TRANSFORMING AND PROCESSING THE MEASUREMENT SIGNALS

SUBSTANTIATION OF THE ANOMALIES OF THE MEASUREMENT RESULTS FOR TRAJECTORIES OF GRAVITATIONAL MANEUVER OF SPACE VEHICLES

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https://doi.org/10.23939/istcmtm2024.01.005

Abstract. In astronomical research, the problem of measuring the trajectories of the gravitational maneuver of space vehicles in the gravitational field of large celestial bodies arises. The known measurement results differ from those predicted by classical celestial mechanics. A practical solution to this anomaly is possible only based on an adequate mathematical model. For this purpose, we have adapted Newton's law of universal gravitation to the case of moving masses in a possible range of speeds in flat space and physical time. At the same time, the finite speed of propagation of the gravitational field is taken into account. The differential equations of motion of cosmic bodies have been obtained. In the heliocentric and planetocentric coordinate systems, transient processes in the cosmic three-mass system are simulated – star, planet, and man-made spacecraft (Sun-Mercury-space probe). To more deeply identify the essence of gravitational interaction, transient processes of both acceleration and deceleration of the space vehicle were simulated depending on the specified space-velocity initial conditions for the differential equations of motion. The results of the simulation of transient processes are attached.

Key words: Gravitational maneuver anomaly, Euclidean space, physical time, Newton's law of gravitation of moving masses, differential equations of motion of celestial bodies, and cosmic three-body system.

1. Introduction

The article is a direct addition to the work [1] published on the pages of this magazine, devoted to the refutation of the anomaly of the movement of the Pioneer spacecraft in the Sun's gravitational field. A similar solution to the problem of the anomaly in the passive gravitational maneuver of spacecraft in the gravitational field of large celestial bodies is proposed here. Gravitational maneuver or slingshot effect is an acceleration, deceleration, or change in the direction of flight of a spacecraft under the influence of the gravitational fields of celestial bodies. It is used to save fuel and achieve high speeds during flights of automatic interplanetary stations to distant planets of the Solar System [2-3]. Thus, the kinetic energy of the spacecraft can be changed without fuel consumption. The most profitable gravitational maneuvers are near giant planets, but most often maneuvers occur near Venus, Earth, Mars, and even the Moon.

It is believed that the first gravitational maneuver was carried out in 1974 by the Mariner-10 spacecraft – it approached Venus, after which the spacecraft headed for Mercury. A complex combination of gravitational maneuvers was used by the automatic interplanetary station Cassini. For acceleration, the device used the gravitational field of three planets – Venus (twice), Earth, and Jupiter. The Russians, as usual, pull the blanket of supremacy over themselves in hindsight. The idea of the gravity maneuver belongs to the Ukrainian scientist Yury Kondratyuk, who was murdered by the same Russians. As early as 1918, in his work "To Those Who Will Read to Build" (printed in 1937), he proposed that a spacecraft traveling between two planets could accelerate at the beginning of its trajectory and slow down at the end of it with the help of gravity heavenly bodies.

2. Goal

Considering the finite speed of propagation of the gravitational field, to develop on a strict mathematical basis the nonlinear differential equations of motion of celestial bodies; based on the results of their integration, to obtain mathematical criteria for the passive gravitational maneuver of space vehicles in the gravitational field of a 3-body space system and to carry out its simulation in dynamics.

3. The equation of motion of celestial bodies

Successful mathematical modeling of transient processes of the gravitational maneuver of spacecraft in Euclidean space and physical time is possible only based on the equations of celestial mechanics, in which Newton's law adapted to the case of moving masses is involved [4-6]. The differential equations of moving n masses interconnected by gravity are the following:

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{m_i} \sum_{k=1;k\neq i}^{n} \mathbf{F}_{ik}; \quad \frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \quad i,k = 1,2,...,n, \qquad (1)$$

where $\mathbf{r}_i, \mathbf{v}_i$ is the radius vector of the trajectory and the velocity vector of the i-th mass m_i; $\mathbf{F}_{ik}$ is the vector of the gravitational interaction of the i-th and the k-th masses; t is the time. Equations (1) require explanation, as they may be about sub-light velocities. Functional dependence $m = m(v)$ is one of the unfortunate misunderstandings of the special theory of relativity, not a mathematical one, but an incorrect physical interpretation. The fact is that the Lorentz coefficient refers to the force interaction of masses, not the masses themselves! This is crystallized in the process of taking into account the finite velocity of gravity propagation.

We write the force vector in the general form [4]:

$$\mathbf{F}_{i,k} = G\frac{m_i m_k}{r_{ik}^2}\left(1+\frac{v_{ik}^2}{c^2}+2\frac{v_{ik}}{c}\mathbf{r}_{ik0}\cdot\mathbf{v}_{ik0}\right)\mathbf{r}_{ik0}, \qquad (2)$$

where r_{ik} is the radius of the distance between the masses; v_{ik} is the mutual instantaneous movement speed; G is the gravity constant; $\mathbf{r}_{ik0}, \mathbf{v}_{ik0}$ are the unit vectors of distance and movement speed. We can define the module of the force vector (2) component by component:

$$F_{Nik} = G\frac{m_i m_k}{r_{ik}^2}, \quad F_{Lik} = G\frac{m_i m_k}{r_{ik}^2}\frac{v_{ik}^2}{c^2},$$
$$F_{Tik} = 2G\frac{m_i m_k}{r_{ik}^2}\left(\frac{v_{ik}}{c}\mathbf{r}_0\cdot\mathbf{v}_0\right) \qquad (3)$$

Here F_{Nik} is Newton's gravitational force, F_{Lik}, F_{Tik} which is the tangential and radial velocity components of the gravitational force. It is clear that at $v_{ik} \to 0$, the modulus of force interaction (2) degenerates into (3). The marginal fate participation in the force interaction of the first and third components (3), based on the speed and orientation characteristics, is obvious:

$$\mathbf{F}_L = (0 \div 1)\mathbf{F}_N; \quad \mathbf{F}_T = ((-2)\div(+2))\mathbf{F}_N. \qquad (4)$$

It was proved in [4, 6] that the second force component (2), determined by the tangential velocity component, effectively coincides with the Lorentz force in the electric field, which in classical electrodynamics represents the force effect of the magnetic field or the so-called relativistic effect in the electric field. Being prolonged for mechanical interaction, it represents the corresponding gravitomagnetic force [7-8].

The functional dependence on the speed of movement of the third component (3) is higher than in (2) because under the condition $v \le c$ the multiplier v/c in the second component is raised to the second power and in the third one to the first power. Finally, it is namely the third component that closes the hitherto unknown triune essence of gravitational forces and makes it possible to solve the given problem on a strictly mathematical basis.

4. Applied part of the problem

For the sake of certainty, let's consider the transient process of the interaction of three cosmic bodies: the Sun m_1, Jupiter m_2, and a man-made spacecraft m_3. The balance of forces (1) in the heliocentric coordinates of the stationary Sun is written as:

$$\frac{d\mathbf{v}_2}{dt} = \frac{1}{m_2}(\mathbf{F}_{21}+\mathbf{F}_{23}); \quad \frac{d\mathbf{r}_2}{dt} = \mathbf{v}_2;$$
$$\frac{d\mathbf{v}_3}{dt} = \frac{1}{m_3}(\mathbf{F}_{31}+\mathbf{F}_{32}); \quad \frac{d\mathbf{r}_3}{dt} = \mathbf{v}_3. \qquad (5)$$

The vectors of the distance between celestial bodies and their relative velocity are found by the results of integration (5):

$$\mathbf{v}_{23} = \mathbf{v}_2 - \mathbf{v}_3; \quad \mathbf{r}_{23} = \mathbf{r}_2 - \mathbf{r}_3. \qquad (6)$$

To simplify the analysis, we solve the problem in 2D space due to the logical orientation of the Cartesian coordinate system with the center coinciding with the center of the star:

$$\frac{dv_{2x}}{dt} = \frac{1}{m_2}(F_{21x}+F_{23x}); \quad \frac{dr_{2x}}{dt} = v_{2x};$$
$$\frac{dv_{2y}}{dt} = \frac{1}{m_2}(F_{21y}+F_{23y}); \quad \frac{dr_{2y}}{dt} = v_{2y};$$
$$\frac{dv_{3x}}{dt} = \frac{1}{m_3}(F_{31x}+F_{32x}); \quad \frac{dr_{3x}}{dt} = v_{3x}; \qquad (7)$$
$$\frac{dv_{3y}}{dt} = \frac{1}{m_3}(F_{31y}+F_{32y}); \quad \frac{dr_{3y}}{dt} = v_{3y}.$$

The projections of gravitational forces according to (2) are the next:

$$F_{21k} = -\frac{Gm_1m_2r_{21k}}{r_{21}^3}\left(1+\frac{v_{2k}^2}{c^2}+2\frac{r_{21x}v_{2x}+r_{21y}v_{2y}}{cr_{21}^2}\right);$$
$$F_{31k} = -\frac{Gm_1m_3r_{31k}}{r_{31}^3}\left(1+\frac{v_{3k}^2}{c^2}+2\frac{r_{31x}v_{3x}+r_{31y}v_{3y}}{cr_{31}^2}\right);$$
$$k = x, y; \qquad (8)$$
$$F_{23k} = -\frac{Gm_2m_3r_{23k}}{r_{23}^3}\left(1+\frac{v_{23k}^2}{c^2}+2\frac{r_{23x}v_{23x}+r_{23y}v_{23y}}{cr_{23}^2}\right);$$
$$F_{32k} = -F_{23k},$$

Where:

$$r_{23k} = r_{21k} - r_{31k}; \quad v_{23k} = v_{21k} - v_{31k}, \quad k = x, y. \qquad (9)$$

$$r_k = \sqrt{r_{kx}^2 + r_{ky}^2}, \quad k = 21, 31, 23;$$
$$v_k = \sqrt{v_{kx}^2 + v_{ky}^2}, \quad k = 2, 3, 23. \qquad (10)$$

Expressions (7)–(10) form a complete system of algebraic-differential equations for the analysis of transient processes in the space system *star–planet–spacecraft*. To obtain the desired unique solution, it is necessary to set constant parameters G, m_1, m_2 and space-velocity initial conditions:

$$r_{2k}(0), r_{3k}(0); \quad v_{2k}(0), v_{3k}(0), \quad k = x, y. \tag{11}$$

As for the mass of the space vehicle, according to (7)-(8), it does not participate in the calculations, since decreases. This is consistent with the principle of equivalence [9].

4.1. Practical analysis

In practical calculations, the system of differential equations (7) can be halved if we accept the logical assumption that the mass of the spacecraft m_3 does not affect the trajectory of the planet's flight (m_2):

$$\frac{dv_{3x}}{dt} = \frac{1}{m_3}(F_{31x} + F_{32x});$$
$$\frac{dr_{3x}}{dt} = v_{3x}; \quad \frac{dv_{3y}}{dt} = \frac{1}{m_3}(F_{31y} + F_{32y}); \tag{12}$$
$$\frac{dr_{3y}}{dt} = v_{3y}.$$

Assuming the planet's orbit is close to circular, the mechanical characteristics of its motion can be dependent on the angle α of the orbital motion:

$$\alpha = \alpha_0 + \omega t, \tag{13}$$

where ω is the orbital angular velocity of the planet, α_0 is the initial value α, as:

$$r_{2x} = r_{12}\cos\alpha; \; r_{2y} = r_{12}\sin\alpha;$$
$$v_{2x} = \omega r_{12}\cos(\alpha + \pi/2); \tag{14}$$
$$v_{2y} = \omega r_{12}\sin(\alpha + \pi/2).$$

To obtain the required unique solution (12)−(14), it is necessary to set constant parameters $G, m_1, m_2, \omega, r_{12}$ and space-velocity initial conditions:

$$\alpha_0, \; r_{3k}(0), v_{3k}(0), \quad k = x, y. \tag{15}$$

4.2. Simulation results

The results of the compatible implementation of (11)-(15) by the numerical method are shown in Fig. 1–8 at the stable parameters

$$Gm_1 = 13{,}27128 \cdot 10^{19},$$
$$Gm_2 = 12.67095 \cdot 10^{16} \, (m^3 s^{-2}),$$
$$\omega = 0.0174532925 \cdot 10^{-8} \; s^{-1},$$
$$r_{12} = 7.783566822 \cdot 10^{11} \, m,$$

corresponding to the Sun, the Mercury, and the spacecraft. All dimensions in the simulation results are in SI. Our numerical results are far from practically justified. We only want to confirm utilizing computer simulation the presence of the gravitational effect and the workability of the theoretical results of its mathematical support.

4.3. Overclocking stage

Fig. 1 shows the time dependence of the hodograph of the spatial radius obtained under the initial spatial-velocity conditions:

$$\alpha(0) = \pi/4; \; v_{3x}(0) = 0; \; v_{3y}(0) = 40000;$$
$$r_{3x}(0) = 5507.358432 \cdot 10^8; \tag{16}$$
$$r_{3y}(0) = 5500.167332 \cdot 10^8.$$

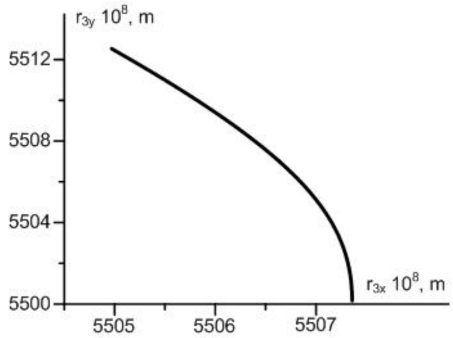

Fig. 1. Curvature of the trajectory of the movement $r_3(t)$ in the heliocentric coordinates of the spacecraft under action gravitational forces of Jupiter during a flying outside its trajectory (approach 366468 km) at a time interval of 26712 s

The duration of the transient process is 26712 s, which corresponds to approximately 7 hours. 25 min. The speed characteristic of the transient process in Fig. 1 is shown in Fig. 2. It illustrates the effect of the gravitational acceleration of the spacecraft at a given time interval in the heliocentric coordinates from 40000 m s^{-1} to 43280 m s^{-1}. Then there was a deceleration to 41495 m s^{-1}. The space navigation effect under the specified space-velocity initial conditions is approximately 1.5 km s^{-1}.

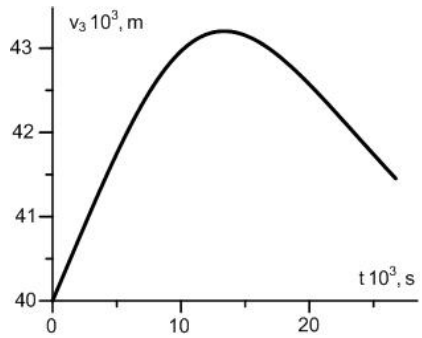

Fig. 2. Time dependence of the speed $v_3(t)$ in the transient process shown in Fig. 1

Please note that this effect strongly depends on the initial conditions. According to [2-3], its maximum value in the gravitational field of Jupiter can reach up to 42.73 km s^{-1}. In the Earth's gravitational field – up to 7.91 km s^{-1}. While it is completely absent in the planetocentric coordinates. Here, the acceleration effect is commensurate with the braking effect, which is demonstrated in Fig. 3.

We draw your attention to the fact that in this time interval, the planet Jupiter also overcomes outer space according to (13)-(14). The corresponding segment of the arc of its circumsolar orbit with a length of 356825 km is shown in Fig. 4.

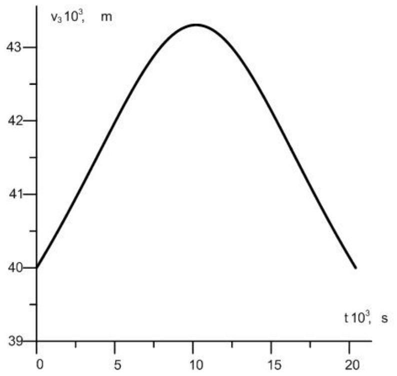

Fig. 3. Time dependence of the velocity $v_3(t)$ in the planetocentric coordinates of Jupiter in the transition process shown in Fig. 1-2

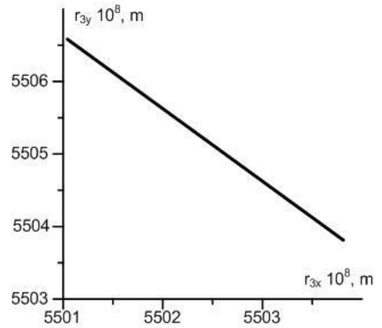

Fig. 4. The arc of Jupiter's trajectory in heliocentric coordinates over time of the transient process corresponding to the curves of Fig. 1-2.

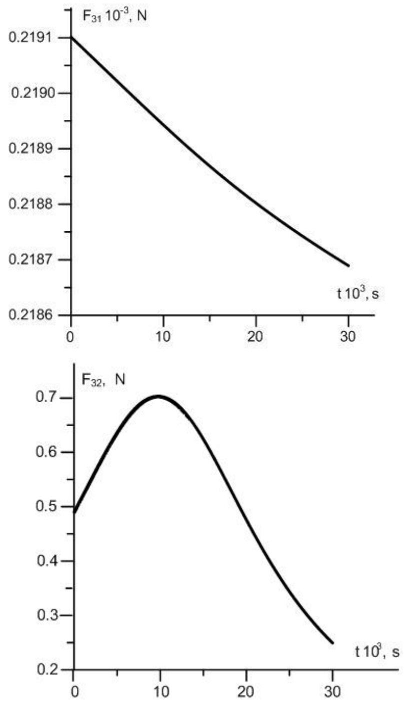

Fig. 5. Power characteristics of the gravitational fields of the Sun $f_1 = F_{31}(t)$ and Jupiter $f_1 = F_{31}(t)$ acting to the spacecraft in the transitional process shown in Fig. 1-2

Fig. 5 shows the power characteristics of the transition process depicted in Fig. 1-2.

4.4. Braking stage

Fig. 6 shows the time dependence of the hodograph of the spatial radius $r_3(t)$, obtained under the space-temporal initial conditions (16) adopted during the simulation of the acceleration of the spacecraft $r_{3x}(0) = 5500.267332 \cdot 10^8$. This was done for the spacecraft to approach the planet from the middle of its orbit. The duration of the transition process is 27334 s, which corresponds to approximately 7 hours. 34 min.

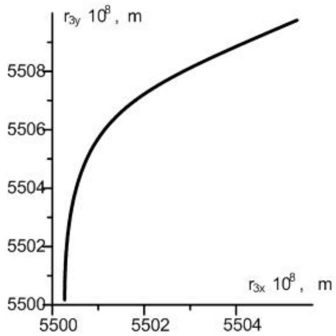

Fig. 6. Curvature of the trajectory $r_3(t)$ of movement in the heliocentric coordinates of the spacecraft under the influence of Jupiter's gravity during a flyby from the inside of its trajectory (minimum approach 351634 km) at a time interval of 27334 s.

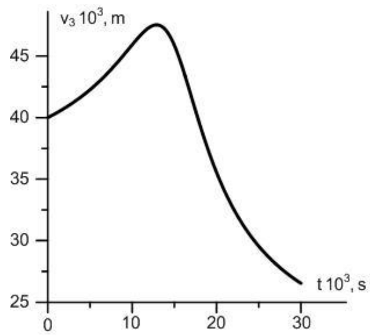

Fig. 7. Time dependence of the speed $v_3(t)$ in the transient process shown in Fig. 6

Fig. 8 shows the time dependence of the distance between the spacecraft and the surface of Jupiter $r_{3J}(t) = r_{32}(t) - r_J$, where $r_J = 0.69911 \cdot 10^8$ m is the radius of Jupiter, in the transient braking process shown in Fig. 6-7.

So, it was rigorously mathematically confirmed the presence of a gravitational effect in heliocentric coordinates of both acceleration (Fig. 2) and deceleration (Fig. 7) of spacecraft in the gravitational field of massive celestial bodies, and at the same time confirmed the absence of this phenomenon in planet-centric coordinates (Fig. 3).

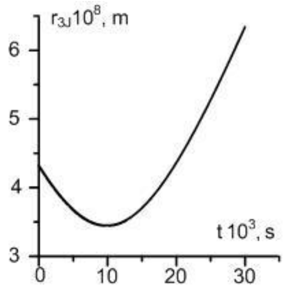

Fig. 8. Time dependence of the flight distance $r_{3J}(t)$
in the transient process shown in Fig. 6 -7.

And is it possible to obtain a reliable result in the practice of cosmonautics? – Yes, but at the same time, it is necessary to know the exact initial space-speed condi- tions laid down by the implementers of the particular space program to which access is not possible recently for me.

5. Conclusions

1. The results of computing the transient processes of the “star-planet-spacecraft’ system illustrate the possibilities of a new approach to solving some fundamental problems of celestial mechanics that cannot be overcome by the methods of classical physics.

2. The known anomalies in the in-flight gravita- tional navigation of spacecraft (discrepancy between the results of measuring trajectories and prediction) are ex- plained not by measurement errors, but by the inapplica- bility of the classical laws of statics to solving the prob- lems of space dynamics. Here, a significant effect is caused by the theoretically discovered third component of the force of the gravitational field, which depends not only on the speed of the moving mass but also on the spatial orientation of its trajectory (in particular, from the radial component of speed).

6. Gratitude

The author is grateful to the Staff of the Department of Information-Measuring Technologies for their mild assistance in designing the article for publication.

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