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FORCE ANALYSIS OF THE BENDING PROCESS OF THE WORKPIECE BY COPIER

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Abstract. The goal of the work is to develop a mathematical model of a workpiece bending process by copier to determine the necessary parameters of the process and increase its efficiency. **Significance.** The quality of the process of workpiece bending by copier depends on many factors and is accompanied by both elastic and plastic deformations. Therefore, a mathematical description and analysis of the workpieces is an urgent issue, because it will allow to justify the parameters of the process and increase its efficiency. **Method.** The dependence between stresses and strains was used for an analysis of the bending process, instead of Hooke's law. Since there is a linear stressed and a volumetric deformed state for the narrow workpiece, then, according to the assumption of the flat cross-sections, a relative deformation of the arbitrary fiber is a linear function of its movement from a neutral layer. It was determined a bending moment in the section of the workpiece having the curvature radius of the neutral layer and considered the equilibrium of the workpiece bent element from the last point of contact with the copier to the point of a force application. As a result, a system of equations was obtained that eliminates the task of elastic-plastic deformation of the workpiece. **Results.** Using the mathematical model of the workpiece bending process by copier allows to determine the necessary parameters of the process, in particular, the clamping force, the size of which, in addition to other factors, is significantly influenced by the gap, with the increase of which the value of the clamping force will decrease. **Scientific novelty.** Mathematical dependencies have been established to determine the main parameters of the workpiece bending process by copier. **Practical significance.** The results of mathematical modelling will allow to increase the efficiency and quality of that process.

Keywords: bending process, stress, deformation, bending moment, copier, clamping force.

Introduction

The metal bending process is a manufacturing operation where sheet metal undergoes deformation, typically using presses or other specialized mechanical devices, to achieve the desired shape or configuration. Bent rectangular and square profiles provide systems' strength and stability, making them crucial components in various industrial and construction sectors. In manufacturing, these systems may be represented as metal frames serving as the bridge foundation, lifting towers, tunnels, and other engineering structures, as well as uprights and beams in building constructions, ensuring the necessary strength and stability, when designing conveyor frames or vehicles. Metal bending is widespread in various industries due to a range of advantages.

The main advantages of the metal bending process include:

- high productivity: bending process can be automated and highly productive, enabling high production volumes;
- material efficiency: metal bending often allows the use of less raw material compared to other metal processing methods;
- versatility with different metals: bending process can be applied to various types of metals, such as steel, aluminum, brass, etc.;
- ability to create complex shapes: metal bending enables the production of parts with complex geometries and shapes;
- time and cost savings: compared to other metal processing methods, bending can be more efficient and faster, reducing production time and costs.

Review of modern information sources on the subject of the paper

Due to high availability, ease of implementation in various industrial sectors, and a range of advantages, the bending process of the workpiece has gained significant popularity. Currently, considerable attention is focused on experimental investigation and prediction of workpiece behavior during bending [1, 2]. Research [1] presents the development of machine learning models for predicting the flexibility and resistance of the cross-section of a double-T profile during bending. Additionally, the application of nonlinear analysis of system behavior using ABAQUS software is extensively discussed for further numerical simulations and data generation for model training. Another modern method of predicting system behavior is the development of models based on creating digital twins [2]. In the study [2], a numerical model of the workpiece bending process by copier was developed, where internal stresses in the workpiece during bending and residual stresses after manufacturing can be investigated based on the obtained data. In work [3], based on the analysis of the experimental data, considerable attention was paid to the critical local banding of plates, and a numerical analysis using the finite element method was conducted to study the influence of a detailed length and properties on their bending for the profile made of a composite material. Also, numerous experimental researches were conducted to analyse the mechanical and structural behaviours of various types of the high-strength steel components including residual stress measurements [4], [5], [6], stub columns [7], [8].

Research methods

For the analysis of the bending process, a power dependence between stresses and strains is utilized. Since there is a linearly stressed and volumetrically deformed state for a narrow workpiece, according to the assumption of plane sections, the relative deformation of any fiber is a linear function of its distance from the neutral axis. Additionally, it is important to analyze the bending moment in the workpiece section, which has a curvature radius of the neutral axis.

Thus, the goal of the study is to develop a mathematical model of the workpiece bending process by copier to determine the necessary process parameters and enhance its efficiency.

Research results

Since both elastic and plastic deformations occur in the workpiece material during its copying deformation, it will be used a power dependence between stresses and strains instead of Hooke's law, namely [9]:

$$\sigma = K\varepsilon^n, \tag{1}$$

with K and n are constants determined by the main mechanical properties of the metal in tension, specifically from the condition of passing the curve through the flow stress (σ_{02}, ε_{02}) and the ultimate tensile strength (σ_{TS}, ε_{TS}) on the σ–ε diagram of the specific material.

$$\begin{cases} \sigma_{02} = K\varepsilon_{02}{}^{n}, \\ \sigma_{TS} = K\varepsilon_{TS}{}^{n}. \end{cases} \quad (2)$$

Solving system (2) for K and n, it is obtained:

$$n = \lg\left(\frac{\sigma_{TS}}{\sigma_{02}}\right)\lg\left(\frac{\varepsilon_{TS}}{\varepsilon_{02}}\right), \quad K = \frac{\sigma_{TS}}{\varepsilon_{TS}{}^{n}}.$$

According to the plasticity theory [10], when the curvature radius of the neutral layer of the workpiece ρ_0 and its height h satisfy the condition $\rho_o/h > 0$, the mutual pressure of material fibers on each other is infinitely small. It is considered that this condition is fulfilled. Therefore, it is assumed the stress component along the *y*-axis (σ_y) is equal to zero.

Workpieces are classified as narrow ($h \approx b$) and wide ($h \ll b$), where b is the workpiece width. It is considered the workpiece to be narrow. Hence, the stress along the *z*-axis is equal to 0 ($\sigma_z = 0$) and there is a linearly stressed and volumetrically deformed state, for which the following dependencies hold:

$$\begin{cases} \sigma_x = \sigma_1, \sigma_y = \sigma_z = 0, \\ \varepsilon_x = \varepsilon_1, \varepsilon_y = \varepsilon_z = -\mu\varepsilon_1 \end{cases}$$

with $\mu = 1/2$ is the Poisson's ratio for the metal.

Subsequently, it is assumed that the bending process occurs in the {x, y} plane (Fig. 1).

Fig. 1. The diagram of the workpiece's cross-section and the bending plane

According to the assumption of plane sections, the strain ε of any fibre is a linear function of its distance y from the neutral axis, i. e.:

$$\varepsilon = \frac{1}{\rho_0} y. \quad (3)$$

Then taking into account (1) and (3), the stress in this fibre will be:

$$\sigma = K\frac{1}{\rho_0{}^{n}} y^{n}.$$

Due to the workpiece symmetry, the neutral layer coincides with the {x, z} plane.

Let's determine the bending moment in the cross-section of the workpiece, which has a curvature radius of the neutral layer ρ_0:

$$M = \frac{2K}{\rho_0^{n}(n+2)}\left[b\left(\frac{h}{2}\right)^{n+2} - (b-2a)\left(\frac{h}{2}-a\right)^{n+2}\right], \quad M = \frac{1}{\rho_0} KJ \quad (4)$$

with J is the moment of inertia of the workpiece cross-section.

In expression (4), the curvature radius ρ_0 of the neutral layer is equal to:

$$\rho_0 = R_c + \frac{h}{2}$$

with R_c is the copier radius.

The moment in any workpiece cross-section with a curvature radius ρ will be determined analogously as in the formula (4), namely:

$$M = \frac{2K}{\rho^n (n+2)}\left[b\left(\frac{h}{2}\right)^{n+2} - (b-2a)\left(\frac{h}{2}-a\right)^{n+2}\right], M = \frac{1}{\rho}KJ. \tag{5}$$

Let's consider the equilibrium of the workpiece bent element with a finite length (Fig. 2), namely the workpiece part from the last point of contact with the copier to the point of the force P application. Let's denote this element as "0–1".

The element will be cut at an arbitrary point Q with coordinates x, y. Let's examine the equilibrium of this segment. The force vector will be denoted by P balancing the force P_1.

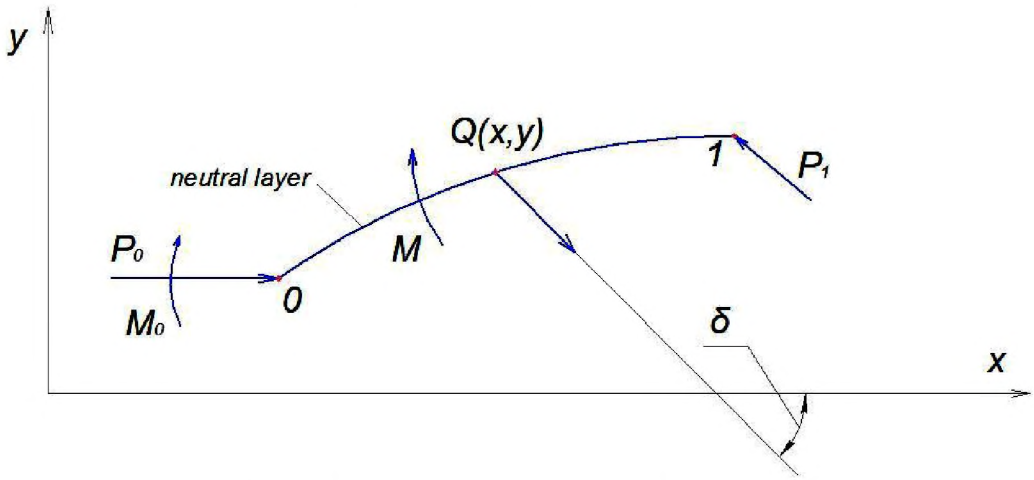

Fig. 2. The equilibrium of the bent workpiece element

The moment M^* balancing the moment of the force P_1, is equal to:

$$M^* = P(y_1 - y)\cdot\cos(\delta) + P(x_1 - x)\cdot\sin(\delta). \tag{6}$$

By differentiating (6) with respect to the arc coordinate S, it is obtained:

$$\frac{dM^*}{dS} = -P\cdot\sin(v+\delta) \tag{7}$$

with v is the angle inclination of the tangent at the point Q.

By differentiating the dependency (5) with respect to the arc coordinate S, it is obtained:

$$\frac{dM}{dS} = \frac{d}{dS}\left[\frac{1}{\rho^n}KJ\right] = KJn\left(\frac{dv}{dS}\right)^{n-1}\cdot\frac{d^2v}{dS^2}. \tag{8}$$

The moment M^* of the external force P must be balanced by the moment M of internal forces, which is determined by the dependency (5), i e., $M = M^*$. Hence, $dM/dS = dM^*/dS$, or taking into account (7) and (8):

$$KJn\left(\frac{dv}{dS}\right)^{n-1}\cdot\frac{d^2v}{dS^2} = -P\cdot\sin(v+\delta). \tag{9}$$

Denoting $v+\delta$ as ξ, multiplying (9) by $d\xi$ and integrating, it is obtained:

$$\left(\frac{d\xi}{dS}\right)^{n+1}=\frac{2(1+n)}{KJn}P\left(C-\sin^2\left(\frac{\xi}{2}\right)\right) \tag{10}$$

with C is the integration constant.

Equation (10) characterizes the change in the curvature of the workpiece neutral layer along the segment “0–1”, since the point $Q(x,y)$ was chosen arbitrarily.

Let’s introduce the following notations:

$$\lambda=\frac{1}{2}\left[\frac{2(1+n)}{nKJ}P\right]^{\frac{1}{1+n}},\ K^2=C,\ K\sin(\varphi)=\sin\left(\frac{\xi}{2}\right)\Rightarrow d\xi=2K\frac{\cos(\varphi)d\varphi}{\sqrt{1-K^2\sin^2(\varphi)}},\ m=\frac{1-n}{1+n}. \tag{11}$$

Substituting (11) into (10), it is obtained:

$$\frac{d\varphi}{dS}=\lambda\left(K\cos(\varphi)\right)^m\sqrt{1-K^2\sin^2(\varphi)}. \tag{12}$$

Separating variables in the expression (12) and integrating them, it is obtained:

$$dS=\frac{1}{K^m}\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\cos^m(\varphi)\sqrt{1-K^2\sin^2(\varphi)}}. \tag{13}$$

Equation (13) connects the external factors λ, S to the elliptic parameters K, φ, which express the curvature of the bent element. Equation (13) contains three unknown parameters, so it needs to be supplemented with two additional equations:

$$\begin{cases}\dfrac{M}{(2\lambda)^n KJ}=\left(K\cos(\varphi)\right)^{\frac{2n}{1+n}},\\ \xi=2\arcsin\left(K\sin(\varphi)\right).\end{cases} \tag{14}$$

Combining (13) and (14), it is obtained a system of equations that will allow to solve the problem of elastic-plastic deformation of the workpiece:

$$\begin{cases}\lambda S=\dfrac{1}{K^m}\displaystyle\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\cos^m(\varphi)\sqrt{1-K^2\sin^2(\varphi)}},\\ M_n=K\cos(\varphi),\\ \xi=2\arcsin\left(K\sin(\varphi)\right)\end{cases} \tag{15}$$

with $M_n=\left[\dfrac{M}{2^n\lambda^n KJ}\right]^{\frac{1+n}{2n}}$.

Next let’s formulate the boundary conditions: at the end point “1” ($S=L$), the bending moment is equal to zero; then from the second equation of the system (15) $M_{n/S=L}=0\Rightarrow K\cos(\varphi_1)=0$ because $K=0,\ \varphi_1=\dfrac{\pi}{2}$.

The active deforming force is normal to the neutral layer at the application point. Therefore, according to Fig. 2, for the end point:

$$\frac{V}{S=L}=\frac{\pi}{2}.$$

Then $\frac{\xi}{S=L}=v+v=\frac{\pi}{2}$, taking into account the third equation of the system (15) it is obtained:

$$\frac{K}{S=L}=\sin\left(\pi/4\right)=\frac{\sqrt{2}}{2}.$$

Thus, for the end point $\varphi_1=\frac{\pi}{2},\ K=\frac{\sqrt{2}}{2}$.

Let's consider the conditions at the initial point "0" ($\xi=0$).

$$\sin\left(\frac{v}{2}\right)=K\sin\left(\varphi_0\right)=\frac{\sqrt{2}}{2}\sin\left(\varphi\right)$$

or $\cos\left(v\right)=\cos^2\left(\varphi_0\right)$.

By integrating the first equation of the system (15) over the entire length of the element, it is obtained:

$$\lambda L=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^m}\int_{\varphi_0}^{\varphi_1=\frac{\pi}{2}}\frac{d\varphi}{\cos^m\left(\varphi\right)\sqrt{1-K^2\sin^2\left(\varphi\right)}}. \tag{16}$$

Let's represent the integral (16) as a sum of two integrals:

$$J_2=\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\cos^m\left(\varphi\right)\sqrt{1-K^2\sin^2\left(\varphi\right)}}=\frac{\sqrt{2}}{4}B\left(p,q\right)=\frac{\sqrt{2}}{4}\frac{\Gamma\left(q\right)\cdot\Gamma\left(p'\right)}{\Gamma\left(q+p'\right)},$$

$$J_3=\int_0^{\varphi_0}\frac{d\varphi}{\cos^m\left(\varphi\right)\sqrt{1-K^2\sin^2\left(\varphi\right)}}=\psi_1\left(K,\varphi_0\right)=M_1F\left(K,\varphi_0\right)+N_1E\left(K,\varphi_0\right)+Q_1\left(K,\varphi_0\right),$$

with $q=\frac{1-m}{4},\ p'=\frac{1}{2},\ \Gamma$ is a Gamma function.

$$M_1=C_1+C_2-\frac{1-K}{K^2}C_3-\frac{\left(1-K^2\right)\left(3K^2-2\right)}{3K^4}C_4,$$

$$N_1=\frac{2\left(2K^2-1\right)}{3K^4}-\frac{1}{1-K^2}C_1+\frac{1}{K^2}C_3,$$

$$F\left(K,\varphi_0\right)=\int\frac{d\varphi}{\sqrt{1-K^2\sin^2\left(\varphi\right)}},$$

$$Q_1\left(K,\varphi_0\right)=\left[\frac{tg\left(\varphi_0\right)}{1-K^2}C_1+\frac{\sin\left(2\varphi_0\right)}{6K^2}C_4\right]\sqrt{1-K^2\sin^2\left(\varphi_0\right)},$$

$$E\left(K,\varphi_0\right)=\int\sqrt{1-K^2\sin^2\left(\varphi\right)}d\varphi,\ K=\frac{\sqrt{2}}{2}.$$

Substituting the values of the integrals into the equation (16), it is obtained:

$$\lambda L=\left(\frac{2}{\sqrt{2}}\right)^m\left(\frac{\sqrt{2}}{4}B\left(p',q\right)-\psi_1\left(K,\varphi_0\right)\right). \tag{17}$$

Dependency (17) combines three independent parameters: the workpiece length L from the last point of the contact with the copier to the application point of the active force P with the amplitude of the elliptic integral φ_0 at the last point of the contact and λ_0.

To uniquely determine them, it is necessary to establish additional dependencies between these parameters. For this purpose, let's write down the displacements of the points of the neutral layer. It will be chosen a moving coordinate system oriented along the force P with the origin coinciding with the fixed coordinate system origin. The moving system is rotated relative to the fixed one by an angle δ (Fig. 3).

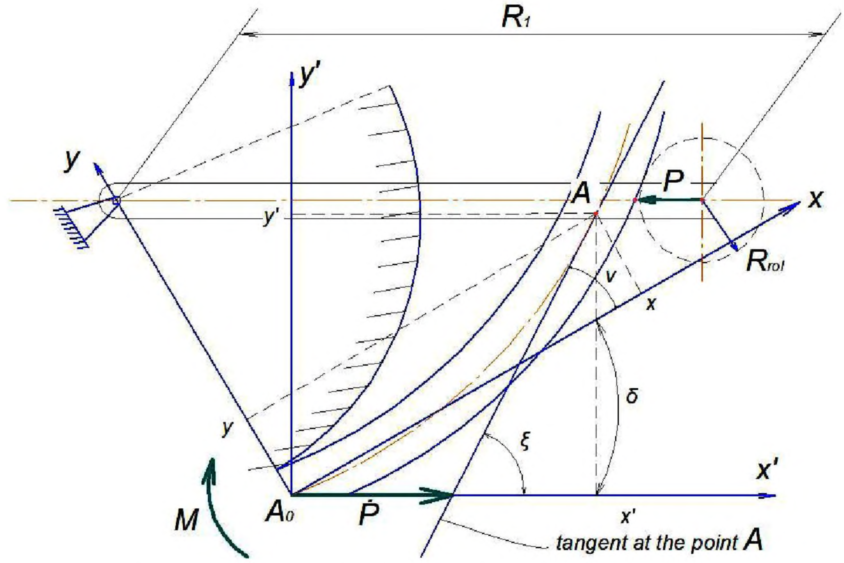

Fig. 3. The diagram of the displacement of points of the workpiece neutral layer

The relation between coordinates is expressed by transition formulas. According to Fig. 3, for an arbitrary point $A(x', y')$, it can be written:

$$\begin{cases} x = x'\cos(\delta) + y'\sin(\delta), \\ y = y'\cos(\delta) - x'\sin(\delta). \end{cases}$$

As the common origin {x, y}, {x', y'} is chosen the point A_0, which is the last point of the neutral layer touching the copier. Therefore, $x'_0 = y'_0 = 0$. Additionally, the fixed system {x, y} is oriented such way the x-axis is along the neutral layer tangent at the point A_0.

In the moving coordinate system {x', y'}, the relations are as follows:

$$\begin{cases} dx = dS\cos(\xi), \\ dy = dS\sin(\xi) \end{cases} \tag{18}$$

with ξ is the angle of inclination of the tangent at point A {x', y'} to the x'-axis. Since

$$\begin{cases} \cos(\xi) = 2\cos^2(\xi/2) - 1 = 1 - 2\sin^2(\xi/2), \\ \xi = 2\arcsin(k\sin(\varphi)), \end{cases}$$

so, the equation (18) can be expressed in the following form:

$$dx' = \cos^2(\varphi)\,dS. \tag{19}$$

From the equation (12), let's express dS and substitute it into (19). As a result, it is obtained:

$$K^m \lambda dx' = \frac{\cos^2(\varphi)\,d\varphi}{\cos^m(\varphi)\sqrt{1 - K^2\sin^2(\varphi)}}. \tag{20}$$

Let's integrate (20) from the initial point ($x_0 = 0$; φ_0) to the final point (x_1'; $\varphi = \frac{\pi}{2}$):

$$K^m \lambda x_1' = \int_{\varphi_0}^{\frac{\pi}{2}} \frac{\cos^2(\varphi)d\varphi}{\cos^2(\varphi)\sqrt{1-K^2\sin^2(\varphi)}}. \quad (21)$$

It is determined the integral of the right-hand side of the expression (21) similarly to the integral in the equation (16). Then the expression (21) takes the form:

$$K^m \lambda x_1' = \frac{\sqrt{2}}{4} B(p', q') - \psi_2(K, \varphi_0),$$

with $q' = \frac{3-m}{4}$.

$$\psi_2(K,\varphi_0) = M_2 F(K,\varphi_0) + N_2 E(K,\varphi_0) + Q_2(K,\varphi_0),$$

$$M_2 = \left(C_1 - \frac{1-K^2}{K^2} C_2 + \frac{2-5K^2+3K^4}{3K^4} C_3 - \frac{8-27K^2+34K^4}{15K^6} - 1 \right) C_4,$$

$$N_2 = \frac{1}{K^2} C_2 + \frac{2(2K^2-1)}{3K^4} C_3 + \frac{8-23K^2+23K^4}{15K^6} C_4,$$

$$Q_2(K,\varphi_0) = \left[\frac{\cos(\varphi_0)}{3K^2} C_3 + \left(\frac{\cos^3(\varphi_0)}{5K^2} + \frac{4(2K^2-1)}{15K^4} \cos(\varphi) \right) C_4 \right] \sin(\varphi_0) \sqrt{1-K^2\sin^2(\varphi_0)},$$

$$K = \frac{\sqrt{2}}{2}.$$

Similarly, it is found y_1':

$$\lambda y_1' = \frac{2K^{1-m}}{1-m} \cos^{1-m}(\varphi_0).$$

Knowing x_1' and y_1', it is determined the values of x and y using the transition formulas. They are expressed as:

$$\lambda x_1 = \frac{1}{K^m} \left[\frac{\sqrt{2}}{4} B(p',q') - \psi_2(K,\varphi_0) \right] \cos(v) + \frac{2K^{1-m}}{1-m} \cos^{1-m}(\varphi_0) \sin(v),$$

$$\lambda y_1 = \frac{2K^{1-m}}{1-m} \cos^{1-m}(\varphi_0) \cos(v) - \frac{1}{K^m} \left[\frac{\sqrt{2}}{4} B(p',q') - \psi_2(K,\varphi_0) \right] \sin(v). \quad (22)$$

Now it becomes possible to solve the following problem: the workpiece is bent by a copier mounted on a stationary axis using a holder with a certain gap. To determine the required pressing force P based on equations (17), (18), (22), it is written down the following system of equations:

$$\lambda L = \begin{cases} \left(\frac{2}{\sqrt{2}} \right)^m \left(\frac{\sqrt{2}}{4} B(p',q) - \psi_1(K,\varphi_0) \right), \\ dx' = dS\cos(\xi), \\ dy' = dS\sin(\xi), \\ \frac{1}{S_0} = 2\lambda (K\cos(\varphi_0))^{\frac{2}{1+n}}, \\ (x_1)^2 + (y_1 - y_{center})^2 = \left(R_1 - R_{rol} - \frac{h}{2} \right)^2. \end{cases} \quad (23)$$

In the system of five equations (23), the unknown parameters are: φ, L, x_1, y_1, λ. By using this system of equations, it can be determined λ and then, using the formula (11), it can be found the pressing force P, which, among other factors, is significantly influenced by the gap, increasing which will decrease the value of P.

Conclusions

Based on the conducted research, dependencies were analysed and established, allowing to estimate the force required to be applied to the workpiece during its bending by the copier, as well as factors influencing this process. These calculated dependencies will enhance the efficiency and quality of the process of bending workpieces by a copier.

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