MATHEMATICAL MODELING AND COMPUTING, Vol. 11, No. 1, pp. 239–249 (2024)

Searching for similar images using Nash game and machine learning

Semmane F. Z.[1,2], Moussaid N.[1], Ziani M.[2]

[1] *LMCSA, FSTM, Hassan II University of Casablanca, PO Box 146, Mohammedia, Morocco*
[2] *LMSA, Department of Mathematics, Faculty of Sciences, Mohammed V University in Rabat, Morocco*

(Received 31 May 2023; Revised 3 March 2024; Accepted 4 March 2024)

The storage of large amounts of digital data, as well as the processing of digital images, are currently expanding significantly across a range of application areas. As a result, effective management of big images databases is necessary, which calls for the employment of automated and cutting-edge indexing techniques. One method used for this is Content-Based Image Retrieval (CBIR), which tries to index and query the picture database using visual aspects of the image rather than its semantic features. In this article, we propose to explore a digital search engine for similar images, based on multiple image representations and clustering, improved by game theory and machine learning methods.

Keywords: *image retrieval; image descriptors; image colors; GIST; Zernike moments; concurrent optimization; Nash game and images clustering.*

2010 MSC: 90C29, 91A05, 58E17, 65K10 **DOI:** 10.23939/mmc2024.01.239

1. Introduction

For computer vision researchers, one of the main challenges is how to automatically index images based on their content. This is a difficult task due to the reality, one or more vectors are used to represent each image. Many approaches have been developed to tackle this issue, including the use of text-based image retrieval (TBIR) methods which rely on textual descriptions of images. However, TBIR has limitations, such as being subjective and depending heavily on the language skills and understanding of the person in charge of indexing the images. This gave rise in the early 1990s to a second generation of methods designated by CBIR (Content Based Image Retrieval) [1,2]. The CBIR approach involves searching a database for one or more images that have been previously indexed based on various descriptors of their content, such as color histograms, SFTA descriptor, and Zernike moments. We know that each image is represented by a vector and not by isolated values, which makes the search for similar images by traditional methods very complicated. To do this, we propose a process for searching similar images using machine learning methods and Nash game [3]. First, we calculate the three representation vectors (color, texture and shape) for each image of the database, after that we go on to the image restoration phase sought. In this step we use a non-cooperative game called the Nash game to solve a multidisciplinary optimization problem; the players tactics are determined by three strategies (color, texture and shape).

In the machine learning phase, at each level of representation, we apply a classification algorithm with the K-means approach [4], which automatically divides the images into various classes. The optimal solution of Nash game will be used to find the member classes at every degree of representation: the outcomes are obtained by applying the KNN algorithm [4], to the point where the membership classes intersect.

2. Features of image

Visual features play a crucial role in understanding and analyzing images. They provide a rich representation of the visual content present in an image, enabling us to describe, classify, search, and retrieve images based on their visual characteristics. These features capture various aspects of an image, such as color, texture and shape, allowing us to extract meaningful information and insights from visual data.

2.1. The color descriptor

Color is the first descriptor used for image searching. Several descriptors exist at when it comes to our situation, we use the condensed color histogram [5], which is depending on the separation of three color levels (Red, Green, Blue). Figure 1 presents the color histogram of three different images.

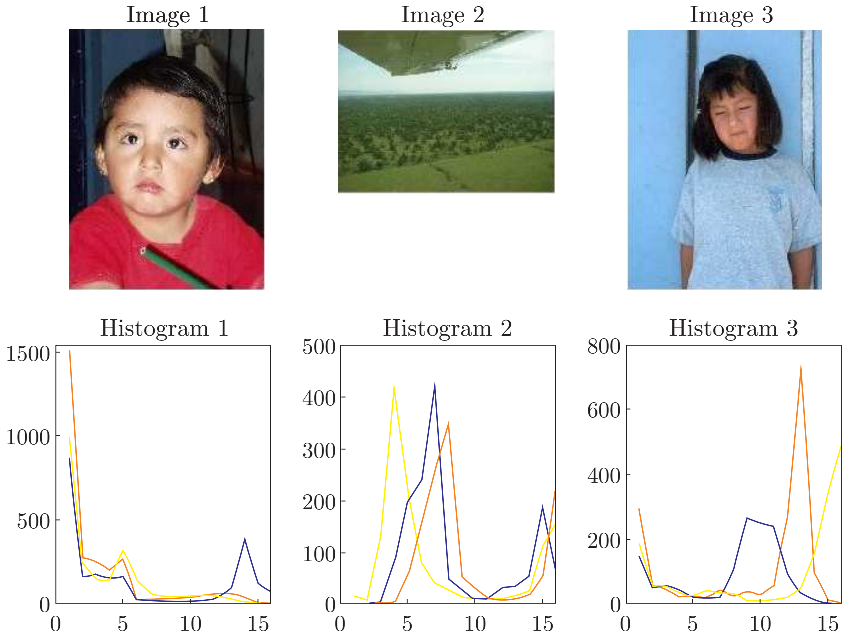

Fig. 1. Color histogram of three images.

2.2. The texture descriptor

Texture descriptor is characterized by the repetition of a reason or certain elements in the image. There are several methods to analyze it. In our case, we use SFTA (image segmentation method by texture analysis based on projective fractals). This method makes possible to synthesize images very close to reality, see [6]. The SFTA descriptor for two different images is represented in Figure 2.

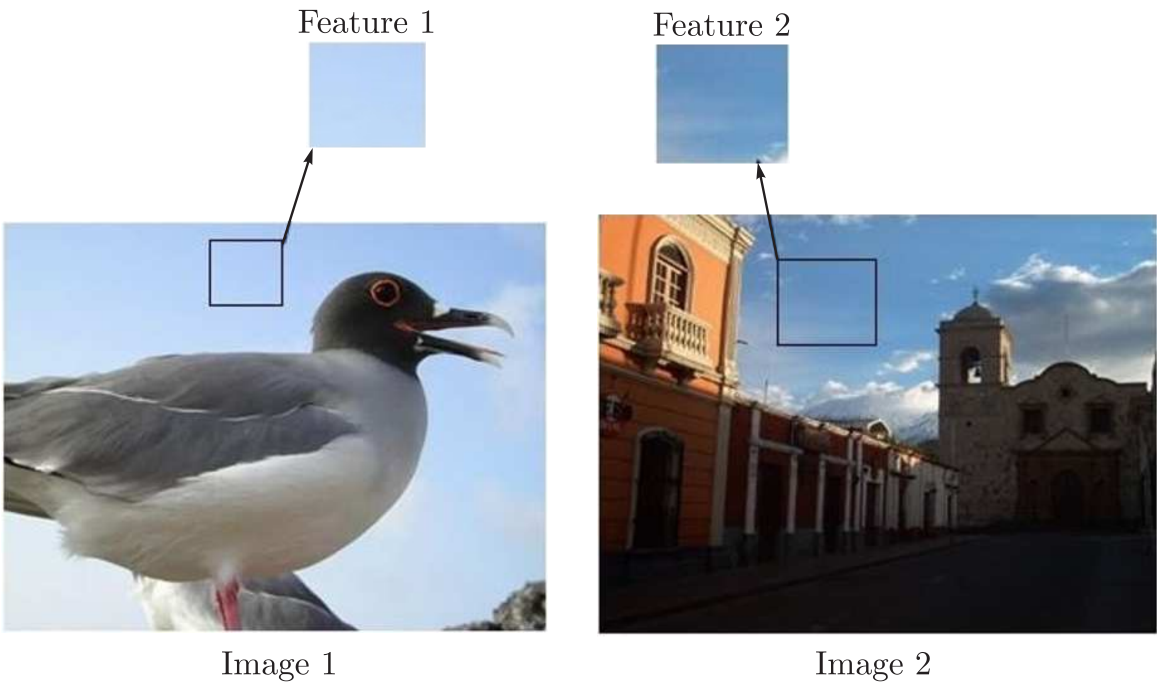

Fig. 2. SFTA descriptor extracted from two images.

2.3. The shape descriptor

The shape descriptor we have adopted is based on Zernike moments. The benefit of using these moments is that they are easy to implement, and are also equipped with good invariance properties by translation and by rotation. For more details on Zernike moments see [7]. Figure 3 presents Zernike moments for three different images.

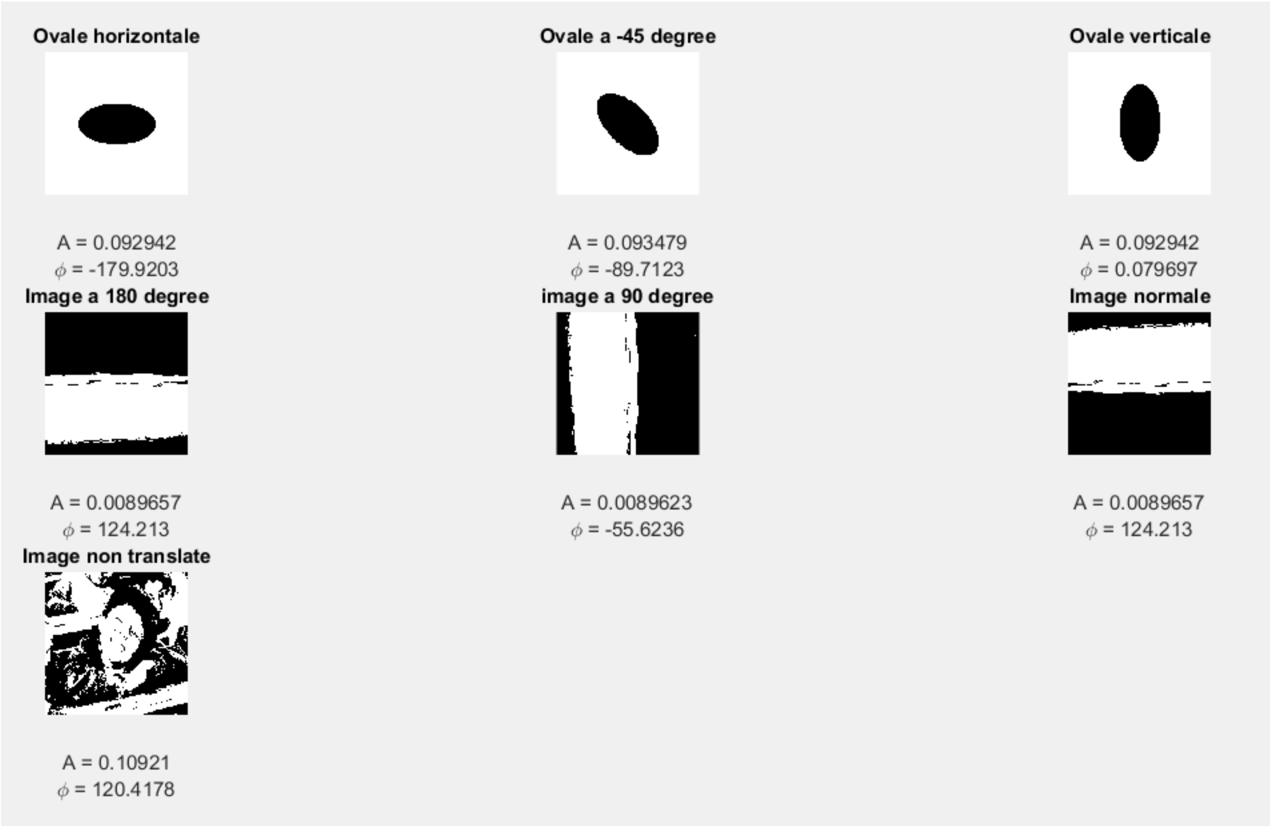

Fig. 3. Zernike moments of three images.

3. The proposed method

In CBIR, the query image I is a restored image in general. Thus we conduct an initial denoising by using Tikhonov regularization algorithm, see [8].

3.1. The query image restoration

Let F be the noisy image such that

$$F = I + m\mu,$$

where μ is a white additive Gaussian noise. We can reconstruct the original image I from the noisy image by minimising the functional

$$J(I) = \frac{1}{2}||I - F||^2 + \frac{\varepsilon}{2}||\nabla(I)||^2, \qquad I \in H_0^1(\Omega), \quad F \in L^2(\Omega),$$

where ε is a parameter to adjust. In our content based image I retrieval systems, the color histogram (C), the SFTA (S) and the Zernike moments (Z) information have been the primitive image descriptors. These descriptors are also used to restore the original image as a Nash equilibrium [9]. Therefore, we split I, the original optimization variable and the noisy image F into three strategies (Color, SFTA and Zernike) and we denote $I = (C, S, Z)$ and $F = (F_C, F_S, F_Z)$.

We consider three players, the first controls the color descriptor C and minimizes its cost function J_C, the second minimizes its J_S cost function by acting on the SFTA. Then the third controls the shape using Zernike moments and minimizes its cost function J_Z. The functions $J_C(C,S,Z)$, $J_S(C,S,Z)$ and $J_Z(C,S,Z)$ are defined by

$$J_C(C,S,Z) = \frac{1}{2}||C - F_C||^2 + \frac{\varepsilon}{2}\left(||\nabla(C)||^2 + ||\nabla(S)||^2 + ||\nabla(Z)||^2\right),$$
$$J_S(C,S,Z) = \frac{1}{2}||S - F_S||^2 + \frac{\varepsilon}{2}\left(||\nabla(C)||^2 + ||\nabla(G)||^2 + ||\nabla(Z)||^2\right),$$
$$J_Z(C,S,Z) = \frac{1}{2}||Z - F_Z||^2 + \frac{\varepsilon}{2}\left(||\nabla(C)||^2 + ||\nabla(S)||^2 + ||\nabla(Z)||^2\right).$$

The resulting Nash game optimization problem is written as

$$\begin{cases} \text{Find } (C^*, S^*, Z^*) \text{ such that} \\ \quad \min\limits_C J_C(C, S^*, Z^*) = J_C(C^*, S^*, Z^*), \\ \quad \min\limits_S J_S(C^*, S, Z^*) = J_S(C^*, S^*, Z^*), \\ \quad \min\limits_Z J_Z(C^*, S^*, Z) = J_Z(C^*, S^*, Z^*). \end{cases} \tag{1}$$

Theorem 1. *There exists a Nash equilibrium (C,S,Z) solution of the problem (1).*

Proof. The functionals J_C, J_S and J_Z are convex and lower semi-continuous. Then we have at least the existence of one Nash equilibrium (C,G,S), for more details, see [10]. ■

The Nash equilibrium can be obtained by solving the final optimization problem (1). The Nash equilibrium is calculated by the subsequent Algorithm 1, see [11].

Algorithm 1 Nash algorithm

Initialization: $n = 0$ and the noisy image $I^{(0)} = (C^{(0)}, S^{(0)}, Z^{(0)})$

Step 1:

Phase 1: Solve the problem $\min\limits_C \; J_C(C, S^{(n)}, Z^{(n)}) \to C^{(n+1)}$

Phase 2: Solve the problem $\min\limits_S \; J_S(C^{(n)}, S, Z^{(n)}) \to S^{(n+1)}$

Phase 3: Solve the problem $\min\limits_Z \; J_S(C^{(n)}, S^{(n)}, Z) \to Z^{(n+1)}$

Step 2:

Define $I^{(n+1)} = (C^{(n+1)}, S^{(n+1)}, Z^{(n+1)})$ and repeat step 1 until convergence

It is convergence when $\|I^{(n+1)} - I^{(n)}\| < \varepsilon$ where ε is to be specified

Figure 4 presents the result of restoring a noisy image using the Nash method.

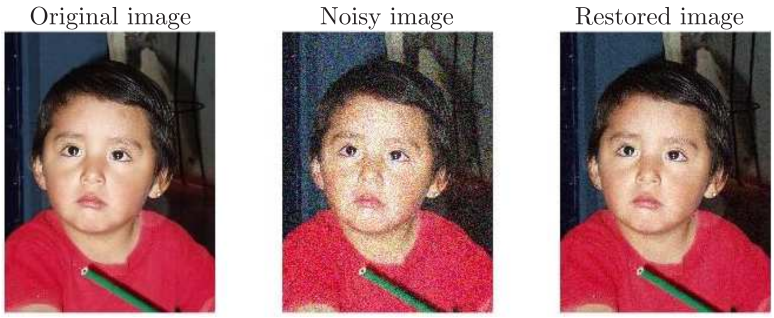

Fig. 4. Restoration by Nash algorithm.

3.2. Searching similar images using Nash game and machine learning

Machine learning plays an important role in the growth of a search engine for similar images because it allows the creation of models that are able to recognize similarities between different images. Using machine learning techniques, one can classify a database of images and also train an algorithm to identify common features in similar images, such as the color, texture and shape. We use the partitioning algorithm for clustering the data known as K-means because it is the most extensively used and well-known clustering algorithm, see [4]. The main steps of the K-means algorithm are presented in Algorithm 2.

Algorithm 2 K-means algorithm

Input: N dataset denoted by X. Desired classes number denoted by K
Output: A partition of K classes $C_1, C_2, \ldots, C_K$
Beginning: Random initialization of K centers of classes
Repeat:
Assignment: Generate a fresh partition by assigning every object to the group whose center is the nearest: $x_i \in C_k$ if $\|x_i - \mu_k\| = \min_j \|x_i - \mu_j\|$ $\forall j$ with μ_k the center of the k class.
Representation: Determine the centers that correspond to the updated section: $\mu_k = \frac{1}{N}\sum_{x_i \in C_k} x_i$ until a stable partition is reached by the procedure

We aim to classify a database of 36 images into three clusters using the K-means algorithm based on their features.

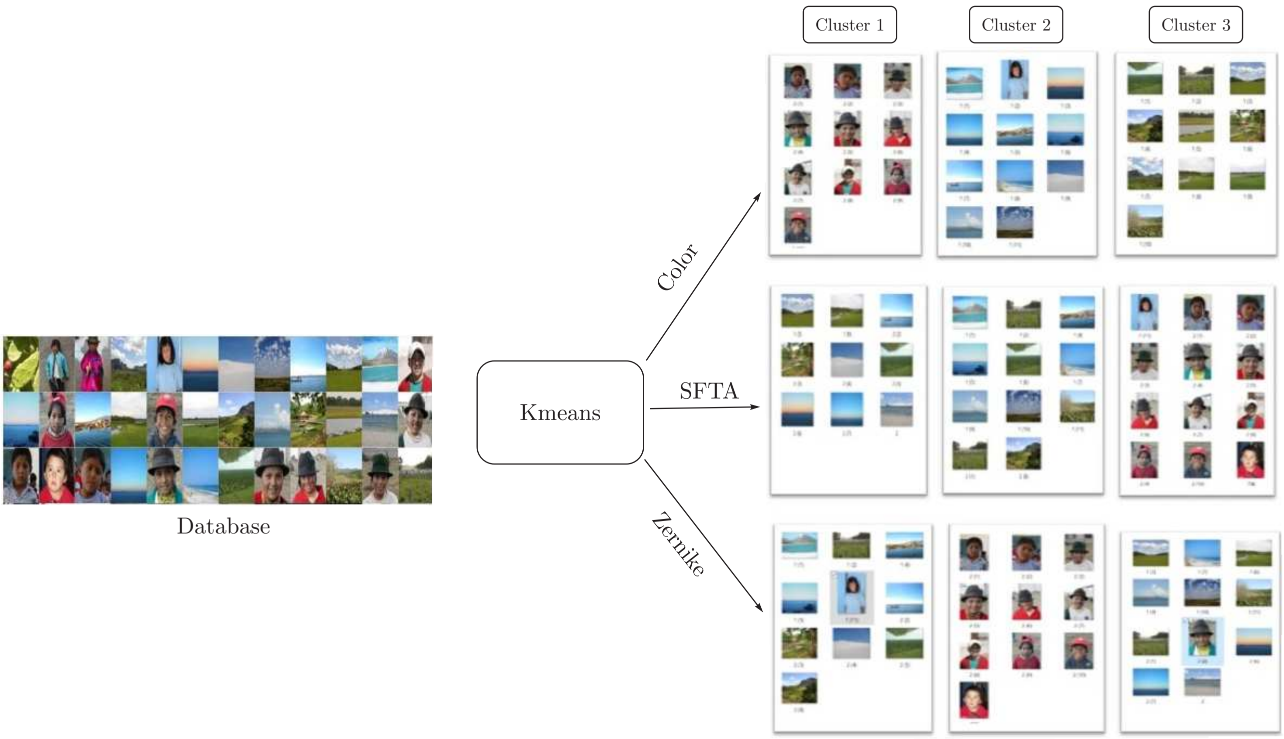

Fig. 5. Example of clustering a database.

Once the classification is done, we use the Nash equilibrium solution to define the classes of membership using the algorithm of KNN (K-nearest neighbours method). It is a non-parametric machine learning algorithm, see [4], that can be used for both classification and regression tasks. In the context of searching for similar images, the KNN method can be used to determine the K several similar images to a given query image within a database of images. To use the KNN method for image search, a distance metric is used to determine the similarity between the features of the query image and the features of the images in the database. Commonly used distance metrics include Euclidean distance, cosine similarity, and Hamming distance, depending on the type of features being used. Figure 6 presents results obtained by using the KNN method.

4. Tests and results

After having defined the new approach in the previous parts, we present the different tests carried out on the methods chosen for the global characterization of an image in order to search for images by their visual content based on the three attributes, namely, color, texture and shape with the use of Nash game strategy and machine learning. To test our methods, we use a database composed of a variety of 3000 images. Sample images from the dataset are presented in Figure 7.

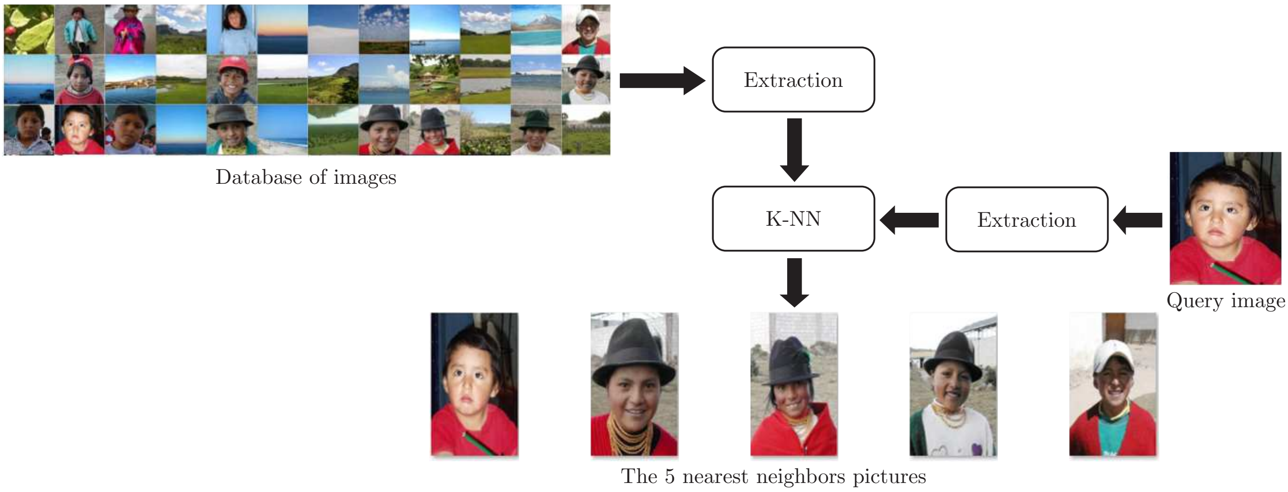

Fig. 6. KNN method applied on database of 36 images.

Fig. 7. Sample images from the database.

The aim of this work is to develop an efficient image search system that can significantly reduce search time. Through the using of machine learning techniques, specifically the K-means and K-nearest neighbor methods, the system is able to reduce the number of images needed. This is achieved by employing $K = 10$ clusters for the color histogram and an equal number of clusters for both the Zernike and SFTA descriptors.

We use the MATLAB software interface to display only eight images generated from our research. The eight images that are most similar to the query image, based on the method used, are displayed in order of score, going from the left to the right and from top to bottom. The query image is positioned at the top-left corner of the resulting eight images. We perform three tests on three different images.

4.1. Test 1

Fig. 8. Similar images retrieval using the color histogram.

Query Image

Fig. 9. Similar images retrieval using the SFTA.

Query Image

Fig. 10. Similar images retrieval using Zernike moments.

Query Image

Fig. 11. Similar images retrieval using proposed method.

Table 1 presents success rate in finding similar images to the query, in test 1, for the different approaches.

Table 1. Table of similarity in Test 1.

Image 1	HC (Fig. 8)	SFTA (Fig. 9)	Zernike (Fig. 10)	Descriptors+machine learning +Nash (Fig. 11)
% similarity	67%	50%	33%	100%

4.2. Test 2

Fig. 12. Similar images retrieval using the color histogram.

Fig. 13. Similar images retrieval using the SFTA.

Fig. 14. Similar images retrieval using Zernike moments.

Fig. 15. Similar images retrieval using proposed method.

Table 2 presents success rate in finding similar images to the query, in test 2, for the different approaches.

Table 2. Table of similarity in Test 2.

Image 2	HC (Fig. 12)	SFTA (Fig. 13)	Zernike (Fig. 14)	Descriptors+machine learning +Nash (Fig. 15)
% similarity	75%	63%	55%	100%

4.3. Test 3

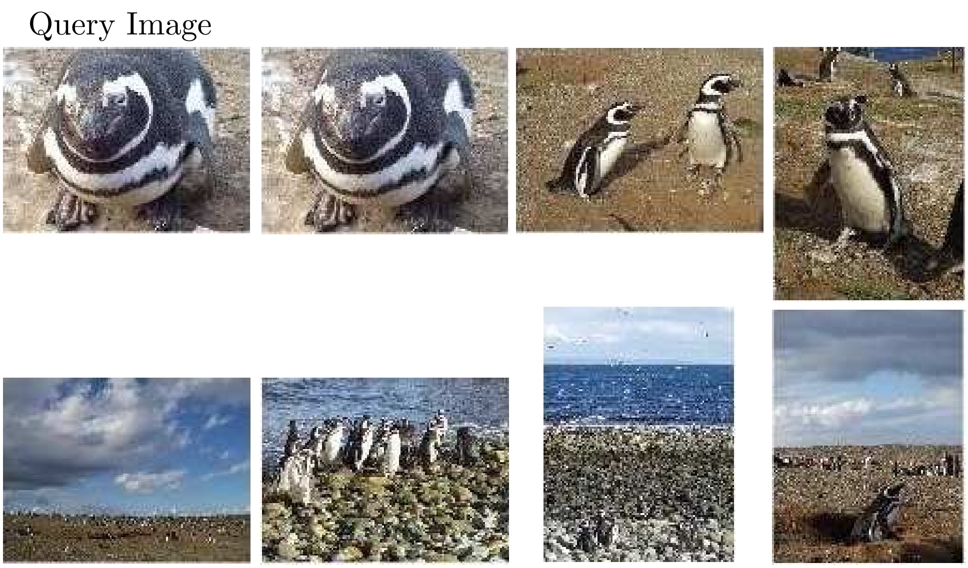

Fig. 16. Similar images retrieval using the color histogram.

Fig. 17. Similar images retrieval using the SFTA.

Fig. 18. Similar images retrieval using Zernike moments.

Fig. 19. Similar images retrieval using proposed method.

Table 3 presents success rate in finding similar images to the query, in test 3, for the different approaches.

Table 3. Table of similarity in Test 3.

Image 3	HC (Fig. 16)	SFTA (Fig. 17)	Zernike (Fig. 18)	Descriptors+machine learning +Nash (Fig. 19)
% similarity	70%	60%	40%	99%

From the results of the previous three tests, we conclude that the use of the descriptors separately may not provide consistent results, but we check the existence of the image sought. The proposed method, based on the Nash equilibrium and machine learning, can address the slightly inconsistent results observed in the previous section (Figures 11, 15 and 19). This approach involves intersecting the results obtained from the color descriptor (Condensed Color Histogram), shape descriptor (Zernike Moments), and texture descriptor (SFTA) to return the most relevant images.

5. Conclusion

The present work entails the development of a search engine for similar images by content depending on a Nash game and machine learning methods. This engine works with descriptors representing the visual and global content of the images obtained by compressed color histogram (for color), Zernike moments (for shape) and SFTA (for texture) as descriptors. The proposed method gives similar images and it provides the existence of the query image.

[1] Shriwas K. M., Raut V. R. Content Based Image Retrieval: A past, Present and New Feature Descriptor. 2015 International Conference on Circuits, Power and Computing Technologies [ICCPCT-2015]. 1–7 (2015).

[2] Mumtaz A., Awais A., Muhammd S., Zahid U. Content Based Image Retrieval (CBIR) Using Materialized Views. International Conference on Computer Science and Information Technology (ICCSIT), Pattaya (2011).

[3] Autisme-Economie.org. Le talon d'Achille de la théorie des jeux (2000).

[4] Azencott C.-A. Introduction to Machine Learning. DUNOD (2018).

[5] Pass G., Zabith R. Histogramme refinement for content-based image retrieval. Proceedings Third IEEE Workshop on Applications of Computer Vision. WACV'96. 96–102 (1996).

[6] Bajscy R. Computer identification of visual surfaces. Computer Graphics and Image Processing. **2** (2), 118–130 (1973).

[7] Teague M. R. Image analysis via the general theory of moments. Journal of the Optical Society of America. **70** (8), 920–930 (1980).

[8] Elmoumen S., Moussaid N., Aboulaich R. Image retrieval using Nash equilibrium and Kalai–Smorodinsky solution. Mathematical Modeling and Computing. **8** (4), 646–657 (2021).

[9] Nasr N., Moussaid N., Gouasnouane O. The Kalai Smorodinsky solution for blind deconvolution. Computational and Applied Mathematics. **41** (5), 222 (2022).

[10] Aubin J. P. Mathematical Methods of Game and Economic Theory. North Holland Publishing Co. Amsterdam, New York (1979).

[11] Gouasnouane O., Moussaid N., Boujena S., Kabli K. A nonlinear fractional partial differential equation for image inpainting. Mathematical Modeling and Computing. **9** (3), 536–546 (2022).

Пошук схожих зображень за допомогою гри Неша та машинного навчання

Семман Ф. З.[1,2], Муссаїд Н.[1], Зіані М.[2]

[1]*LMCSA, FSTM, Університет Хасана II Касабланки, п.с. 146, Мохаммедія, Марокко*
[2]*LMSA, кафедра математики, факультет природничих наук, Університет Мухаммеда V у Рабаті, Марокко*

Зберігання великих обсягів цифрових даних, а також обробка цифрових зображень наразі значно розширюються в ряді областей застосування. Як наслідок, потрібне ефективне керування базами даних великих зображень, що потребує використання автоматизованих та найсучасніших методів індексування. Одним із методів, який використовується для цього, є пошук зображень на основі вмісту (CBIR), який намагається індексувати та запитувати базу даних зображень, використовуючи візуальні аспекти зображення, а не його семантичні характеристики. У цій статті запропоновано дослідити цифровий пошуковий механізм для подібних зображень, який заснований на множинному представленні зображень і кластеризації, який вдосконалений за допомогою теорії ігор і методів машинного навчання.

Ключові слова: *пошук зображення; дескриптори зображень; кольори зображення; GIST; моменти Церніке; одночасна оптимізація; гра Неша та кластеризація зображень.*