A comparative study of game theory techniques for blind deconvolution

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The aim of this study is to lay emphasis on the potential of the use of Game theory to deal with Blind image Deconvolution. We consider a static game of two players. Player one controls the image intensity while the player two controls the blur kernel. In this game each player aims at minimizing his own functional. The outcome of the game is a pair of strategies: a deblurred image and an estimation of the blur kernel, that minimizes two functionals. We determine the optimal image deblurring using two particular game theoretic approaches, recently introduced: the Nash method [Meskine D., Moussaid N., Berhich S. Blind image deblurring by game theory. Proceedings of the 2nd International Conference on Networking, Information Systems & Security (NISS '19). 31 (2019)] and the Kalai–Smorodinsky solution method [Nasr N., Moussaid N., Gouasnouane O. The Kalai Smorodinsky solution for blind deconvolution. Computational and Applied Mathematics. **41**, 222 (2022)] We evaluate the performance of two techniques through numerical experiments and using some objective quality metrics.

Keywords: *deblurring; game theory; multi-objective optimization.*

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1. Introduction

Blind deconvolution is a process of retrieving a clean image from its blurry version without any knowledge of the specific blurring process that was applied i.e. the PSF: Point Spread Function. The blurring can occur due to different factors for instance: out-of-focus blur, motion blur, or atmospheric turbulence.

Blind deconvolution is not a well-posed problem. An ill-posed problem is a problem in which the solution is sensitive to small changes in the input data. The solution may not exist, or it may not be unique or stable.

In the blind deconvolution's case, the input data is the blurred image, and the unknowns are the sharp image and the blur kernel. The problem is ill-posed because small errors in the observed image or the estimated PSF can cause large errors in the reconstructed image, and there may be many possible combinations of the original image and the PSF that could have generated the blurred image.

To conquer the ill posedness of the blind deconvolution problem, additional constraints or assumptions need to be introduced to ensure that the solution is unique and stable. These constraints can take the form of regularization, which involves incorporating new assumptions about the image and the PSF into the reconstruction process. Regularization techniques such as Tikhonov regularization [1–3], total variation regularization [4–8], and sparsity-based regularization [9, 10] have been used in image blind deconvolution to stabilize the solution and to improve its quality.

Blind deconvolution is far from being a simple problem due to the presence of noise and other artifacts that may be introduced during the blurring process. To overcome these challenges, different methods have been proposed, including statistical methods, optimization-based methods, and deep learning-based methods.

Statistical methods, such as the Maximum A Posteriori (MAP) estimation [11–13] and Expectation Maximization (EM) algorithm [14,15], estimate the unknown blurring process and the underlying image simultaneously. Optimization-based methods, such as the Total Variation (TV) regularization [16,17], exploit sparsity and smoothness of images to estimate the unknown PSF and the underlying image.

Deep learning-based methods have also shown promising results for blind deconvolution of images [18–21]. These methods typically use a neural network to estimate the unknown blurring kernel and the underlying image simultaneously.

The corruption process of an image is described as

$$g = v \otimes u + \eta, \tag{1}$$

where g represents the damaged image, v is the clean unknown image, u is the point spread function (PSF) modeling the blurring process, $\otimes$ is an operator of convolution and η is an additive noise.

Blind deconvolution has numerous applications in different fields for instance medical imaging, astronomy, and surveillance. It can be used to recover high-quality images from blurred or low-resolution images, which can be valuable for applications such as medical diagnosis, surveillance, and security.

Deblurring refers to the process of removing or reducing the blurriness or distortion from an image. When an image is captured or transmitted, it can become blurred due to various factors mentioned above. Deblurring aims to restore the image to its original sharpness and clarity.

The original image v refers to the clean and undistorted version of the image that we want to recover. It represents the image as it was intended to be seen without any blurring or degradation.

The blurred corrupted image g is the result of applying blurring or distortion to the original image. It represents the image that has been affected by factors such as motion or imperfections, resulting in a loss of sharpness and clarity. This image is often referred to as the degraded or observed image.

The goal of deblurring is to reconstruct or recover the original image v from the blurred corrupted image g. This involves developing algorithms and techniques that can estimate or reverse the blurring process to restore the image to its intended quality.

2. Blind deconvolution via Game Theory

2.1. Blind deconvolution via Nash game

Game theory provides a powerful framework for modeling and solving multiobjective optimization problems, where there are multiple objectives to be optimized simultaneously. Game theory provides a framework for modeling such situations as games where the players have conflicting objectives, and the payoff of each player is the corresponding objective function value [22]. The goal is to find a solution that is optimal for all objectives. The solution concept used in game theory for such problems is the Nash equilibrium. Following this the authors of [23] suggested to solve the Blind Deconvolution problem using a Nash game. They define a static Nash game, of two players: the first is "Deblurring" and the second is "PSF", the Deblurring player controls the image intensity while the PSF player controls the Blur kernel. Each player's goal is to minimize his own functional, the solution of this game is a pair of an optimal deblurred image and an optimal estimation of the blur kernel. This pair is none other than the Nash Equilibrium. In [23] the authors associated two functionals $f_v(v,u)$ and $f_u(v,u)$ with the Deblurring and PSF respectively:

$$f_v(v,u) = \frac{1}{2}\|u \otimes v - g\|^2_{L^2(\Omega)} + \int_\Omega \beta(x)|Dv|dx, \tag{2}$$

$$f_u(v,u) = \frac{1}{2}\|u \otimes v - g\|^2_{L^2(\Omega)} + \int_\Omega (1-\beta(x))|Du|dx, \tag{3}$$

where $\beta(x)$ is defined by $\beta(x) = \frac{1}{1+\alpha|\nabla G_\sigma \otimes g|^2}$, α is a threshold parameter, and $G_\sigma(x) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|x|^2}{2\sigma^2}\right)$ denotes the Gaussian filter with the parameter σ.

To compute the Nash equilibrium the authors of [23] conceived an alternating minimization algorithm, they fix one variable and update the other: first they solve

$$\min_v f_v(v,u^i) \to v^{i+1},$$

then they alternate

$$\min_u f_u(v^i,u) \to u^{i+1},$$

i is the iteration number. In [24], we tested the effectiveness of this algorithm on various blurred gray scale images. The following Figures 1 and 2 show an experiment on a blurred Lifting body and Cameraman image.

a (Blurred image) ***b*** (Nash method (PSNR = 37.3))

Fig. 1. The Nash method experiment on the Lifting Body image.

a (Blurred image) ***b*** (Nash method (PSNR = 29.6))

Fig. 2. The Nash method experiment on the Cameraman image.

2.2. Blind deconvolution using the Kalai–Smorodinsky solution

In [25] the authors went for a similar approach and solved the Blind Deconvolution problem using a solution originating from game theory specifically: the Kalai–Smorodinsky solution using the famous NBI method (Normal Boundary Intersection Method) [26].

Normal boundary intersection. The NBI method was developed by Das and Dennis in 1998 [26]. It is a technique used in multi-objective optimization to find a compromise solution between different conflicting objectives.

Imagine you have a set of goals or objectives that you want to achieve, but these objectives may be in conflict with each other. For example, you may want to maximize your profits while minimizing your environmental impact. These objectives are often referred to as "objectives" or "criteria" in optimization problems.

The NBI method works by finding a balance between these conflicting objectives. It does this by defining a boundary, which represents the trade-offs between the objectives. This boundary is called the "Pareto front".

The NBI method is successful in producing an evenly distributed set of points on the Pareto surface. This means that the method provides a good coverage of solutions across the trade-offs between objectives. This even distribution allows decision-makers to have a comprehensive understanding of the possible trade-offs and choose among a diverse set of solutions, for this reason the NBI method has an advantage over two common multi-objective approaches: the weighting method and the ε-constraint method.

Blind deconvolution. Using the NBI method, the authors [25] approach is based on the following multiobjective optimization problem:

$$\min_{v,u} P(v,u), \tag{4}$$

where

$$P(v,u) = \begin{pmatrix} f_v(v,u) \\ f_u(v,u) \end{pmatrix} \tag{5}$$

unlike single objective optimization problems the concept of a single solution that minimizes two functions $f_v(v,u)$ and $f_u(v,u)$, does not exist hence we look for an agreement, i.e. the Pareto front of optimal solutions. To determine the Pareto front the authors used the NBI method (Normal Boundary Intersection) [26]. It is known for generating a uniformly distributed points on the Pareto front unlike other popular gradient-based methods such as the ε-constraint method and the weighted sum method.

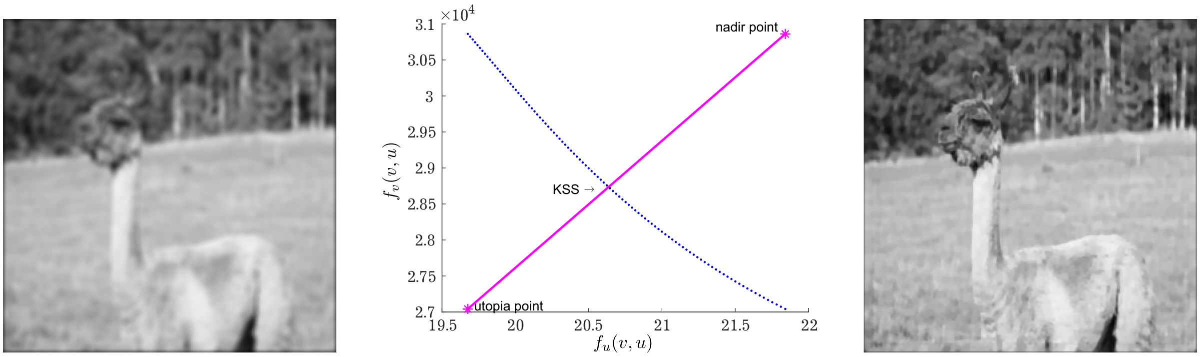
The NBI method identifies the Pareto optimal solutions represented on the Pareto front, by solving K subproblems. The subproblem to be solved is expressed as the following

$$\begin{cases} \min\limits_{v,u} \overline{f_v(v,u)}, \\ \text{St. } \overline{f_v(v,u)} - \overline{f_u(v,u)} + 2\alpha_j - 1 = 0. \end{cases} \tag{6}$$

- $\overline{f_v}$ and $\overline{f_u}$ are the normalized versions of f_v and f_u, respectively
$$\overline{f_v} = \frac{f_v - f_v^*}{f_v^N - f_v^*} \quad \text{and} \quad \overline{f_u} = \frac{f_u - f_u^*}{f_u^N - f_u^*}, \tag{7}$$
- f_v^N and f_u^N represent the worst possible value for the objective function f_v and f_u called the nadir points. To find the nadir point, we maximize each objective function independently of the other.
- f_v^* and f_u^* represent the best possible value for the objective functions f_v and f_u called the utopia points. A solution that is closer to the utopia point is considered better. To find the utopia point, we minimize each objective function without regard to the other one.
- $\alpha_k = \frac{k}{K}$, $k \in \{0, 1, \ldots, K-1\}$, where K is the number of points we identify in order to construct the Pareto front.

The process of constructing the Pareto front is an iterative one: to generate the Pareto optimums we need to solve the subproblem 6 above K times. Each Pareto solution has as coordinates the deblurred image and an estimation of the PSF.

To determine the KSS solution, we draw a line connecting the nadir point and the utopian ideal point. The point where the line intersects the Pareto front represents the Kalai–Smorodinsky solution, as shown in the following Figure 3 (the blue curve is the Pareto front and the connecting line is the pink line).



a (Blurred image) ***b*** (Pareto front) ***c*** (KSS method (PSNR = 30.73))

Fig. 3. The KSS method experiment on the llama image.

3. Comparison

For the aim of comparing the effectiveness of the two game theory based methods in deblurring, we carry out various numerical experiments in MATLAB.

a (Original image)

b (Blurred image)

c (Nash method (PSNR = 28.95))

d (KSS method ($PSNR = 32.85$))

Fig. 4. Onion experiment.

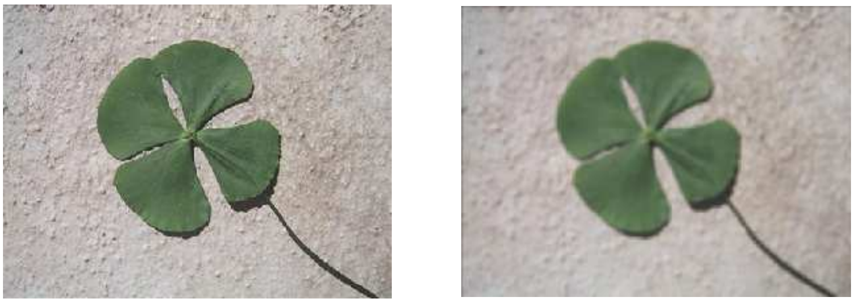
a (Original image)

b (Blurred image)

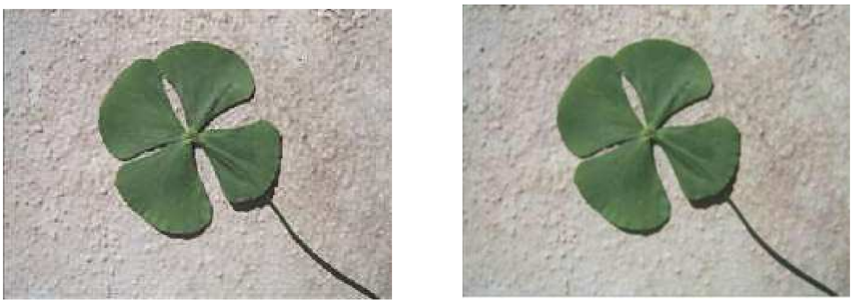
c (Nash method (PSNR = 28.86))

d (KSS method ($PSNR = 33.61$))

Fig. 5. Lena experiment.

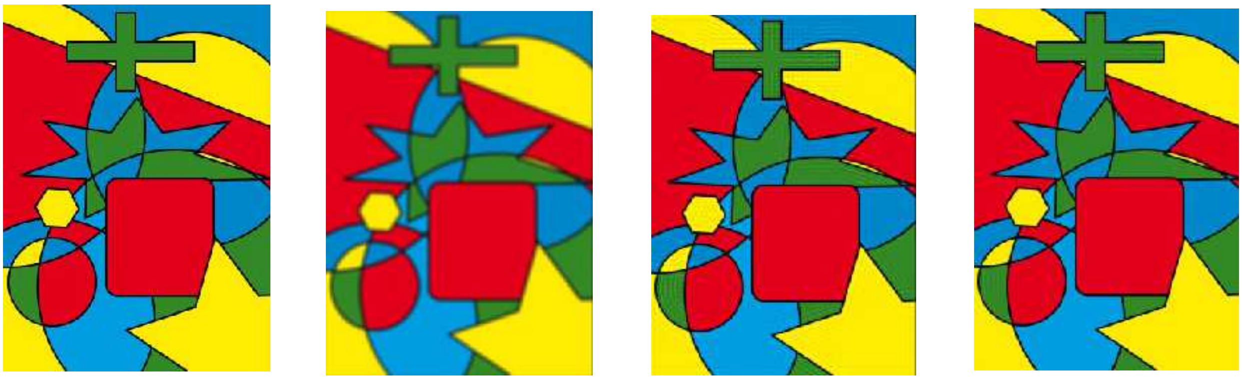


a (Original image) ***b*** (Blurred image)



c (Nash method (PSNR = 30.94)) ***d*** (KSS method (PSNR = 32.46))

Fig. 6. Clover experiment.



a (Original image) ***b*** (Blurred image) ***c*** (Nash method (PSNR = 30.81)) ***d*** (KSS method (PSNR = 33.29))

Fig. 7. 4 colors experiment.

Table 1. SSIM comparison.

Image	Nash	KSS
Lena	0.935971230724093	0.968978731456625
Clover	0.910370776782113	0.913171924262159
4 colors	0.994648952490064	0.996897202200127
Onion	0.973217306067959	0.990127380492750

Table 2. PSNR comparison.

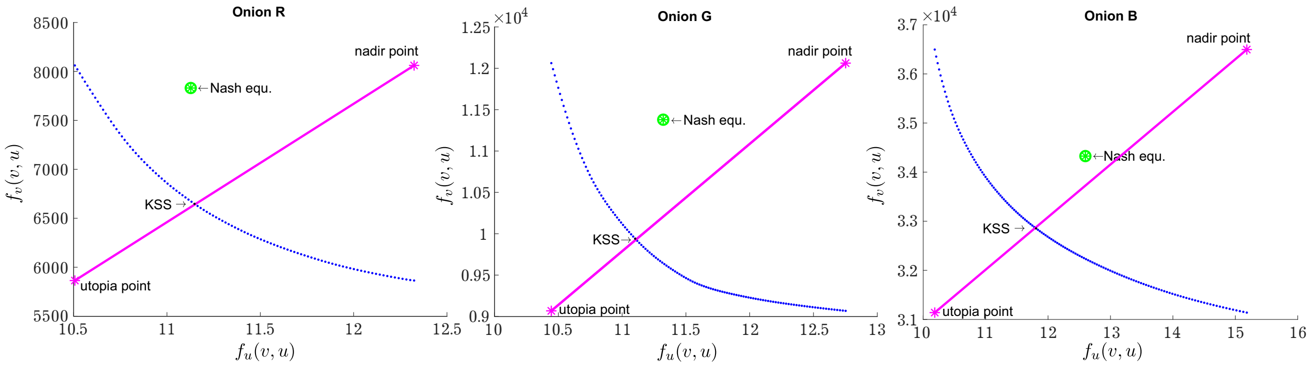
Image	Nash	KSS
Lena	28.860235325232660	33.613569411605750
Clover	30.946577901048187	32.463748346461870
4 colors	30.816323567981130	33.296178879817650
Onion	28.950770064622727	32.851751967518375

To assess the quality of the deblurring we use three popular image quality metrics, RMSE: Root Mean Square Error, SSIM: Structural Similarity Index Measure and PSNR: Peak Signal to Noise Ratio. For the simulation we consider four colored images: Lena, Onion, Clover and 4 colors as shown in Figures 4, 5, 6 and 7.

Table 3. RMSE comparison.

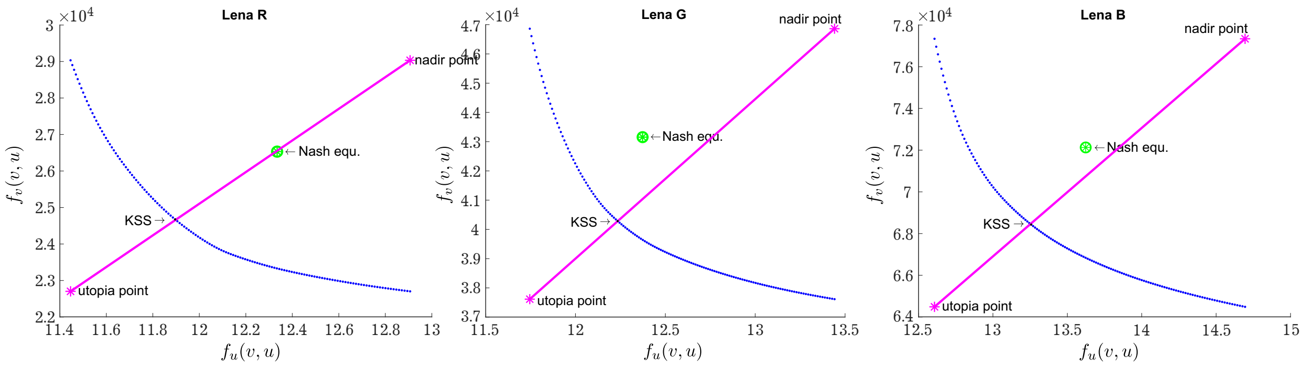
Image	Nash	KSS
Lena	0.001300099129425	4.351540790196359e-04
Clover	8.041595240948711e-04	5.670549754181097e-04
4 colors	8.286433381309662e-04	4.681468570135449e-04
Onion	0.001273277291196	5.185907943846604e-04

The Blind Deconvolution quality of the results are illustrated in Table 1, 2 and 3.



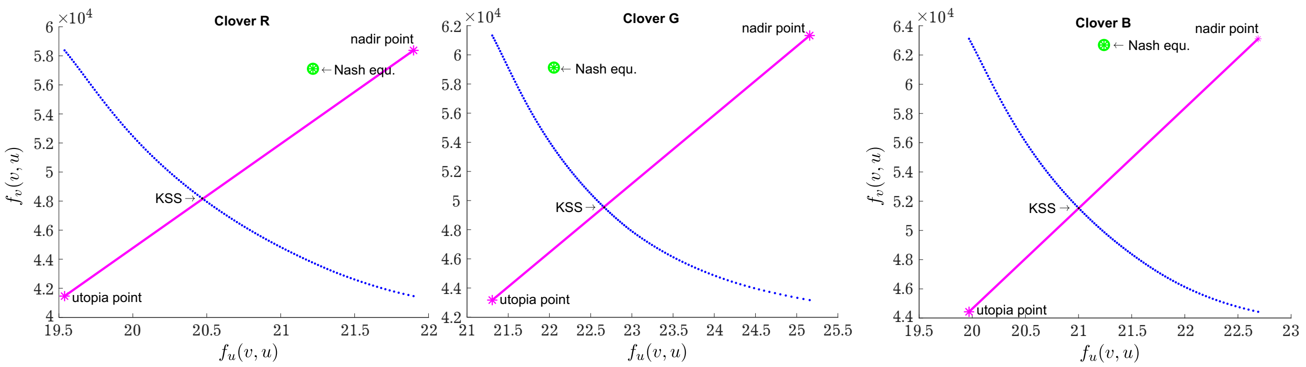
a (Red color component) ***b*** (Green color component) ***c*** (Blue color component)

Fig. 8. Pareto front of the Onion image.



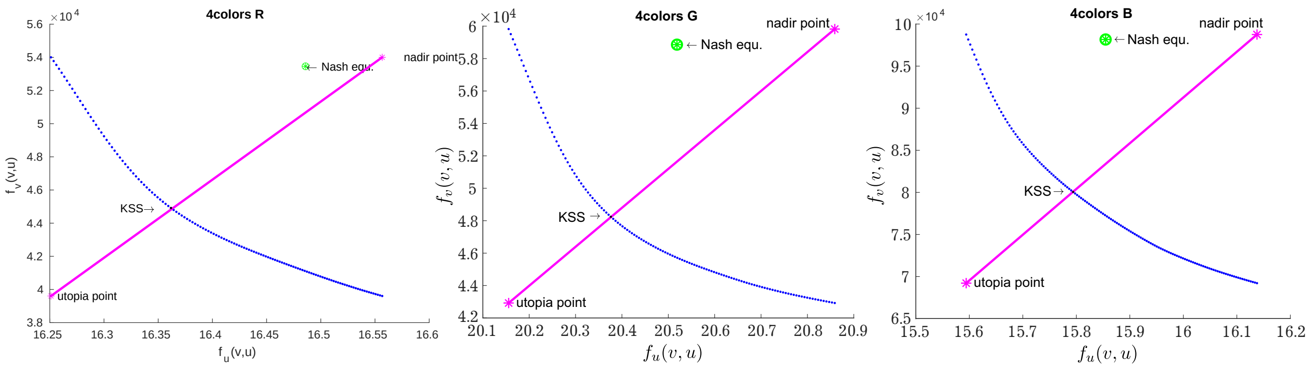
a (Red color component) ***b*** (Green color component) ***c*** (Blue color component)

Fig. 9. Pareto front of the Lena image.



a (Red color component) ***b*** (Green color component) ***c*** (Blue color component)

Fig. 10. Pareto front of the Clover image.



a (Red color component) ***b*** (Green color component) ***c*** (Blue color component)

Fig. 11. Pareto front of the 4 colors image.

Figures 8, 9, 10 and 11 below represent the KSS solution on the Pareto front. We look for the KSS solution for each component (Red, Green and Blue) of the colored images. We drew the Nash solution in green.

4. Conclusions and discussion

In this study we compare two game theory based Blind Deconvolution methods: one is based on the Nash equilibrium and the other is based on the Kalai–Smorodinsky solution. Comparison of two approaches on color images: Onion, Lena, Clover and 4 colors show that the Kalai–Smorodinsky solution method produces the better results with regard to the image quality indicators SSIM, RMSE and PNSR.

We also observed in Figures 8, 9, 10 and 11 that the Nash equilibrium rarely make it on the Pareto front and the Nash and the Kalai–Smorodinsky solutions never coincide in all experiments.

Intuitively we say that the KSS deblurring method performs better than the Nash deblurring method in four experiments because all Nash equilibria are inefficient in the Pareto sense as seen in Figures 8, 9, 10 and 11.

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Порівняльне дослідження методів теорії ігор для сліпої деконволюції

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Мета цього дослідження полягає в тому, щоб підкреслити потенціал використання теорії ігор для роботи зі сліпою деконволюцією зображень. Розглядається статична гра двох гравців. Перший гравець контролює інтенсивність зображення, а другий гравець контролює ядро розмиття. У цій грі кожен гравець прагне мінімізувати власний функціонал. Результатом гри є пара стратегій: розмитість зображення та оцінка ядра розмиття, що мінімізує два функціонали. Визначається оптимальне зменшення розмитості зображення за допомогою двох конкретних підходів теорії ігор, нещодавно представлених: метод Неша [Meskine D., Moussaid N., Berhich S. Blind image deblurring by game theory. Proceedings of the 2nd International Conference on Networking, Information Systems & Security (NISS '19). 31 (2019)] та метод розв'язування Калаї–Смородінського [Nasr N., Moussaid N., Gouasnouane O. The Kalai Smorodinsky solution for blind deconvolution. Computational and Applied Mathematics. **41**, 222 (2022)]. Оцінюється продуктивність двох методів за допомогою числових експериментів і використання деяких об'єктивних показників якості.

Ключові слова: *зменшення розмитості; теорія ігор; багатоцільова оптимізація.*