USE OF CONTROL CHARTS OF CUMULATIVE AMOUNTS FOR THE METHOD OF EXTENDING THE SERVICE LIFE OF THE IONIZING RADIATION SOURCE ACCORDING TO CALIBRATION RESULTS

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Abstract. The article forms and substantiates the application of control charts for the method of extending the service life of the source of ionizing radiation (SIR). An algorithm for its aim was developed. The main types of control charts and the features of their application were considered. A methodology for constructing a cusum map for extending the service life of SIRs based on the results of their calibrations, as well as a methodology for checking the results of SIR calibration for statistical controllability, were developed. Extrapolation methods were considered, and the method of least squares was chosen as the most optimal for this technique.

Key words: Source of ionizing radiation, calibration, control charts, method of least squares, forecasting.

1. Introduction

The service life of SIRs can be extended [1] following the procedure established by law. The term of service is the calendar period of using the product for its intended purpose, starting from its introduction into circulation or after repair, during which the contractor guarantees its safety and is responsible for significant defects that arose due to his fault [2]. Currently, the main document that regulates the extension of the "exploitation period" of the SIR is the Test Methodology of MI 12-01:2014 of the State Enterprise "KYIVOBLS STANDARTMETROLOGY" titled as "Instructions for maintenance of sources of ionizing radiation". This document regulates the technical maintenance of SIR and applies to "closed" radionuclide sources – alpha-, beta-, gamma-, and X-ray radiation of metrological purposes with expired passport service life.

2. Drawbacks

The decision to extend the service life should be based on facts confirmed by metrological inspections during the service life, as well as provide for the determination and establishment of an additional time interval during which this source can be operated.

Test methodology MI 12-01:2014 proposes to determine this time interval according to reliability indicators based on statistical analysis of the properties of SIR samples. The duration of the extended SIR "exploitation period" and the "limital period of operation" are determined following the algorithm for evaluating the reliability indicators of similar SIRs. The assessment is based on the accumulated data of studying the latter. So, this method was recommended to apply expediently to estimate the residual resource of similar SIRs according to the factors of radiation resistance, natural aging, and wear and tear carried out under State Standard of Ukraine 3004-95 "Reliability of equipment. Methods of estimating reliability indicators based on experimental data". Therefore, for a single SIR instance, the use of this method is impractical.

3. Goal

The purpose of the current paper is to develop a method of extending the service life of a particular source of ionizing radiation and to determine the period during which it can be operated after ending its pre-announced service life.

4. Formation of requirements and justification of the use of control charts regarding the method of extending the service life of the source of ionizing radiation

To solve the problems, the requirements for the method of extending the service life of the source of ionizing radiation were formulated. Namely, the method should be able to:

- Application for certain SIRs, manufactured in single copies or small batches;
- Considering information about metrological characteristics and their changes during the SIR's service life;
- Predicting changes in the SIR's metrological characteristics, in particular, identifying irregularities in the process of metrological drift. By applying control cards, one can fulfill the specified requirements. This enables [3] to:

  – determine the stability stage in the metrological characteristics of a particular SIR, i.e. its ability to stabilize metrological characteristics within the specified limits during a given time;

  – evaluate the own variability [4] of the process of changes in the metrological characteristics under the external or internal impact factors;

  – identify, and investigate the influence of special causes of variability, which can lead to unacceptable changes in metrological characteristics;

− use data on its variability (presence of a series trend, cycles, etc.) to analyze the alterations in metrological characteristics;

− determine whether changes in metrological characteristics are predictable and stable and whether it is possible to assess the compliance of the process with the established requirements.

To analyze changes in the SIR's characteristics, it is possible to study the application of control charts presented in Table 1 [5].

Table 1. Main types of control charts and features of their application

No	Main types of control charts	Features of application of control charts
1	Control chart of arithmetic averages	Quantitative control chart designed to assess process variability based on arithmetic averages in subgroups.
2	Control chart of moving averages	A control chart designed to assess the state of the process based on the arithmetic averages of the last *n* observations. Such a map is particularly useful when only one observation in a subgroup is available. The disadvantage of the map is the lack of weighting factors when calculating the arithmetic mean, which takes into account the composition of the *n* points used.
3	Control chart of medians	Quantitative control chart designed to assess process variability based on subgroup median values.
4	Control chart of exponentially weighted moving averages	A control chart designed to estimate process variability by exponentially smoothed moving average arithmetic values.
5	Swing control chart	Quantitative control chart designed to assess process variability by scope in subgroups. The range value in the subgroup *R* is the difference between the largest and smallest observations in the subgroup.
6	Control chart of sliding swings	A quantitative control chart designed to assess process variability throughout *n* consecutive observations.
7	Control card of cumulative sums, cusum-card	A control chart showing the cumulative sum of deviations of the statistics of consecutive samples from the target value to detect changes in process characteristics. A Cusum map can be used to monitor, diagnose, and predict the behavior of an observed characteristic. While applying a cusum map for monitoring, it is interpreted using masks (for example, *V*-masks) superimposed on it. A process violation signal occurs when the cumulative sum line crosses the mask boundary or touches it.

The analysis of the features of the application of control charts showed the feasibility of using control charts of cumulative sums for the method of extending the service life of a separate SIR. It is due to analysis, providing an opportunity to make a decision based on preliminary information about the SIR metrological characteristics. As a result, we become able to define the exit of the process from the state's statistical controllability.

The control charts of cumulative sums are the most sensitive to the process shift since all the data of the accumulated sums of sample statistics are used to assess the state of the technological process, that is, both current and previous sample data can be considered. Thus, decisions made based on obtained information are more reliable than decisions based on the results of only one sample.

A distinctive feature of the method of cumulative sums is the fact that the decision regarding the correctness of the process is made on previous information. This scheme of using sampled control results provides a significant reduction in the average length of a series of samples. It means that the disorder of the process can be revealed much faster than with the classic scheme of using sample statistics, representing independent control results. Thus, the distinctive feature of cusum maps is that the plotted points do not correspond to the particular observations or statistics, such as the mean or range calculated from a single sample, but represent information from the first to the last observation inclusively.

When using cusum maps for RIS analysis, a variant is possible when the calibration data for the life cycle of the source coincide with the half-life law, but the control maps would demonstrate that the process was in a state of statistical uncontrollability at a certain moment. That is, the source is sealed, and there is no loss of radioactive material. The activity obtained during calibration corresponds to the half-life law, and therefore the process was brought out of statistical controllability by other impact factors.

4.1. Algorithm of actions for prolonging the service life of the SIR based on the calibration results

We suggest using the following algorithm:

1. Carrying out planned calibrations of the SIR during the service period.

2. Formation of a database of SIR calibrations for the entire period of service.

3. Construction of a cusum map based on the results of SIR calibrations for the entire service life, considering the actual value of SIR activity.

4. Construction of a truncated V mask taking into account the time intervals of research.

5. Checking for statistical controllability by superimposing a truncated V mask on the graph of the cusum map.

6. Forecasting the activity of SIR radionuclides for a given period using the method of least squares and the method of trend extrapolation.

7. Checking the predicted values for statistical controllability to decide on extending the service life of the source for a certain period.

4.2. Methodology of a cusum map design for extending the service life of RIS based on calibration results

Information on periodic calibrations of radionuclide activity for the entire service life of the RIS is analyzed using maps of cumulative sums [6]. The deviation ΔX_i of the obtained measurement result X_i from the reference value T is:

$$\Delta XX_i = XX_i - TT \quad (1)$$

By the reference value, we understand the activity of the radionuclide according to the passport. The value of the cumulative sum of deviations of the actual values from the target ΔX_Σ is calculated [7]:

$$\Delta XX_\Sigma = \sum_{i=1}^{n} \Delta XX_i \quad (2)$$

A graphical assessment of cusum maps is carried out using a truncated V-mask. Here, the processing of cusum maps is based on a template with a cut in the form of a truncated letter “V”, which forms the boundaries of the deviation area.

A truncated general-purpose V-mask is shown in Figure 1. It contains a reference starting point, marked O. Two vertical segments OB and OC of length $5\sigma_e$ each are located on either side of the reference starting line (i.e., h = $5\sigma_e$). These two segments are called the decision intervals. Two inclined segments BA and CD, called permission lines, span the cusum graph. The size of EO is equal to ten observation intervals, and the vertical segments of EA and ED are equal to $10\ \sigma_e$ (i.e. F = 0.5 σ_e).

$$h = h_{OB} = h_{OC} = 5\sigma_e\ ;$$
$$2h = h_{EA} = h_{ED} = 10\sigma_e; \quad (3)$$
$$l_{OE} = 10d;$$

where d is an observation interval, σ is the standard deviation of the process; σ_e is the standard error. Based on the values of these intervals, we construct the resolution lines l_{CD} and l_{BA}.

To use the mask, the starting point is indicated on the reference line, located horizontally on the kusum map. In a situation where the management process is ongoing, this is the last point. If your cusum-grafic is located in the middle of the allowed mask lines (or their extension to points A and D), then no significant shift on average was found on this graph. In this case, the process is in a state of statistical control relative to the reference value. If the graph changes beyond the mask dividing lines, this indicates a significant deviation from the target value. In this case, the process is uncontrolled.

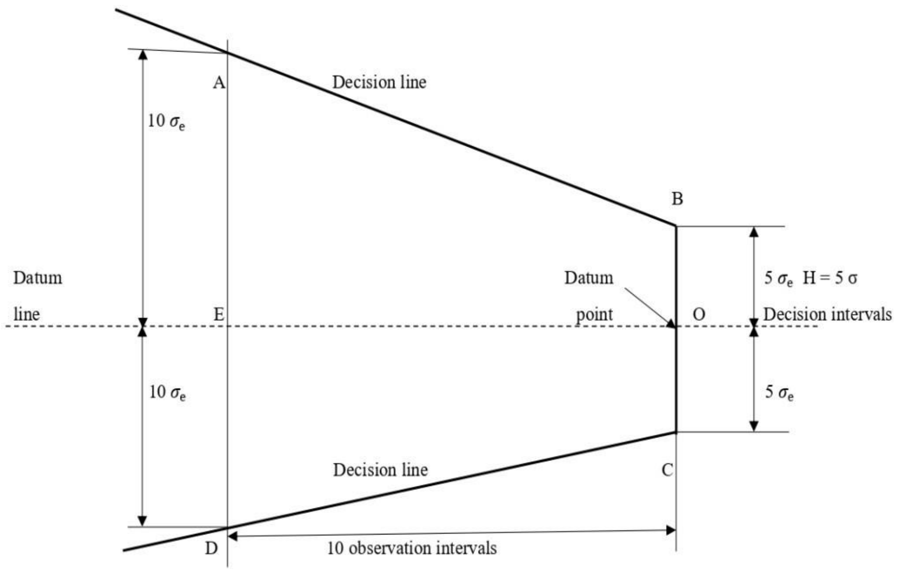

Fig. 1. Parameters of a truncated V-mask

In the presence of only the upper or the lower limit of permissible values, unilateral control is applied. In this case, a half-mask can be used. When monitoring shifts up/down, only the relevant half of the mask is needed. The usage of cusum maps is justified when the process continues for a long time or when a change in the average value of data or measurements can be expected [8].

4.3. The method of checking the results of SIR calibration for statistical controllability

After checking the condition and stability of the results of checking the SIR activity and the tightness, we proceed to the selection of the extrapolation method. Among the existing methods of extrapolation: the method of smoothing by the exponent, the method of harmonic weights, the method of dispersion analysis, correlation-regression analysis, and the method of trend extrapolation – the last seems to be the better. Since the SIR's activity study is the half-life law, the calibration and leak testing must take place under normal conditions. Then the impact factors can be considered as constant. Moreover, the trend extrapolation method is the most convenient one for high-mentioned conditions. Its essence is that based on a dynamic series of statistical data, the main trend of alteration, which extends to the forecasting period, is determined. A rule of thumb is usually followed, according to which the forecast period should not exceed a third of the duration of the forecast base. For example, for a 1-year forecast, it is desirable to have statistical data for at least 3 years [9].

The application of extrapolation is possible only under the following conditions: there is a statistically significant trend in the time series; the researched process is inertial, i.e. patterns that existed in the past will be preserved in the future; the factors determining the development of the process remain unchanged. Extrapolation based on the average level of the series of dynamics is used if the series does not note a statistically significant trend of development. Then the levels of the series fluctuate around the average value. So, the forecast is calculated as the arithmetic mean of all levels of the series. If the dynamic series of the forecasted indicator notes a steady upward or downward trend, and fluctuations around this trend are insignificant, then its extrapolation is carried out according to the average rate of change:

$$Y_{t+1} = Y_t \times \bar{T}, \quad (4)$$

where, Y_{t+1} is the forecast level of the series; Y_t is the last level of the series under investigation; $\bar{T}$ is the average rate of change in the levels of the series:

$$\bar{T} = \sqrt[n-1]{\frac{Y_t}{Y_n}}, \quad (5)$$

where Y_n is the initial level of the series under investigation; n is the number of levels of the series.

Complex extrapolation methods are based on the analytical alignment of the series using the method of least squares. At the same time, the graph of the dynamic series should demonstrate a certain clear trend of growth or decline of the studied indicator. For example, there is a series trend that can be described by the equation of a linear function: $Y=a+bT$. Here, it is possible to determine the constants a and b and apply the resulting equation to predict the studied indicator. The following formulas help us in calculations of the constants:

$$b = \frac{\sum TF - n\,\bar{T}\bar{Y}}{\sum T^2 - n\,(\bar{T})^2} \quad (6)$$

$$a = \bar{Y} - b\,\bar{T}, \quad (7)$$

where a, and b are the constants of the equation that determines the relationship between the serial number of the time T and the value of the studied indicator in this period Y; Y – is the arithmetic mean of the studied indicator.

The method of least squares (LSM) [10] is used, in particular, to approximate experimental data by a certain function. The LSM's problem is solved by parametric estimation of the regression function, which describes the dependence of one value Y, the value of which (y_i) is observed with random errors θ_i, on a group of non-random values x_1, x_2, ..., x_k. In general, a simple sample regression model is written as:

$$\hat{y} = a, \quad (8)$$

where y is a vector of observations on the dependent variable; x is a vector of observations on the independent variable; a, and b are unknown parameters of the regression model. Equation (8) builds a linear regression model. Formulas for determining the unknown parameters a and b can be written as a regression of y dependent on x, in which the parameters are calculated by the LSM. We obtain:

$$a = exp\left(-\frac{1}{n}\sum lny_i - \frac{b}{n}\sum lnx_i\right) \quad (9)$$

$$b = \frac{n\sum(lnx_i\, lny_i) - \sum lnx_i \sum lny_i}{n\sum ln^2 - (\sum lnx_i)^2} \quad (10)$$

After determining the unknown parameters of the regression model, we estimate the density of the connection between the dependent variable y and the independent variable x. That is, we are trying to answer the question of how significant is the influence of variable x on y. The simplest criterion that gives a quantitative assessment of the relationship between two indicators is the correlation coefficient:

$$r_{xy} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right)\left(n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right)}} \quad (11)$$

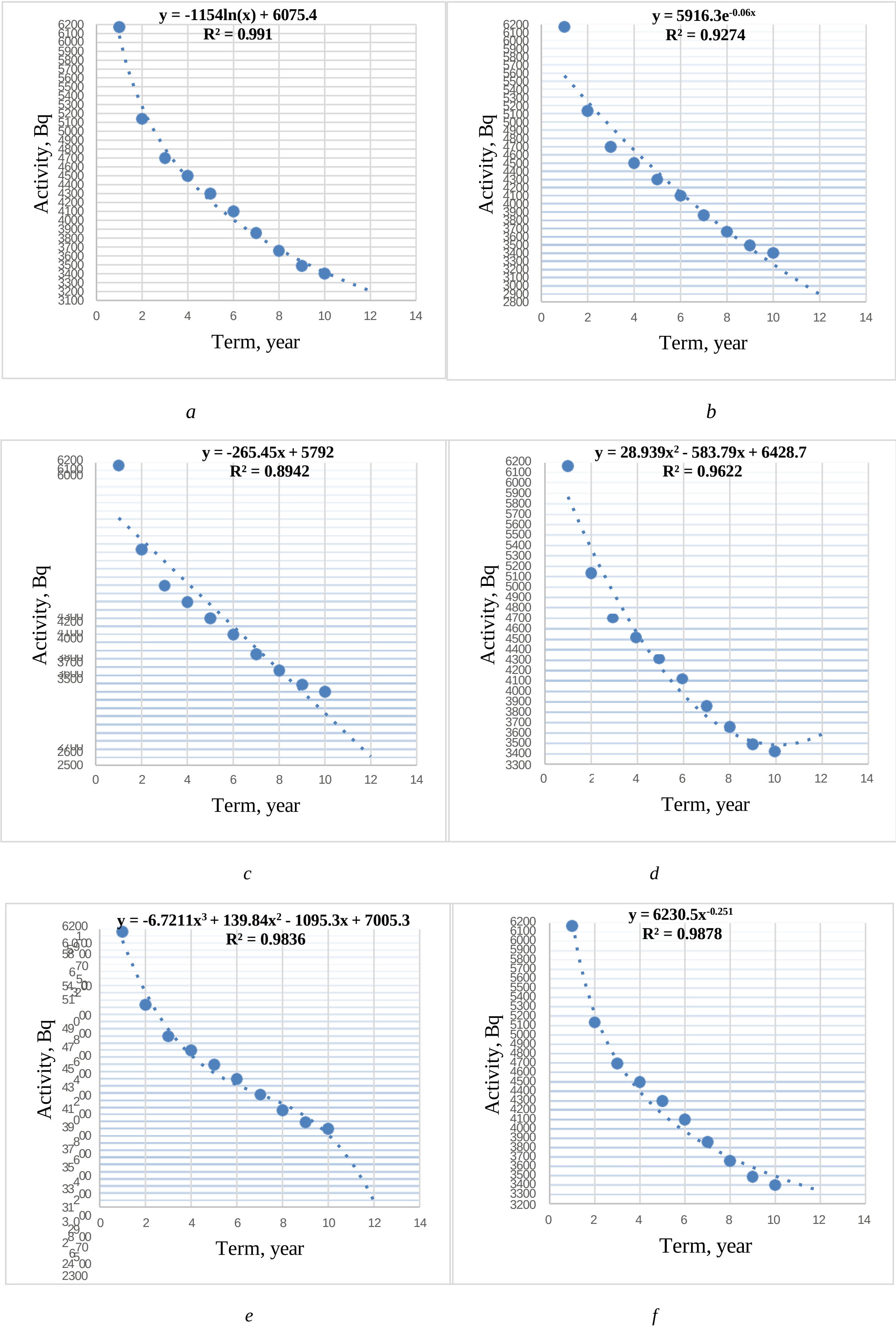

Figure 2. Modeling of SIR's activity based on calibration results using regression functions with forecasting of its change for 2 years

Along with it, another criterion is used, which also measures the density of the connection between two or more indicators and checks the adequacy of the constructed regression model to the reality. That is, an answer is given to the question of whether the change in the value of *y* de- pends linearly on the change in the value of *x*, and it does not occur under the impact of random factors. Such a crite- rion is the coefficient of determination denoted as R^2.

The part of the variance that explains the regression is called the coefficient of determination which serves as a criterion for the adequacy of the model; it is a measure of the explanatory power of the independent variable *x* calcu- lated as:

$$R_{xy}^2 = r^2 \quad (12)$$

For adequate models, it is necessary to assess their accuracy, which is characterized by the deviation of actual data from estimated data [11]. To extend the SIR service life, every activity value obtained over the entire life cycle has to be approximated by a regression function. The last one, which has the best coefficient of determination, has to be chosen to predict the extension of the SIR's operation. Next, the predicted part is checked for statistical controlla- bility using cumulative sum maps.

4.4. Examples of function selection for describing SIR changes activity based on calibration.

Figure 2 shows the simulation of SIR's activity based on the results of calibration using functions of change prediction for 2 years. The latter does not exceed 30% of the SIR's service life, which is equal to 10 years.

Table 2. Obtained coefficients of determination for various regression functions

Function	Coefficient of determination, R^2
Logarithmic (a)	0.991
Exponential (b)	0.927
Linear (c)	0.894
Polynomial 2 (d)	0.962
Polynomial 3 (e)	0.983
Power (f)	0.988

So, in our case, we choose a logarithmic function, since the value of the coefficient of determination of this function is inherent in the smallest deviation from unity.

5. Conclusions

The following conclusions were reached based on research on extending the service life of ionizing radiation sources.

1. Since the activity of the source of ionizing radia- tion is subject to the half-life law, the calibration and leak testing of the source are carried out under normal condi- tions, correlation-regression analysis is the most convenient when choosing a process-description model.

2. Adhering to the empirical rule, according to which the forecasting period should not exceed a third of the duration of the forecast base, checking the forecasted part with the help of cumulative sum maps, and making sure that the process is in a statistically controlled state, it is shown that we have the right to extend the service life of the ionizing source radiation for two years.

3. To achieve the goal of extending the service life when describing the process, the optimality of using Cusum maps has been proven, which contributes to quickly and accurate- ly determining the area of process changes and the moment of implementation of corrective actions.

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7. Mutual claims of authors

The authors declare the absence of any financial or other potential conflict related to the work.

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