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The impact of rumors on the success of Covid-19 vaccination programs in a Coronavirus-infected environment: optimal control approach

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In this paper, we propose a mathematical model that describes the effect of rumors on the success of vaccination programs against Covid-19 in an environment infected by the coronavirus. The aim of this study is to highlight the role of addressing the spread of rumors regarding vaccination risks and booster doses in the success of vaccination programs and in achieving herd immunity. Additionally, we formulate an optimal control problem by proposing several strategies, including awareness and anti-rumor programs, to assist country officials in achieving successful vaccination programs with optimal effort. The existence of optimal controls is investigated, and Pontryagin's maximum principle is used to characterize them. The optimality system is solved using an iterative method. Finally, we conduct numerical simulations to verify the theoretical analysis using Matlab.

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1. Introduction

On May 14, 1796, English doctor Edward Jenner performed the first vaccination of a young boy with cowpox pus (or vaccinia), which immunized him against the disease [1]. Jenner became the first to scientifically experiment with 'vaccination', although the concept had been practiced in various forms prior to his work. Chinese writings from the 16th century mention the practice of inoculation, involving deliberately injecting smallpox taken from a weakly ill patient to immunize others. This suggests that the origins of this practice might extend back to the Middle Ages.

Since Jenner's methods caught on in Europe, opposition to vaccines was actually based on religious and medical beliefs at that time, which led early to rumors spreading about risks and dangers of vaccination. French health historian Patrick Zylberman states that "The contestation of vaccination is as old as vaccination itself" [2]. Therefore, whenever a vaccine against a new disease appears, several rumors surface about the risks of vaccination, as is the case with vaccines against Covid-19.

Rumor is usually defined as the unconfirmed elaboration or annotation of things, events, or issues of public interest that spread through various channels, being neither true nor false [1, 3, 4]. In the past, rumors were disseminated orally, in magazines, and newspapers. However, with the advent of the Internet and social media, rumors have a greater potential to spread more widely and faster than ever before.

Rumors can shape public opinion and influence the opinions and beliefs of individuals, which may lead to changes in individuals' attitudes towards various health, economic, political and social issues. Therefore, understanding the dynamics of the spread of rumors and how to effectively control and curb their spread is a topic worth investigating.

Since the announcement vaccines production against Covid-19 and their approval by the World Health Organization [5], many rumors have emerged about vaccination and its health risks. For example, we cite the following rumors: "I have already had Covid-19, so I do not need to get the vaccine", "the vaccine may cause infertility", "the mRNA vaccine will alter my DNA.", "the vaccines were rushed and there has not been enough testing." and "the vaccines are not safe for people with allergies". Rumors spread faster more than truth [6], which has a negatively affects for the success of vaccination programs.

The statistics depicted in (Figure 1) illustrate the progress of vaccinations in selected countries around the world.

Through this graph, we see discrepancies in the speed of progress of vaccination programs, particularly in developing countries and in certain African countries, where illiteracy and ignorance are prevalent. Such conditions facilitate the spread of rumors about the risks of vaccination and booster doses. This has been confirmed by many officials of these countries during their statements and press conferences. Rumors spread in these countries due to many reasons including: lack of confidence, fear of the unknown, illiteracy and lack of awareness, and also lack of information.

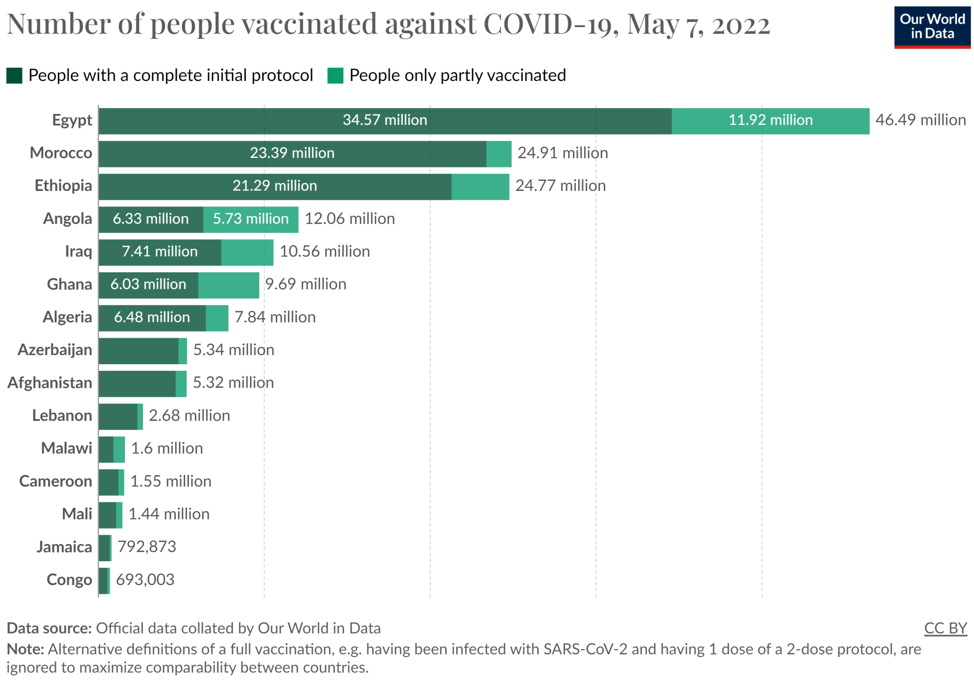

Fig. 1. Share of people vaccinated against Covid-19 in some countries.

From these data and other statistics, we conclude that besides the efforts made to provide the vaccine and facilitate access to it, it is also necessary to work on resisting and dealing with the rumors that impedes citizens' from getting vaccination and take booster doses. If rumors are not adequately combated, they will inevitably hamper the objective of achieving collective immunity.

The classical mathematical model of rumor spreading was introduced by Daley and Kendal [7, 8]. Based on the SIR epidemic model, the population is subdivided into three groups: those who are unaware of the rumor (ignorants), those who spread the rumor (spreaders), and those who are aware of the rumor but choose not to spread it (stiflers). Following Daley and Kendal's work, several studies and mathematical models have been introduced to address this issue from different angles using various approaches ([9–16] and the references mentioned therein). Several mathematical modeling studies have emerged that aim to use different methods and approaches to understand the coronavirus, describe its dynamics, and also propose optimal strategies in order to succeed in the vaccination programs. However, they have not taken into account the impact of the spread of rumors on the success of vaccination programs. We cite for example, [17–22].

In this study, we propose a mathematical model that describes the spread of rumors about vaccination and the risks of the booster doses in an environment infected by Covid-19. The aim is to emphasize the importance of combining two measures: dealing with the spread of rumors and awareness programs about the necessity of vaccination. We propose optimal control strategies in order to maximize the number of vaccinated individuals during the time interval $[t_0, t_f]$ and also to minimize the cost spent on this strategy. To achieve this objective, we include two controls u_1 and u_2. The control u_1 represents the effort of combating the spread of rumors about the vaccination against Covid-19. The control u_2 represents the effort of awareness campaigns to encourage people to get vaccinated and complete the doses. The optimal control problem is formulated and the existence of the optimal controls is investigated. The Pontryagin's maximum principle is used to characterize the optimal controls and the optimality system is solved by an iterative method. Finally, some numerical simulations are performed to verify the theoretical analysis using Matlab.

The paper is organized as follows. In section 2, we present the proposed mathematical model and we give some basic properties of the model. In section 3, we present the optimal control problem for the proposed model where we give some results concerning the existence of the optimal controls and we characterize these optimal controls using Pontryagin's maximum principle. Also, in section 4, numerical simulations are presented. Finally, we conclude the paper in section 5.

2. Mathematical model

We consider a mathematical model that describes the spread of rumors about vaccination and booster doses in an environment where Corona virus is transmitted among individuals. We also consider the impact of rumors spread on the success of the vaccination programs.

We divide the population denoted by N into seven compartments:

- The compartment S_p represents the individuals who are likely to be infected with Covid-19 and at the same time they spread rumors about the risks of vaccination and booster doses. This compartment is increased by a recruitment rate Λ and by a portion of individuals who decide not to be vaccinated by booster doses, either due to a personal conviction at a rate α_8, or due to effective contact with individuals of the compartment S_p at a rate δ_8. The S_p compartment is decreased by a portion of susceptible individuals who have been infected with Covid-19 due to effective contact with individuals of I_{S_p} and I_{S_t} respectively at a rate β_1 and β_2. Also, this compartment is decreased by a natural death rate μ and by the portion of individuals who decide to stop sharing rumors about vaccination, either by personal will at a rate α_1, or by a positive effect of individuals of S_t on the compartment S_p at a rate δ_1.
- The compartment S_t represents the individuals who are likely to be infected with Covid-19 and at the same time they stop the spread of rumors about vaccination. This compartment is increased by the individuals of S_p who decide to stop sharing rumors at rates α_1 and δ_1. It is decreased by a natural death rate μ and by a rate α_7 of individuals who decide to be vaccinated. Also, this compartment is decreased by the portion of individuals of susceptible individuals who have been infected with Covid-19 due to effective contact with individuals of I_{S_p} and I_{S_t} respectively at a rate β_3 and β_4.
- The compartment I_{S_p} represents the individuals infected by Corona virus and at the same time they are spreaders of rumors about vaccination and booster doses. This compartment is increased by the new infected individuals of compartment S_p, and decreased by a natural death rate μ and the infected people who decide to stop spreading the rumors. It is also decreased by the positive effect of individuals of the compartments I_{S_t}, S_t and R_{S_t} a rate respectively δ_2, δ_3 and δ_4.
- The compartment I_{S_t} represents the individuals infected by Corona virus and are stiflers of rumors about vaccination and booster doses. This compartment is increased by the individuals infected by Corona virus who stop the spread of the rumors. It is decreased by recovered individuals at rate α_5 and also by a natural death rate μ.

- The compartment R_{S_p} represents the recovered individuals who still spread rumors about vaccination. This compartment is increased by the newly recovered of I_{S_p} at rate α_4 and it is decreased by the individuals who decide to stop spreading rumors due to a personal conviction at rate α_3, or due to the effect of individuals of the compartments R_{S_t}, I_{S_t} and S_t respectively at rates δ_5, δ_6 and δ_7. It is also decreased by a natural death rate μ.
- The compartment R_{S_t} represents the recovered individuals who are also stifler of rumors about vaccination and booster doses. This compartment is increased by the individuals of R_{S_p} who stop the spread of the rumors. It is decreased by a natural death rate μ and by the individuals moving to vaccination program at a rate α_6.
- The compartment V represents the individuals vaccinated with the first dose or with the booster doses. This compartment is increased by the individuals of S_t and R_{S_t} who are decided to be vaccinated at rate respectively α_7 and α_6. It is decreased by a natural death rate μ, and by a portion of individuals who decide not to be vaccinated by booster doses, either due to personal conviction at a rate α_8, or due to the effect of individuals of the compartment S_p at a rate δ_8.

The variables $S_p(t)$, $S_t(t)$, $I_{S_p}(t)$, $I_{S_t}(t)$, $R_{S_p}(t)$, $R_{S_t}(t)$, and $V(t)$ are the numbers of the individuals in the seven classes at time t, respectively. The unit of time can correspond to periods, years, months; it depends on the frequency of the survey studies as needed. The graphical representation of the proposed model is shown in Figure 2.

Where $\psi_1 = \beta_1 I_{S_p} + \beta_2 I_{S_t}$, $\psi_2 = \beta_3 I_{S_p} + \beta_4 I_{S_t}$, $\varphi_1 = \delta_2 I_{S_t} + \delta_3 S_t + \delta_4 R_{S_t}$ and $\varphi_2 = \delta_5 R_{S_t} + \delta_6 I_{S_t} + \delta_7 S_t$.

The total population size at time t is denoted by $N(t)$ with $N(t) = S_p(t) + S_t(t) + I_{S_t}(t) + I_{S_p}(t) + R_{S_p}(t) + R_{S_t}(t) + V(t)$. The dynamics of this model are governed by the following nonlinear system of differential equations:

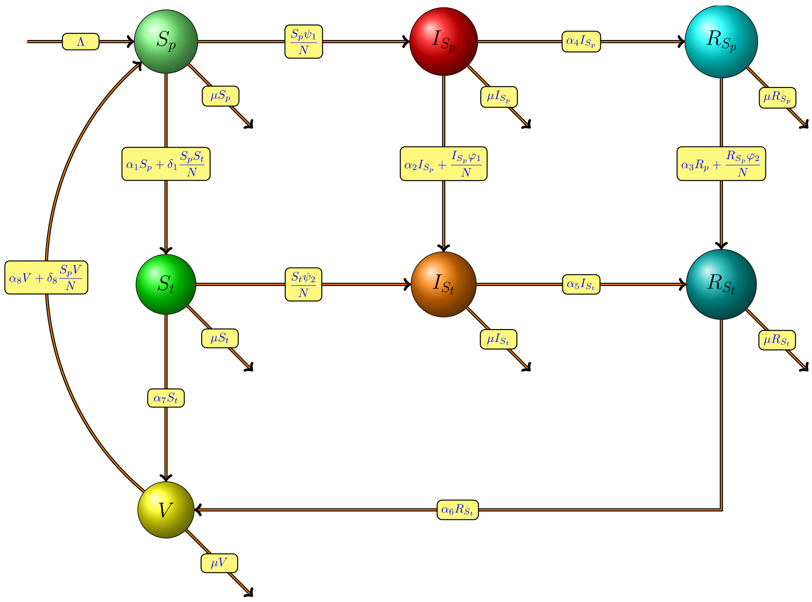

Fig. 2. Diagram transfer.

$$\begin{cases} \dot{S}_p(t) = \Lambda - \dfrac{S_p(t)}{N}\left[\psi_1(t) + \delta_1 S_t(t) - \delta_8 V(t)\right] - (\mu + \alpha_1) S_p(t) + \alpha_8 V(t), \\ \dot{S}_t(t) = \dfrac{S_t(t)}{N}\left[\delta_1 S_p(t) - \psi_2(t)\right] + \alpha_1 S_p(t) - (\mu + \alpha_7) S_t(t), \\ \dot{I}_{S_p}(t) = \dfrac{S_p(t)}{N}\psi_1(t) - \dfrac{I_{S_p}(t)}{N}\varphi_1(t) - (\mu + \alpha_2 + \alpha_4) I_{S_p}(t), \\ \dot{I}_{S_t}(t) = \dfrac{S_t(t)}{N}\left[\psi_2(t)\right] + \dfrac{I_{S_p}(t)}{N}\varphi_1(t) + \alpha_2 I_{S_p}(t) - (\mu + \alpha_5) I_{S_t}(t), \\ \dot{R}_{S_p}(t) = \alpha_4 I_{S_p}(t) - \dfrac{R_{S_p}(t)}{N}\varphi_2(t) - (\mu + \alpha_3) R_{S_p}(t), \\ \dot{R}_{S_t}(t) = \alpha_3 R_{S_p}(t) + \dfrac{R_{S_p}(t)}{N}\varphi_2(t) + \alpha_5 I_{S_t}(t) - (\mu + \alpha_6) R_{S_t}(t), \\ \dot{V}(t) = \alpha_6 R_{S_t}(t) + \alpha_7 S_t(t) - V(t)\left[\delta_8 \dfrac{S_p(t)}{N} + \alpha_8\right] - \mu V(t), \end{cases} \tag{1}$$

where $S_{p0} \geqslant 0$, $S_{t0} \geqslant 0$, $I_{S_p0} \geqslant 0$, $I_{S_t0} \geqslant 0$, $R_{S_p} \geqslant 0$, $R_{S_t} \geqslant 0$, and $V_0 \geqslant 0$ are the given initial states.

2.1. Some proprieties of the model

2.1.1. Boundedness of trajectories

The trajectories of the system (1) are bounded. Indeed, by adding all equations in (1), we obtain
$$\frac{dN}{dt} \leqslant \Lambda - \mu N.$$

Thus,
$$N(t) \leqslant N(0)\exp(-\mu t) - \frac{\Lambda}{\mu}\left(1 - \exp(-\mu t)\right),$$
where $N(0)$ represents the initial values of the total population. Thus $\lim\limits_{t\to\infty} \sup N(t) = \frac{\Lambda}{\mu}$. It implies that all possible solutions of the system (1) enter the region
$$\Omega = \left\{ \left(S_p(t), S_t(t), I_{S_p}(t), I_{S_t}(t), R_{S_p}(t), R_{S_t}(t), V(t)\right) \in \mathbb{R}^7_+, 0 \leqslant N(t) \leqslant \frac{\Lambda}{\mu} \right\}.$$

2.1.2. Existence of solutions

The system (1) can be rewritten as follows
$$\begin{aligned} \dot{X}(t) &= AX(t) + B(X(t)) \\ &= F(X(t)), \end{aligned}$$
where
$$A = \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 & a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & a_4 & 0 & 0 & 0 \\ 0 & 0 & \alpha_4 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & \alpha_5 & \alpha_3 & a_6 & 0 \\ 0 & \alpha_7 & 0 & 0 & 0 & \alpha_6 & -\mu \end{bmatrix}, \qquad \begin{cases} a_1 = -\left(\mu + \alpha_1\right), \\ a_2 = -\left(\mu + \alpha_7\right), \\ a_3 = -\left(\mu + \alpha_2 + \alpha_4\right), \\ a_4 = -\left(\mu + \alpha_5\right), \\ a_5 = -\left(\mu + \alpha_3\right), \\ a_6 = -\left(\mu + \alpha_6\right) \end{cases}$$
and
$$B(X(t)) = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 \end{bmatrix}^T,$$
$$\begin{cases} b_1 = \Lambda - \dfrac{S_p(t)}{N}\left[\beta_1 I_{S_p}(t) + \beta_2 I_{S_t}(t) + \delta_1 S_t(t) - \delta_8 V(t)\right], \\ b_2 = \dfrac{S_t(t)}{N}\left[\delta_1 S_p(t) - \beta_3 I_{S_p}(t) - \beta_4 I_{S_t}(t)\right] + \alpha_1 S_p(t), \\ b_3 = \dfrac{S_p(t)}{N}\left[\beta_1 I_{S_p}(t) + \beta_2 I_{S_t}(t)\right] - \dfrac{I_{S_p}(t)}{N}\left[\delta_2 I_{S_t}(t) + \delta_3 S_t(t) + \delta_4 R_{S_t}(t)\right], \\ b_4 = \dfrac{S_t(t)}{N}\left[\beta_3 I_{S_p}(t) + \beta_4 I_{S_t}(t)\right] + \dfrac{I_{S_p}(t)}{N}\left[\delta_2 I_{S_t}(t) + \delta_3 S_t(t) + \delta_4 R_{S_t}(t)\right], \\ b_5 = -\dfrac{R_{S_p}(t)}{N}\left[\delta_5 R_{S_t}(t) + \delta_6 I_{S_t}(t) + \delta_7 S_t(t)\right], \\ b_6 = \dfrac{R_{S_p}(t)}{N}\left[\delta_5 R_{S_t}(t) + \delta_6 I_{S_t}(t) + \delta_7 S_t(t)\right], \\ b_7 = -V(t)\left[\delta_8 \dfrac{S_p(t)}{N} + \alpha_8\right], \end{cases}$$
$$X(t) = \begin{bmatrix} S_p & S_t & I_{S_p} & I_{S_t} & R_{S_p} & R_{S_t} & V \end{bmatrix}^T.$$

The function B satisfies
$$\left\|B\left(X_1(t)\right) - B\left(X_2(t)\right)\right\| \leqslant M \left\|X_1(t) - X_2(t)\right\|,$$
where M is a positive constant.

Moreover,
$$\left\|F\left(X_1(t)\right) - F\left(X_2(t)\right)\right\| \leqslant L \left\|X_1(t) - X_2(t)\right\|,$$
where $L = \max\left(\|A\|, M\right)$.

Thus, it follows that the function F is uniformly Lipschitz continuous, we conclude that the solution of the system (1) exists (see [23]).

3. Optimal Control Problem

3.1. Problem statement

Immediately after the emergence of vaccination against Covid-19, several countries rushed to launch vaccination programs to achieve collective immunity against Corona virus. However, given the statistics indicated in the introduction, many countries still suffer from low citizen demand for vaccination or even completion of the second and third dose. Several factors result in this low demand, including the significant spread of rumors about vaccination and its health complications.

Therefore, successful vaccination programs require resisting the spread of rumors in parallel with awareness campaigns to encourage people to be vaccinated.

Achieving this objective necessitates developing optimal strategies for awareness programs and anti-rumors programs that help countries to achieve successful vaccination programs with optimal effort. The effort made always involves time, logistics, money, and human resources. So, our objective in this proposed strategy of control is to maximize the number of vaccinated $V(t)$ during the time interval $[t_0, t_f]$ and also to minimize the cost spent on this strategy.

In the model (1), we include two controls $u_1(t)$ and $u_2(t)$ for $t \in [t_0, t_f]$. The control u_1 represents the effort provided to combat the spread of rumors about the vaccination against Covid-19. The control u_2 represents the effort of awareness campaigns to encourage people to vaccination programs and complete the doses.

So, the controlled mathematical system is given by the following system of differential equations:

$$
\begin{cases}
\dot{S}_p(t) = \Lambda - \dfrac{S_p(t)}{N}\left[\psi_1(t) + \delta_1 S_t(t) - \delta_8(1-u_2(t))V(t)\right] - (\mu + \alpha_1 + u_1(t))S_p(t) \\
\qquad +\alpha_8(1-u_2(t))V(t), \\
\dot{S}_t(t) = \dfrac{S_t(t)}{N}\left[\delta_1 S_p(t) - \psi_2(t)\right] + (\alpha_1 + u_1(t))S_p(t) - (\mu + \alpha_7 + u_2(t))S_t(t), \\
\dot{I}_{S_p}(t) = \dfrac{S_p(t)}{N}\psi_1(t) - \dfrac{I_{S_p}(t)}{N}\varphi_1(t) - (\mu + \alpha_2 + \alpha_4 + u_1(t))I_{S_p}(t) \\
\dot{I}_{S_t}(t) = \dfrac{S_t(t)}{N}[\psi_2(t)] + \dfrac{I_{S_p}(t)}{N}\varphi_1(t) + (\alpha_2 + u_1(t))I_{S_p}(t) - (\mu + \alpha_5)I_{S_t}(t), \\
\dot{R}_{S_p}(t) = \alpha_4 I_{S_p}(t) - \dfrac{R_{S_p}(t)}{N}\varphi_2(t) - (\mu + \alpha_3 + u_1(t))R_{S_p}(t), \\
\dot{R}_{S_t}(t) = (\alpha_3 + u_1(t))R_{S_p}(t) + \dfrac{R_{S_p}(t)}{N}\varphi_2(t) + \alpha_5 I_{S_t}(t) - (\mu + \alpha_6 + u_2(t))R_{S_t}(t), \\
\dot{V}(t) = \alpha_6 R_{S_t}(t) + \alpha_7 S_t(t) - (1-u_2(t))V(t)\left[\delta_8 \dfrac{S_p(t)}{N} + \alpha_8\right] - \mu V(t) \\
\qquad +u_2(t)(S_t(t) + R_{S_t}(t)),
\end{cases} \tag{2}
$$

where $S_{p0} \geqslant 0$, $S_{t0} \geqslant 0$, $I_{S_{p0}} \geqslant 0$, $I_{S_{t0}} \geqslant 0$, $R_{S_{p0}} \geqslant 0$, $R_{S_{t0}} \geqslant 0$, and $V_0 \geqslant 0$ are the given initial states.

Then, the problem is to minimize the objective functional

$$
J(u_1, u_2) = -V(t_f) + \int_{t_0}^{t_f} \left[-V(s) + \frac{A}{2}u_1^2(s) + \frac{B}{2}u_2^2(s)\right] ds.
$$

Where the parameters A and B are the strictly positive cost coefficients. They are selected to weight the relative importance of u_1 and u_2 at time t; t_f is the final time.

In other words, we seek the optimal controls u_1 and u_2 such that

$$
J(u_1^*, u_2^*) = \min_{(u_1,u_2)\in U_{ad}^2} J(u_1, u_2).
$$

Where U_{ad} is the set of admissible controls defined by

$$
U_{ad} = \left\{u_i(t) \colon 0 \leqslant u_i \leqslant 1, \text{ for } i = 1,2 \text{ and } t \in [t_0, t_f]\right\}.
$$

3.2. Existence of the optimal controls

The existence of the optimal controls can be obtained using a result by Fleming and Rishel [24] (see Corollary 4.1).

Theorem 1. *Consider the control problem with system (2). There exists an optimal control* $(u_1^*, u_2^*) \in U_{ad}^2$ *such that* $J(u_1^*, u_2^*) = \min\limits_{(u_1,u_2)\in U_{ad}^2} J(u_1, u_2)$. *If the following conditions are met:*

1) *The set of controls and corresponding state variables is nonempty.*
2) *The control set* U_{ad} *is convex and closed.*
3) *The right-hand side of the state system is bounded by a linear function in the state and control variables.*
4) *The integrand,* $L(S_p, S_t, I_{S_p}, I_{S_t}, R_{S_p}, R_{S_t}, V, u_1, u_2)$, *of the objective functional is convex on* U_{ad} *and there exist constants* $c_1, c_2 \succ 0$ *and* $\beta \succ 1$ *such that*

$$L(S_p, S_t, I_{S_p}, I_{S_t}, R_{S_p}, R_{S_t}, V, u_1, u_2) \geqslant -c_1 + c_2\big(|u_1|^2 + |u_2|^2\big)^{\beta/2}.$$

Proof. Condition 1:

To prove that the set of controls and corresponding state variables is nonempty, we use a simplified version of an existence result ([25] Theorem 7.1.1).

Let $\dot{S}_p = F_{S_p}(t; S_p, \ldots, V), \ldots, \dot{V} = F_V(t; S_p, \ldots, V)$, where the $F_{S_p}, \ldots, F_V$ form the right hand side of the system of equations (2). Let $u_i(t) = C_i$ for $i = 1, 2$ for some constant, and since all parameters are constants and $S_p, S_t, I_{S_p}, I_{S_t}, R_{S_p}, R_{S_t}$ and V are continuous, then $F_{S_p}, \ldots, F_V$ are also continuous.

Additionally, the partial derivatives $\partial F_{S_p}/\partial S_p, \ldots, \partial F_{S_p}/\partial V; \ldots; \partial F_V/\partial S_p, \ldots, \partial F_V/\partial V$ are all continuous. Therefore, there exists a unique solution $(S_p, S_t, I_{S_p}, I_{S_t}, R_{S_p}, R_{S_t}, V)$ that satisfies the initial conditions. Then, the set of controls and corresponding state variables is nonempty and condition 1 is satisfied.

Condition 2: By definition, U_{ad} is closed. Take any controls $u, v \in U_{ad}$ and $\lambda \in [0, 1]$, then $0 \leqslant \lambda u + (1 - \lambda)\, v$.

Additionally, we observe that $\lambda u \leqslant \lambda$ and $(1-\lambda)v \leqslant (1-\lambda)$, then $\lambda u + (1-\lambda)v \leqslant \lambda + (1-\lambda) = 1$. Hence,

$$0 \leqslant \lambda u + (1-\lambda)v \leqslant 1 \quad \text{for all} \quad u, v \in U_{ad} \quad \text{and} \quad \lambda \in [0, 1].$$

Therefore, U_{ad} is convex and condition 2 is satisfied.

Condition 3: From the system of differential equations (2), we have

$$\frac{dN}{dt} \leqslant \Lambda - \mu N.$$

Then

$$\limsup_{t\to\infty} N(t) \leqslant \frac{\Lambda}{\mu}$$

Therefore, all solutions of the model (2) are bounded. So, there exist positive constants B_1, B_2, B_3, B_4, B_5, B_6, and B_7 such that, $\forall t \in [t_0, t_f]$:

$$S_p(t) \leqslant B_1,\ S_t(t) \leqslant B_2,\ I_{S_p}(t) \leqslant B_3,\ I_{S_t}(t) \leqslant B_4,\ R_{S_p}(t) \leqslant B_5,\ R_{S_t}(t) \leqslant B_6, \text{ and } V(t) \leqslant B_7.$$

We consider,

$$\begin{cases} \dot{S}_p(t) = F_{S_p} \leqslant \Lambda + K_1, \\ \dot{S}_t(t) = F_{S_t} \leqslant S_p(t)K_2 + u_1(t)B_1, \\ \dot{I}_{S_p(t)} = F_{I_{S_p}} \leqslant I_{S_p}(t)K_3 + K_4, \\ \dot{I}_{S_t}(t) = F_{I_{S_t}} \leqslant I_{S_t}(t)K_5 + K_6 + u_1(t)B_3, \\ \dot{R}_{S_p}(t) = F_{R_{S_p}} \leqslant I_{S_p}(t)K_7 + u_1(t)B_5, \\ \dot{R}_{S_t}(t) = F_{R_{S_t}} \leqslant R_{S_t}(t)K_8 + K_9, \\ \dot{V}(t) = F_V \leqslant S_t(t)K_{10} + K_{11} + u_2(t)(B_2 + B_6), \end{cases}$$

where

$$\begin{cases} K_1 = \delta_8 \dfrac{B_1 B_8}{N} + \alpha_8 B_8, \\ K_2 = \delta_1 \dfrac{B_2}{N} + \alpha_1, \\ K_3 = \beta_1 \dfrac{B_1}{N}, \\ K_4 = \beta_2 \dfrac{B_1 B_4}{N}, \\ K_5 = \beta_4 \dfrac{B_2}{N} + \delta_2 \frac{B_3}{N}, \end{cases} \qquad \begin{cases} K_6 = \dfrac{1}{N}\left[2\delta_3 B_3 B_2 + \delta_4 B_3 B_5 + \alpha_2 B_3\right], \\ K_7 = \alpha_4, \\ K_8 = \delta_5 \dfrac{B_5}{N}, \\ K_9 = \dfrac{1}{N}\left[\delta_6 B_5 B_4 + \delta_7 B_5 B_2 + \alpha_3 B_5\right], \\ K_{10} = \alpha_7, \\ K_{11} = \alpha_6 B_7. \end{cases}$$

So, we can rewrite the system (2) in a matrix form:

$$F(t; S_p, \dots, V) \leqslant \bar{\Lambda} + AX(t) + BU(t),$$

where

$$\begin{aligned} F(t; S_p, \dots, V) &= \begin{bmatrix} F_{S_p} & F_{S_t} & F_{I_{S_p}} & F_{I_{S_t}} & F_{R_{S_p}} & F_{R_{S_t}} & F_V \end{bmatrix}^T, \\ \bar{\Lambda} &= \begin{bmatrix} \Lambda + K_1 & 0 & K_4 & K_6 & 0 & K_9 & K_{11} \end{bmatrix}^T, \\ X(t) &= \begin{bmatrix} S_p & S_t & I_{S_p} & I_{S_t} & R_{S_p} & R_{S_t} & V \end{bmatrix}^T, \\ U(t) &= \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T \end{aligned}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_5 & 0 & 0 & 0 \\ 0 & 0 & K_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_8 & 0 \\ 0 & K_{10} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ B_1 & 0 \\ 0 & 0 \\ B_3 & 0 \\ B_5 & 0 \\ 0 & 0 \\ 0 & B_2 + B_6 \end{bmatrix},$$

which gives a linear function of controls vector and state variables vector. Therefore we can write

$$\begin{aligned} \|F(t; S_p, \dots, V)\| &\leqslant \left\|\bar{\Lambda}\right\| + \|A\| \, \|X(t)\| + \|B\| \, \|U(t)\| \\ &\leqslant \varphi + \phi\left(\|X(t)\| + \|U(t)\|\right), \end{aligned}$$

where $\varphi = \left\|\bar{\Lambda}\right\|$ and $\phi = \max\left(\|A\|, \|B\|\right)$. Hence, we see the right hand side of the state system is bounded by a sum of state and control vector. Therefore, condition 3 is satisfied.

Condition 4: The integrand in the objective functional (2) is convex on U_{ad}. It rests to show that there exist constants $c_1, c_2 \succ 0$ and $\beta \succ 1$ such that the integrand $L(S_p, \dots, V, u_1, u_2)$ of the objective functional satisfies

$$\begin{aligned} L(S_p, \dots, V, u_1, u_2) &= -V(t) + \frac{M}{2} u_1^2 + \frac{N}{2} u_2^2 \\ &\geqslant -c_1 + c_2 \left(|u_1|^2 + |u_2|^2\right)^{\beta/2}. \end{aligned}$$

The state variables being bounded, let $c_1 = \sup\limits_{t \in [t_0, t_f]} (V(t))$, $c_2 = \inf\left(\frac{M}{2}, \frac{N}{2}\right)$ and $\beta = 2$ then it follows that

$$L(S_p, \dots, V, u_1, u_2) \geqslant -c_1 + c_2 \left(|u_1|^2 + |u_2|^2\right)^{\beta/2}. \qquad \blacksquare$$

3.3. Characterization of the optimal controls

To characterize the optimal controls for our problem, we apply the Pontryagin's Maximum Principle [26]. The key idea is introducing the adjoint function to attach the system of differential equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts the problem of finding the control to optimize the objective functional subject to the state differential equations with initial condition to find the control to optimize Hamiltonian pointwise (with respect to the control).

Now, we have the Hamiltonian H in time t, defined by

$$H(t) = -V(t) + \frac{M}{2}u_1^2(t) + \frac{N}{2}u_2^2(t) + \sum_{i=1}^{7} \lambda_i f_i,$$

where f_i is the right side of the system of differential equations (2) of the i^{th} state variable.

Theorem 2. *Given an optimal control $u^* = (u_1^*, u_2^*) \in U_{ad}^2$, and solutions S_p^*, S_t^*, $I_{S_p}^*$, $I_{S_t}^*$, $R_{S_p}^*$, $R_{S_t}^*$ and V^* of corresponding state system (2), there exist adjoint functions, λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, and λ_7 satisfying*

$$\begin{cases}
\dot{\lambda}_1 = \lambda_1 \left\{ \frac{1}{N}\left[\psi_1(t) + \delta_1 S_t(t) - \delta_8(1-u_2(t))V(t)\right] + (\mu + \alpha_1 + u_1(t)) \right\} \\
\quad -\lambda_2 \left\{ \delta_1 \frac{S_t(t)}{N} + (\alpha_1 + u_1(t)) \right\} - \lambda_3 \left\{ \frac{1}{N}\psi_1(t) \right\}, \\
\dot{\lambda}_2 = \lambda_1 \left\{ \delta_1 \frac{S_p(t)}{N} \right\} - \lambda_2 \left\{ \frac{1}{N}\left[\delta_1 S_p(t) - \psi_2(t)\right] - (\mu + \alpha_7 + u_2(t)) \right\} + \lambda_3 \left\{ \delta_3 \frac{I_{S_p}(t)}{N} \right\} \\
\quad -\lambda_4 \left\{ \frac{1}{N}\psi_2(t) + \delta_3 \frac{I_{S_p}(t)}{N} \right\} + \lambda_5 \left\{ \delta_7 \frac{R_{S_p}(t)}{N} \right\} - \lambda_6 \left\{ \delta_7 \frac{R_{S_p}(t)}{N} \right\} - \lambda_7 \left\{ \alpha_7 + u_2(t) \right\}, \\
\dot{\lambda}_3 = \lambda_1 \left\{ \beta_1 \frac{S_p(t)}{N} \right\} + \lambda_2 \left\{ \beta_3 \frac{S_t(t)}{N} \right\} - \lambda_3 \left\{ \beta_1 \frac{S_p(t)}{N} - \frac{1}{N}\varphi_1(t) - (\mu + \alpha_2 + \alpha_4 + u_1(t)) \right\} \\
\quad -\lambda_4 \left\{ \beta_3 \frac{S_t(t)}{N} + \frac{1}{N}\varphi_1(t) + (\alpha_2 + u_1(t)) \right\} - \lambda_5 \left\{ \alpha_4 \right\}, \\
\dot{\lambda}_4 = \lambda_1 \left\{ \beta_2 \frac{S_p(t)}{N} \right\} + \lambda_2 \left\{ \beta_4 \frac{S_t(t)}{N} \right\} - \lambda_3 \left\{ \beta_2 \frac{S_p(t)}{N} - \delta_2 \frac{I_{S_p}(t)}{N} \right\} \\
\quad -\lambda_4 \left\{ \beta_4 \frac{S_t(t)}{N} + \delta_2 \frac{I_{S_p}(t)}{N} - (\mu + \alpha_5) \right\} + \lambda_5 \left\{ \delta_6 \frac{R_{S_p}(t)}{N} \right\} - \lambda_6 \left\{ \delta_6 \frac{R_{S_p}(t)}{N} + \alpha_5 \right\} \\
\dot{\lambda}_5 = \lambda_5 \left\{ \frac{1}{N}\varphi_2(t) + (\mu + \alpha_3 + u_1(t)) \right\} - \lambda_6 \left\{ (\alpha_3 + u_1(t)) + \frac{1}{N}\varphi_2(t) \right\} \\
\dot{\lambda}_6 = \lambda_3 \left\{ \delta_4 \frac{I_{S_p}(t)}{N} \right\} - \lambda_4 \left\{ \delta_4 \frac{I_{S_p}(t)}{N} \right\} + \lambda_5 \left\{ \delta_5 \frac{R_{S_p}(t)}{N} \right\} \\
-\lambda_6 \left\{ \delta_5 \frac{R_{S_p}(t)}{N} - (\mu + \alpha_6 + u_2(t)) \right\} \qquad - \lambda_7 \left\{ \alpha_6 + u_2(t) \right\} \\
\dot{\lambda}_7 = -1 - \lambda_1 (1 - u_2(t)) \left\{ \delta_8 \frac{S_p(t)}{N} + \alpha_8 \right\} + \lambda_7 \left\{ (1 - u_2(t)) \left[\delta_8 \frac{S_p(t)}{N} + \alpha_8 \right] + \mu \right\}
\end{cases}$$

with the transversality conditions at time t_f

$$\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = \lambda_6(t_f) = 0 \;\text{ and }\; \lambda_7(t_f) = -1.$$

Furthermore, for $t \in [t_0, t_f]$ the optimal controls $u_1^*(t)$ and $u_2^*(t)$ are given by

$$u_1^*(t) = \min\left(1, \max\left(0, \frac{1}{M}\left[(\lambda_1 - \lambda_2)S_p(t) + (\lambda_3 - \lambda_4)I_{S_p}(t) + (\lambda_5 - \lambda_6)R_{S_p}(t)\right]\right)\right),$$

$$u_2^*(t) = \min\left(1, \max\left(0, \frac{1}{N}\left[(\lambda_2 - \lambda_7)S_t(t) + (\lambda_6 - \lambda_7)R_{S_t}(t) + (\lambda_1 - \lambda_7)\left\{ V(t)\left[\delta_8 \frac{S_p(t)}{N} + \alpha_8\right]\right\}\right]\right)\right).$$

Proof. The Hamiltonian in time t is given by

$$\begin{aligned}
H &= -V(t) + \frac{M}{2}u_1^2(t) + \frac{N}{2}u_2^2(t) \\
&+ \lambda_1 \left\{ \Lambda - \frac{S_p(t)}{N}\left[\psi_1(t) + \delta_1 S_t(t) - \delta_8(1-u_2(t))V(t)\right] - (\mu + \alpha_1 + u_1(t))S_p(t) + \alpha_8(1-u_2(t))V(t) \right\} \\
&+ \lambda_2 \left\{ \frac{S_t(t)}{N}\left[\delta_1 S_p(t) - \psi_2(t)\right] + (\alpha_1 + u_1(t))S_p(t) - (\mu + \alpha_7 + u_2(t))S_t(t) \right\}
\end{aligned}$$

$$
\begin{aligned}
&+\lambda_3\left\{\frac{S_p(t)}{N}\psi_1(t)-\frac{I_{S_p}(t)}{N}\varphi_1(t)-(\mu+\alpha_2+\alpha_4+u_1(t))I_{S_p}(t)\right\}\\
&+\lambda_4\left\{\frac{S_t(t)}{N}[\psi_2(t)]+\frac{I_{S_p}(t)}{N}\varphi_1(t)+(\alpha_2+u_1(t))I_{S_p}(t)-(\mu+\alpha_5)I_{S_t}(t)\right\}\\
&+\lambda_5\left\{\alpha_4 I_{S_p}(t)-\frac{R_{S_p}(t)}{N}\varphi_2(t)-(\mu+\alpha_3+u_1(t))R_{S_p}(t)\right\}\\
&+\lambda_6\left\{(\alpha_3+u_1(t))R_{S_p}(t)+\frac{R_{S_p}(t)}{N}\varphi_2(t)+\alpha_5 I_{S_t}(t)-(\mu+\alpha_6+u_2(t))R_{S_t}(t)\right\}\\
&+\lambda_7\left\{\alpha_6 R_{S_t}(t)+\alpha_7 S_t(t)-(1-u_2(t))V(t)\left[\delta_8\frac{S_p(t)}{N}+\alpha_8\right]-\mu V(t)+u_2(t)\left(S_t(t)+R_{S_t}(t)\right)\right\}.
\end{aligned}
$$

For $t\in[t_0,t_f]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle given in [26] such that

$$
\begin{cases}
\dot{\lambda}_1=-\dfrac{\partial H}{\partial S_p}, & \lambda_1(t_f)=0,\\
\dot{\lambda}_2=-\dfrac{\partial H}{\partial S_t}, & \lambda_2(t_f)=0,\\
\dot{\lambda}_3=-\dfrac{\partial H}{\partial I_{S_p}}, & \lambda_3(t_f)=0,\\
\dot{\lambda}_4=-\dfrac{\partial H}{\partial I_{S_t}}, & \lambda_4(t_f)=0,\\
\dot{\lambda}_5=-\dfrac{\partial H}{\partial R_{S_p}}, & \lambda_5(t_f)=0,\\
\dot{\lambda}_6=-\dfrac{\partial H}{\partial R_{S_t}}, & \lambda_6(t_f)=0,\\
\dot{\lambda}_7=-\dfrac{\partial H}{\partial V}, & \lambda_7(t_f)=-1.
\end{cases}
$$

For $t\in[t_0,t_f]$, the optimal controls $u_1^*(t)$ and $u_2^*(t)$ can be solved from the optimality condition,

$$\frac{\partial H}{\partial u_1}=0 \quad\text{and}\quad \frac{\partial H}{\partial u_2}=0.$$

that is

$$
\begin{aligned}
u_1(t)&=\frac{1}{M}\left[(\lambda_1-\lambda_2)S_p(t)+(\lambda_3-\lambda_4)I_{S_p}(t)+(\lambda_5-\lambda_6)R_{S_p}(t)\right],\\
u_2(t)&=\frac{1}{N}\left[(\lambda_2-\lambda_7)S_t(t)+(\lambda_6-\lambda_7)R_{S_t}(t)+(\lambda_1-\lambda_7)\left\{V(t)\left[\delta_8\frac{S_p(t)}{N}+\alpha_8\right]\right\}\right].
\end{aligned}
$$

By the bounds in U_{ad} of the controls, it is easy to obtain $u_1^*(t)$ and $u_2^*(t)$ in the form of (2). ■

4. Numerical simulation

In this section, we present the results obtained by solving the optimality system numerically. In our control problem, we have initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with separated boundary conditions at times step $i=t_0$ and $i=t_f$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration, we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in Matlab using the following data: $S_p(0)=8\cdot10^6$, $S_t(0)=4\cdot10^6$, $I_{S_p}(0)=300$, $I_{S_t}(0)=700$, $R_{S_p}(0)=0.4\cdot10^6$, $R_{S_t}(0)=0.6\cdot10^6$, and $V(0)=1\cdot10^6$, $\Lambda=756$, $\mu=0.054$, $\alpha_1=0.15$, $\alpha_2=0.02$, $\alpha_3=0.01$, $\alpha_4=0.09$, $\alpha_5=0.21$, $\alpha_6=0.40$, $\alpha_7=0.40$, $\alpha_8=0.75$, $\beta_1=0.30$, $\beta_2=0.15$, $\beta_3=0.45$, $\beta_4=0.20$, $\delta_1=0.10$, $\delta_2=0.02$, $\delta_3=0.03$, $\delta_4=0.04$, $\delta_5=0.06$, $\delta_6=0.07$, $\delta_7=0.07$, $\delta_8=0.85$.

We note here that it is possible to use statistical data for a specific country or region in order to derive conclusions specific to that country or region. This matter requires statistical studies related to the subject being studied, which can be worked on in a future study. In this work, we only give general and arbitrary values for the variables and parameters. Eventually, the purpose is to put forward general conclusions and recommendations and examples of controls strategies that can be adopted as well as to compare between them and their effects.

The proposed control strategies in this work help country officials to achieve successful vaccination programs with optimal effort in order to achieve collective immunity and thus approaching to the elimination of the Corona virus:

Strategy 1: The awareness campaigns about the importance of vaccination.

In this strategy, we focus on the effort of awareness campaigns to encourage people to be vaccinated and complete the booster doses. We use only the optimal control u_2.

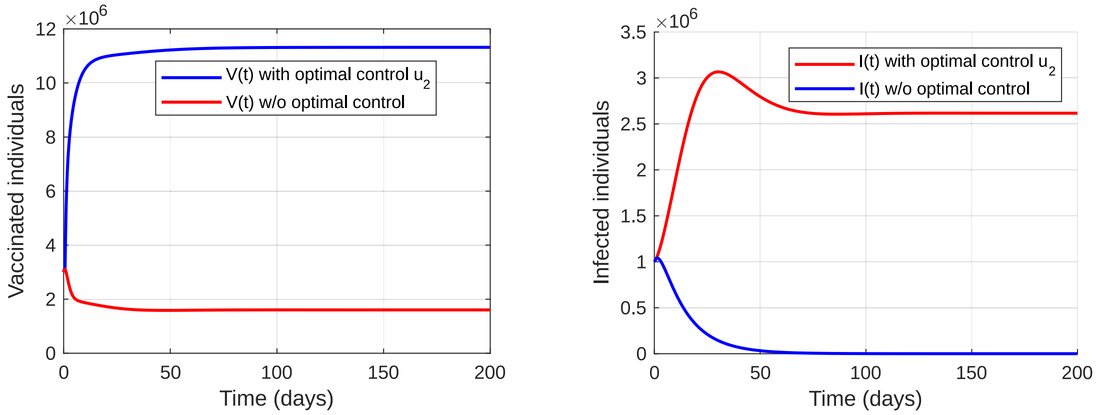

Fig. 3. The evolution of the number of vaccinated and the infected individuals with optimal control u_2.

From these figures, we observe that the number of vaccinated individuals has increased from $1 \cdot 10^6$ to $6.08 \cdot 10^6$ at the end of this awareness campaign (Figure 3) by a difference of $5.03 \cdot 10^6$ vaccinated individuals between case with control and the case without control. Also, the number of infected individuals decreased by 38.06% after introducing the optimal control. These changes are important but not sufficient to make the vaccination programs successful and combat the spread of the Corona virus. Therefore, we must also target the reasons that prevent people from getting vaccinated, especially the spread of rumors by this strategy of control.

Strategy 2: Combining awareness campaigns and confronting rumors.

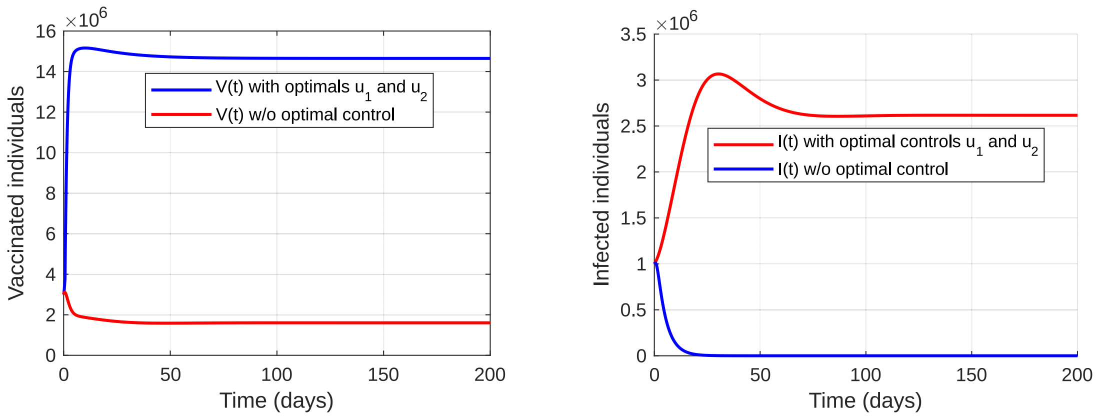

Fig. 4. The evolution of the number of vaccinated and the infected individuals with optimal controls u_1 and u_2.

In this strategy, we focus, beside the awareness campaigns, on confronting the spread of rumors about vaccination and booster doses. We use two optimal controls u_1 and u_2 at the same time in order to improve the results of strategy 1 about the vaccinated and infected rates. The optimal control u_1 represents the effort provided to combat the spread of rumors about the vaccination against Covid-19.

From Figure 4, we can see that the number of vaccinated individuals increased more significantly from $1 \cdot 10^6$ to $11.2 \cdot 10^6$. The number of the vaccinated people at the end of the strategy reaches 80% of the population, which can lead to collective immunity. Also, Figure 4 demonstrates that the number of infected decreased by 97.69% at the end of this optimal strategy.

Finally, we conclude that the proposed strategy becomes more effective and can lead to collective immunity when we combined the awareness campaigns about the importance of vaccination and the programs of confronting the spread of rumors about risks of vaccination and booster doses.

5. Conclusion

In this work, we formulated a mathematical model that describes the effect of rumors on the success of the vaccination programs against Covid-19 in an environment infected by Corona virus. The objective is to highlight the importance of confronting the spread of rumors about risks of vaccination and booster doses, in order to better execute the vaccination programs and achieve collective immunity. Also, we proposed optimal strategies for awareness programs and anti-rumors programs that help country officials to achieve successful vaccination programs with optimal effort. We introduced two controls; the first control represents the effort provided to combat the spread of rumors about the vaccination against Covid-19. The second control represents the effort of awareness campaigns to encourage people to be vaccinated and complete the doses. The optimal control problem is formulated and the existence of the optimal controls is investigated. Pontryagin's maximum principle was used to characterize the optimal controls and the optimality system was solved by an iterative method.

Consequently, the proposed mathematical modeling and the optimal control strategies confirm that to better execute a vaccination program and to reach collective immunity against the Corona virus, we must combine awareness campaigns about the importance of vaccination and the program of confronting the spread of rumors about risks of vaccination and booster doses.

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Вплив чуток на успіх програм вакцинації проти Covid-19 у зараженому коронавірусом середовищі: підхід оптимального керування

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У цій статті запропоновано математичну модель, що описує вплив чуток на успіх програм вакцинації проти Covid-19 у середовищі, яке заражене коронавірусом. Мета цього дослідження полягає в тому, щоб підкреслити роль боротьби з поширенням чуток щодо ризиків вакцинації та бустерних доз в успіху програм вакцинації та досягненні колективного імунітету. Крім того, сформульовано задачу оптимального керування, пропонуючи декілька стратегій, включаючи програми підвищення обізнаності та боротьби з чутками, щоб допомогти офіційним особам країни досягти успішних програм вакцинації з оптимальними зусиллями. Досліджено існування оптимальних керувань і для їх характеристики використано принцип максимуму Понтрягіна. Система оптимальності розв'язується ітераційним методом. Нарешті, здійснено чисельне моделювання для перевірки теоретичного аналізу за допомогою Matlab.

Ключові слова: *математичне моделювання; оптимальний контроль; чутки; щеплення; Covid-19.*