracking, it is possible to reduce tensile stresses by decreasing the moisture gradient along the section. This can be achieved by softening the regime at the beginning of drying and conducting heat treatment at the middle and end of the process.

In a two-stage regime, during the constant drying rate stage, high relative steam elasticity and relatively low temperature T_c are maintained. During the falling drying rate stage, the dry-bulb temperature increases to a maximum, while the relative steam elasticity decreases. The moisture content of the capillary-porous material when transitioning from one stage to another should be equal to the saturation moisture content of the material fibres.

During high-temperature drying, it should be noted that the moisture movement rate from the inside to the surface is proportional to the moisture conductivity coefficient. The moisture conductivity coefficient increases with temperature, and the moisture gradient increases with decreasing relative humidity, as the surface moisture content decreases. The change in moisture gradient during high-speed high-temperature treatment requires strict control over preservation of efficiency, and safety of the drying regime.

The roughness of the regime is characterized by the psychrometric difference $T_c - T_m$ and plays a crucial role in high-speed high-temperature drying. The roughness of the regime should be regulated throughout the drying process at high initial moisture content and periodically adjusted during low-temperature drying. Efficiency is characterized by the ratio of normal drying process time to high-temperature regime application time. Safety is determined by the ratio of the strength limit to the maximum stresses generated under the given regime.

The main obstacle to high-speed drying is cracking. The appearance of cracks is caused by the development of a stressed state in the material, which exceeds the permissible limit. This stressed state is created by unacceptable shrinkage resulting from non-uniform distribution of moisture content and temperature inside the material. Cracking can be avoided by reducing tensile stresses through decreasing the moisture gradient at the beginning of drying and conducting heat treatment in the middle and end of the process. Assuming that local failure occurs in the elastic region under the action of normal tensile stresses, then in a plate of thickness $2L$, the moisture gradient between the central and surface layers, equals $\frac{L}{3}(u_c - u_p)$, where u_c is the moisture content at the central surface, u_p is the moisture content at the outer surface, can be used to determine the moisture content based on boundary normal stresses. However, in the work [11], it is noted that the calculated stresses exceed the experimental ones by 5–10 times. The reason for this is considered the inapplicability of Hooke's law because local failure occurs in the elastic-plastic region. It is also indicated that the formation of cracks is due to shear stresses, and the conditions for failure are such that the intensity of tangential stresses at a given point of the specimen reaches critical values, and tensile stresses cannot cause local rupture. In real bodies, local ruptures occur due to defects, weak spots, and cracks.

Crack formation occurs in the elastic-plastic region under the action of plastic deformations, which occur before the appearance of cracks and at the moment of appearance constitute 60–80 percent of all deformation, depending on the moisture content of the material. The values of crack formation stresses and ultimate destructive stresses depend on the loading rate, increasing with increasing stress intensity by 2 or more times. At high moisture contents, the ultimate destructive tangential stress and crack formation stress are approximately the same. With decreasing moisture content, the ultimate rupture stress increases more than the crack formation stress [3]. The optimal drying modes of capillary-porous materials are influenced by material shrinkage due to changes in moisture content. Shrinkage development varies for different materials. For example, in cement stone, shrinkage increases only when the material moisture content is less than 30 percent. There are materials in which shrinkage begins immediately after a decrease in moisture content (clay). Let us consider this influence using the example of wood. If the moisture content in some part of the cross-section becomes lower than the hygroscopicity limit, the fibres tend to decrease in size, hindered by fibres with higher moisture content. As a result, internal stresses arise. Drying fibres, unable to contract, become stretched, while adjacent fibres, due to the action of drying, become compressed. As moisture of hygroscopic nature is removed from the material, shrinkage development gradually slows down. When internal hairline cracks occur in the material, shrinkage may slow down. If cracking within the material does not develop, then after some time, shrinkage may progress at the previous rates, and then after some time, it may stop. This

cessation is the result of the material reaching an average moisture content close to the equilibrium value for the given temperatures and relative humidity of the drying agent. The shrinkage development law may follow various dependencies, such as exponential decay. It should be noted that wood material is anisotropic, and drying in it occurs below the fibre saturation point.

The dependence of the modulus of elasticity on moisture is inversely proportional to moisture until saturation point and can be approximated by straight lines, with different slopes before and after saturation. Maintaining the required value of surface moisture can ensure the specified values of permissible stresses. Surface moisture is regulated by changing the moisture of the drying agent.

When regulating the drying process, it is considered that the moisture distribution across the thickness satisfies the expression [12] $\frac{u_c - u_y}{u_c - u_p} = \left(\frac{y}{L}\right)^n$, where u_c , u_p , u_y are the moisture content values at the centre, on the surface, and at a distance y from the centre; n is the factor characterizing the rate of change of the moisture relative distribution in the thickness. At the beginning of drying, $n \rightarrow \infty$, at the end $n \rightarrow 0$, and shrinkage is determined by the exponential dependence on time $\varepsilon = \varepsilon_k (1 - e^{-\tau/T})$, where ε is the current shrinkage value, ε_k is the shrinkage value at the end of the process, T is the process time constant.

The moisture distribution across the thickness at the beginning of the falling drying rate period is parabolic $n = 2$.

For internal stresses in the layer y

$$\sigma_y = \frac{A}{3} u_y (2u_c - u_p - 3u_y).$$

Further deepening of the evaporation zone is taken into account by a sinusoidal distribution for $n = 1.5$

$$\sigma_y = \frac{A}{2.5} u_y (1.5u_c - u_p - 2.5u_y).$$

Then the distribution is assumed to be linear $n = 1$

$$\sigma_y = \frac{A}{2} u_y (u_c - u_p - 2u_y).$$

The multiplier $A = k [\varepsilon_k (1 - e^{-\tau/T}) + u_p]$ is determined by the properties of the specific material and defines the averaging program set by the control system. By maintaining the required value of surface moisture, the specified permissible stress values can be ensured. Thus, to obtain a safe drying process, it is necessary to determine the optimal values of the velocity of movement, the corresponding relative saturation, the temperature of the drying agent, and also the time for changing the regulation stages.

The change in moisture content due to the change in material temperature in the function of time depends on the relationship between the processes of moisture and heat change within the moist material, as well as on the processes of heat and mass transfer of the body surface with the surrounding environment.

3. Approaches to drying optimization

Approaches to drying optimization vary. If the primary requirements include crack absence, then the drying regime proceeds in accordance with the change in the structural-mechanical properties of the material. If crack formation occurs in the initial period, the criterion for crack formation can be the relative moisture content difference between the average moisture content \bar{u} and the local moisture content u relative to the initial moisture content \bar{u}_0 , denoted as $K = \frac{\bar{u} - u}{\bar{u}_0}$. If crack formation occurs on the surface of the body in the initial drying period, then $u = u_p$. For the parabolic law of moisture distribution, $K = \frac{2}{3} \frac{(u_c - u_p)}{\bar{u}_0}$, where $u_c - u_p$ is the difference in moisture content between the central and surface layers. The main criterion for moisture transfer in the drying process is the Kirpichov criterion Ki' , which equals the ratio of the surface moisture gradient ∇u_p to the initial moisture content \bar{u}_0 multiplied by the characteristic size of the body R . The obtained values of Ki' are related to certain values of surface moisture u_p and temperature t_p on the body's surface.

For the constant velocity period with a parabolic law of moisture distribution and temperature, Liukov found that $\frac{(u_c - u_p)}{\bar{u}_0} = \frac{1}{2}Ki' + \frac{t_p - t_c}{t_c}Pn$, where $Ki' = \frac{q'R}{a'\gamma_0\bar{u}_0}$ is the Kirpichov criterion for moisture transfer, $Pn = \frac{\delta t_c}{u_0}$ is the Postnov criterion. Here q' is the evaporation intensity, a' is the coefficient of mass transfer potential conductivity $a' = \frac{\lambda'}{c'\gamma_0}$, γ_0 is the skeleton density, R is half of the plate's thickness, λ' and c' are the moisture conductivity and volumetric moisture content, respectively.

Under intensive drying conditions, heat and mass transfer patterns change. In this case, the temperature at any point of the material, starting from the surface, increases. Meanwhile, the decrease in moisture content at any point of the material occurs linearly. This indicates that the drying rate becomes constant, while the temperature increases, including near the surface. Additionally, a temperature gradient is observed. Initially, the temperature near the body's surface equals the wet-bulb temperature, but then continuously increases. Towards the end of the process, the air and body temperatures become equal. Near the surface, the temperature distribution follows a linear law, then transits to a curve, asymptotically approaching the ambient temperature. Close to the surface, molecular transfer (vapour diffusion in inert gas) occurs, while as distance from the surface increases, molecular heat transfer changes to molar (convective), gradually strengthening. The thickness of the conditional boundary layer remains constant during the initial drying period, then increases approximately linearly. The air humidity decreases both near and away from the surface from the beginning of drying. Near the surface, similar to temperature, a linear distribution of air humidity is observed, which then transitions to a curve, approaching the humidity of the surrounding environment. The conditional thickness of the boundary layer is $\delta = \frac{\lambda}{\alpha}$, where λ is the thermal conductivity coefficient and α is the heat transfer coefficient. The drying process varies for different materials.

In the temperature range of 0 – 150°C during heating of the product (for example, bricks), hygroscopic moisture is removed, accompanied by a significant amount of water vapour. The water vapour pressure inside the product reaches a significant level already at 70°C, increasing with temperature. Therefore, if the rate of steam formation inside the material exceeds the rate of steam filtration through its thickness, the pressure inside the material can lead to the appearance of cracks and delamination. Moreover, the material surface, having dried quickly, continues to heat up significantly. Inside the material, the temperature quickly rises to 100°C, remaining at this level until all moisture is removed, resulting in significant temperature differences between the surface and the internal part of the product, and consequently, significant stresses and the appearance of cracks.

Experiments show that if the material drying-up is carried out by increasing the gas flow rate with moderate temperature elevation (around 50 – 80 degrees per hour), then this process occurs with high intensity with minimal temperature gradients throughout the thickness and without compromising product quality. The drying temperature range is of crucial importance. If it is short, then a gradual temperature rise and extended exposure are predominant. Holding the products at the maximum temperature is used to even out the temperature throughout the thickness of the product.

Drying is completed by removing moisture from the surface into the surrounding environment. The intensity of moisture removal depends on the difference in partial pressures of the vapour in the air layer above the wood surface and in the surrounding air volume, as well as the speed of the air flowing over the wood. The greater the difference in partial pressures and the air velocity, the more intensively moisture evaporates from the material surface.

4. The Nusselt criteria in heat and mass transfer problems

The criteria of heat and mass transfer are expressed through Nusselt criteria:

$$Nu = \frac{\alpha l}{\lambda_g} = \frac{ql}{\lambda_g \Delta t}, \quad Nu' = \frac{\alpha' l}{\lambda'_g} = \frac{q' l}{\lambda'_g \Delta p}, \quad (1)$$

where l is the characteristic linear dimension.

If we replace the complex mechanism of transfer with conditional molecular transfer, we obtain a conditional boundary layer of thickness δ . In this case, the coefficients of heat and mass transfer, as

well as the Nusselt coefficients, are determined from experimental curves according to the formulas:

$$\alpha = \frac{\lambda}{\delta_t}, \quad \alpha' = \frac{\lambda'}{\delta_p}, \quad Nu = \frac{l}{\delta_t}, \quad Nu' = \frac{l}{\delta_p}. \quad (2)$$

Continuing the direct distribution of air humidity to the horizontal line, we can find the conditional depth of the evaporation surface ξ [3].

In the first period, the drying intensity becomes constant with continuous deepening of the evaporation surface, which is explained by the increase in the mass transfer coefficient λ' due to the displacement of the diffusion mechanism of vapour transfer by effusion in the surface layer of the body (evaporation zone). In the first period, $q' = \frac{\lambda'_\phi}{\delta_\phi + \xi} (\phi - \phi_c)$. In the first approximation, δ_ϕ becomes constant and larger than ξ . The increase in the denominator due to the increase in ξ is compensated by the increase in λ' . Therefore, the flux density becomes constant. In the second period, δ_ϕ increases, resulting in a decrease in the drying intensity [1].

In intensive regimes, the heat flux density can be determined as $q = r_k q' + \bar{c} \gamma_0 \frac{V}{F} \frac{d\bar{t}}{d\tau}$, where $\frac{V}{F}$ is the volume-to-surface ratio, r_k is the specific heat of evaporation. Due to additional expenses in calculating the heat flux density, Nusselt criteria are determined by the conditional thickness of the boundary layer. Studies show that in the first period, the Nusselt criteria remain constant, and in the second period, they decrease with decreasing moisture content. Empirical relations have been obtained: $\frac{Nu}{Nu_n} = \left(\frac{u}{u_{kr}}\right)^n$, where u_{kr} is the critical moisture content, Nu_n is the Nusselt thermal criterion during constant drying speed, equal to $Nu_n = A \left(\frac{T_r}{T_m}\right)^2 \left(\frac{T_m}{T_c}\right)^{0.4} Re^{0.5}$. For wood, $A = 0.5$, $n = 0.3$. Similar relations are obtained for mass transfer: $\frac{Nu'}{Nu'_n} = \left(\frac{u}{u_{kr}}\right)^{n'}$, where n' is close to n . Thus, the change in heat and mass transfer occurs according to the same law.

Calculations of the Nusselt criterion Nu' using Eq. (1) and by the thickness of the conditional boundary layer showed that the criterion Nu' calculated by Eq. (1) in the first period is significantly larger than Nu' calculated by the thickness of the conditional boundary layer. The difference between them decreases as the moisture content decreases [3].

This difference is all the greater, the higher the intensity of drying. This is explained by the mechanism of vapour transfer through the evaporation zone. During evaporation, the evaporation surface deepens into the body. Transfer from the inner evaporation zone occurs not only by diffusion but also by effusion (molecular flow) if the radius of the body's capillaries is less than 10^{-5} cm and the pressure equals atmospheric one.

5. Accounting for effusion in calculating the Nusselt numbers

The molecular flow occurs when gas from less heated parts of the capillary moves to more heated ones at the same pressure p . In the drying process, the body's surface has a higher temperature compared to the temperature of the surface in the evaporation zone. Therefore, this temperature difference accelerates molecular flow, as the potential of effusive flow $\frac{p}{\sqrt{T}}$ is higher. In the case of diffusive transfer, the presence of a temperature gradient in the evaporation zone reduces the transfer rate. If the body contains capillaries with a radius larger than 10^{-5} cm, a diffusive-effusive mechanism of transfer occurs.

In the presence of macro-capillaries, diffusive vapour transfer is complicated by thermal creep. This phenomenon occurs as follows: if there is a temperature gradient along the walls of a macro-capillary, circulation currents of moist gas arise. The gas near the surface of the capillary moves against the heat flow, while along the axis of the capillary, it moves in the direction of the heat flow. Since the capillaries near the surface of evaporation have a lower temperature compared to the capillaries of the surface layer of the body, thermal creep facilitates vapour transfer through the evaporation zone.

Transfer of vapour from the body's surface to the surrounding environment occurs primarily by molecular means. The mass transfer coefficient λ' in the evaporation zone – the boundary layer near the body's surface – is much larger than the molecular mass transfer coefficient of vapour. Therefore, Nusselt criteria are elevated compared to criteria determined by the thickness of the boundary layer, which provides more accurate results.

The new vapour mass transfer mechanism affects both heat transfer and the hydrodynamics of heat exchange. The increase in the Nusselt criterion during drying is explained as follows: as the evaporation surface deepens, vapour passes through this zone by a diffusive-effusive path, complicated by thermal creep and the presence of a total pressure gradient. Upon exiting the vapour from the surface layer of the body, the total pressure gradient instantly relaxes, serving as a turbulent factor for the laminar flow of the surface layer of the vapour-gas mixture. As a result, the heat transfer process intensifies and the values of Nu increase. As moisture is removed, the drying intensity decreases, and the heat transfer coefficient diminishes, approaching the value of the pure heat transfer coefficient.

There is a relationship between the criteria Ki' and Nu' , given by $Ki' = Nu' \cdot \frac{R}{T} \frac{\lambda_g}{a' \lambda_0 \bar{u}_0} (p_{nm} - p_c)$. The criterion Ki' is an analogue of the Biot criterion $Bi = \frac{\alpha R}{\lambda} = Nu \frac{R}{T} \frac{\lambda_g}{\lambda}$ for heat transfer. Here, λ is the thermal conductivity coefficient of the body, R is the characteristic size of the body (for an infinite plate, it is half the thickness), or the hydraulic radius $R_v = \frac{V}{S}$, which is equal to the ratio of the body's volume to its surface area.

6. Temperature and humidity control in dryers

Temperature and humidity control in dryers is achieved by regulating the temperature using dry T_c and wet T_m bulb thermometers of the drying agent. The temperature measured by the dry thermometer varies at different stages of the drying process. During the material heating stage, the temperature changes rapidly. During the isothermal heating stage across the entire cross-section, the temperature increases slightly. Temperature uniformity across individual layers throughout the material thickness is achieved. In the final stage, temperature changes measured by the dry thermometer indicate intensive moisture removal and drying potential alteration.

Moisture evaporation from the surface of the wet bulb thermometer occurs more intensely with lower drying agent humidity. Measuring humidity based on readings from both dry and wet bulb thermometers is reliable under the condition of hydrodynamic equilibrium $S\alpha(T_c - \tilde{T}_m) = CS(P_m - P_v)r$, where S is the surface of wet bulb thermometer evaporation, α is the heat transfer coefficient, C is the intensity coefficient of moisture evaporation from the wet bulb thermometer surface, $P = 745$ mm Hg is the total barometric pressure, P_v is the vapour pressure in the air, and $r = 595$ cal/kg is the latent heat of evaporation. From this relationship, $P_v = P_m - A(T_c - \tilde{T}_m)$, where $A = \frac{\alpha}{Cr}$. Once the partial pressure of the vapour is determined, the relative humidity of the drying agent can be found $\phi = \frac{P_v}{P_m}$. The coefficient A is determined by an empirical formula $A = 0.00001 (65 + \frac{6.75}{v})$, where v is the velocity of the air-vapour mixture in the chamber.

Therefore, readings from dry and wet bulb thermometers determine the humidity and temperature of the drying agent, essential information for ensuring the quality of the drying process. During the heating period, it is considered that the humidity of the drying agent is unequivocally determined by the temperature measured by the wet bulb thermometer and is independent of its temperature when $T_c = \text{const}$.

7. Drying regime optimization

Thus, let us consider how to control the moisture transfer mechanism by means of control the diffusion and thermodiffusion processes. During drying with heated air, the total moisture flow equals the difference in moisture gradients, determined by the difference in gradients of moisture content and temperature: $q_m = a_m \left| \frac{\Delta u}{\Delta x} \right| - a_m \delta \left| \frac{\Delta t}{\Delta x} \right|$. Under the influence of the temperature gradient, moisture moves within the material. The moisture flow directed to the surface of the material decreases by the amount of moisture flow caused by thermodiffusion. The temperature gradient acts as a barrier to the movement of liquid from central layers to the surface. Under constant drying intensity, conditions are created that facilitate liquid evaporation inside the material. Thermodiffusion reduces the moisture gradient and decreases the rate of liquid movement and the amount of water-soluble substances on the material surface. Changing the magnitude and direction of the temperature gradient alters the

conditions of moisture and substance migration, leading to changes in the material's physicochemical properties.

In the case of drying with heated air, the temperature gradient in the material leads to a significant gradient in moisture content, creating large stresses that cause cracking. By rapidly heating and cooling the material from the surface, such values of the Postnov criterion $Pn = \frac{\delta \Delta t}{\Delta u}$ can be achieved where the moisture content distribution will be close to uniform. In this case, moisture and temperature stresses in the material do not occur, and the material dries fairly quickly and without cracks.

By using the laws of diffusion and thermodiffusion, substance transfer mechanisms can be controlled. Since diffusion is determined by the diffusion coefficient and thermodiffusion by the thermodiffusion coefficient, it is important to establish the dependence of these coefficients on moisture content and temperature.

8. Conclusions

Optimization of the drying processes of capillary-porous materials requires, first of all, the selection of optimal process parameters. The ambient temperature, air humidity in the chamber, and airflow velocity should be chosen considering the technological changes of the material being dried. If the primary technological standard of drying is minimal heat and electricity consumption, the absence of cracks in the product during drying is another requirement. To meet these requirements, the drying regime should be aligned with the changes in the structural and mechanical properties of the material. If crack formation occurs in the initial drying period, the Kirpichev criterion for moisture transfer is used as the criterion of crack formation. During the constant drying rate period, the temperature of the material can be assumed to equal the wet-bulb temperature. If the critical permissible Kirpichev criterion is known and the dependency graphs for various moisture contents are constructed, an entire range of regimes can be obtained, from which the regime with the highest drying intensity is selected. Simultaneously, the drying apparatus should have minimal heat and electricity consumption. Increasing the airflow velocity enhances the intensity in the first period and significantly less in the final period. The ratio between capillary-bound and adsorbed moisture substantially affects the drying intensity in the second period and has little impact on the intensity in the first period.

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Методи оптимізації та регулювання процесів конвективного сушіння матеріалів у сушильних установках

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У цій роботі метод оптимізації процесу сушіння зерна передбачає керування механізмом вологоперенесення шляхом впливу на дифузійні та термодифузійні процеси. На основі критерію Кірпічова забезпечується кількісний контроль динаміки перенесення вологи, а числа Нуссельта допомагають контролювати градієнт температури та ефективно видалення вологи. Крім того, у статті пропонується використовувати емпіричні зв'язки між числами Нуссельта та параметрами задачі, такими як вологовміст, температура та швидкість повітряного потоку обдуву. Оптимізація параметрів сушіння на основі цих рівнянь сприяє покращенню якості сушіння, без виникнення у матеріалі надмірних напружень або деформацій. Запропоновані методи контролю градієнта вологи та температури в матеріалі є стійкими та дозволяють досягти рівномірного висушування продукції за коротший час та дозволяють знизити споживання електроенергії.

Ключові слова: *сталий розвиток; технологія сушіння; перенесення вологи; термодифузія; числа Нуссельта; критерій Кірпічова; оптимізація.*