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DEVELOPING A MATHEMATICAL MODEL FOR DESCRIBING THE MOTION OF A VIBRATORY TYPE SEPARATOR WITH SEQUENTIAL SCREENS

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Abstract. The developed mathematical model of a vibratory separator with sequential screen placement and spring suspension is nonlinear, unified, and parametric. It was derived based on the utilization of the second-order Lagrange equation and asymptotic methods of nonlinear mechanics. The model is intended for investigating the dynamics of the separator during its design and subsequent operation, particularly for selecting its optimal design parameters and determining the conditions of steady-state and transient operating modes. It is presented as a set of analytical dependencies representing the motion laws of arbitrary points of the separator's working body in terms of its geometric, mass-inertial, and kinematic parameters with the ability to cover a wide range of variations. The model can be integrated into systems for automated design and calculation of various vibratory separators, differing in the number of screens, arrangement, and complexity of the drive, suspension, working body, etc.

Key words: separator; unbalance; nonlinear model; vibratory drive; sequential screens; spring suspension.

Introduction and problem statement

One of the effective technologies widely applied in various production fields is separation technology, which involves the extraction and sorting of individual components from mixtures of different types and states based on specific physical and mechanical characteristics. The separation process is an integral part of the overall manufacturing process of many products, and the quality of separation significantly influences the quality of the final product.

The spectrum of separator designs is extremely diverse. Therefore, much attention is paid to researching the principles of their operation, increasing productivity, and creating new separator prototypes. An effective approach to designing a new separator involves creating its mathematical model, investigating it, and selecting optimal parameters for future development. However, such an approach to developing new separator designs is not widely applied in practice. This can be explained by the fact that such research requires the use of complex mathematical apparatus, advanced mathematical methods, and computing technology. To adequately reflect all dynamic phenomena occurring in the separating system, mathematical models describing it must be nonlinear. The linearity of models leads to the inability to fully reflect the influence of parameters of the future separator design on its efficiency of operation.

Review of modern information sources on the subject of the paper

The range of separator types is extremely diverse. Separators can be classified based on their structural features, particularly the movement of the separating surface, into separators where the separating surface is stationary, with movable separating elements [1], [2].

The most productive separators are those equipped with a movable screen, and there can be multiple screens [1], [2]. When these screens are arranged one above the other with decreasing cell size towards the bottom and provided with oscillating motion, the mixture will discharge onto the top screen, which has the largest cell. Particles of the largest size will remain on it, while others, depending on the cell size and the particle itself, will be distributed across the screens throughout the height of the separator.

The type of separation described above is implemented in vibratory separators [2–5]. A distinctive feature of these separators is that by using the drive and the arrangement of the screens, different amplitudes and trajectories of their movement can be achieved. Additionally, the screen of the separator can have a rotational motion. Such separation is implemented in drum-type separators [4, 5]. Drum separators consist of screens of conical or cylindrical shape, which rotate about their axes.

The considered vibratory and drum separators are characterized by high productivity and reliability. Vibratory separators are driven by vibrational energy, which can be controlled, providing a wider range and flexibility of separation modes. Drum separators have a large area of screens, which can be arranged concentrically and axially.

For better mixing and transportation of material mixtures that need to be separated from each other (and these two conditions carry significant weight in determining the productivity of the separator), we propose to use a vibratory drive to mobilize the drum separator with sequential screen placement. In other words, to synthesize a new type of separator from two existing types – a drum vibratory separator, which will combine the advantages of both original types.

A vibratory separator with sequential screen placement represents a complex dynamic system, the adequate description of which can only be rationalized through modeling. This type of research has been conducted by the authors to study the dynamics of machines implementing vibrational bulk processing of products and vibratory separators [6–7]. To design an optimal construction of such a separator, it is necessary to conduct theoretical investigations of its dynamics to understand how its parameters affect the intensity of the separation process for bulk mixtures. Specifically, the development of a nonlinear model describing the motion of the separator will enable the realization of this task.

Main material presentation

The conducted research aims to develop a nonlinear model of a vibratory separator with sequential screen placement and spring suspension to study dynamic phenomena in the separator and determine the influence of separator parameters on the factors of separation of bulk mixtures in it.

Based on the description, the rotational drum-type separators exhibit a certain versatility due to their circular screens. However, rotational motion alone cannot provide sufficient separation productivity compared to vibrational motion. Therefore, we install a vibratory drive on the circular screen of cylindrical or conical shape. This will facilitate better mixing of the mixture during separation and improve its contact and interaction with the screen. Additionally, the vibratory drive can ensure vibrational transportation of the mixture along the screen, thus increasing the productivity of the separation process.

In such a separator, a working chamber for mounting the screens is not required. The enclosed screens of cylindrical or conical shape will have sufficient rigidity to serve as the working supporting element. The number of screens installed at the end of a vibratory separator with sequential screen placement can be arbitrary, for example, N. Then, the number of fractions separated by this separator from the mixture will be $N+1$. It is important to emphasize that a receiver for the separated fraction must be installed under each screen. The spring suspension of the separator can be of a spring, pendulum (leaf spring), or pneumatic balloon type.

The versatility of such a separator lies in the following aspects: its screens are independent of each other and can have different lengths; the size and shape of the screen cells can vary; this separator can be assembled like a construction set by changing the number of screens; the open form of the screens and absence of rotation (only oscillatory motion) allow for the creation of openings for loading mixtures of different types, as well as designing U-shaped screens with loading capability; the open form of the screens allows for subsequent drying of the separated fractions of the mixture or washing them; the large length of the separator, due to the arrangement of elongated screens, allows for the use of the separator as a vibratory conveyor; to improve the process of transporting the mixture along the axis of the separator, it is possible to easily implement an inclination of the axis of all screens (the entire screen system) at a certain angle to the horizon; the vibratory unbalanced drive of the separator is simple in design, installation, and maintenance.

Based on the considerations provided, Fig. 1 illustrates the schematic diagram of the investigated vibratory separator with sequential screen placement, unbalanced drive, and spring suspension. The diagram depicts: 1 – the working body of the separator – a system of sequential screens (the number of which can be arbitrary); 2 – adjustable unbalanced node; 3 – elastic coupling; 4 – frame; 5 – spring suspension (in this case, a system of helical cylindrical springs); 6 – drive motor; 7 – collectors of separated fractions (their number is equal to the number of screens, they are attached to the frame of the separator); 8 – mixture loading zones (can be conducted separately into each screen along the length of the separator).

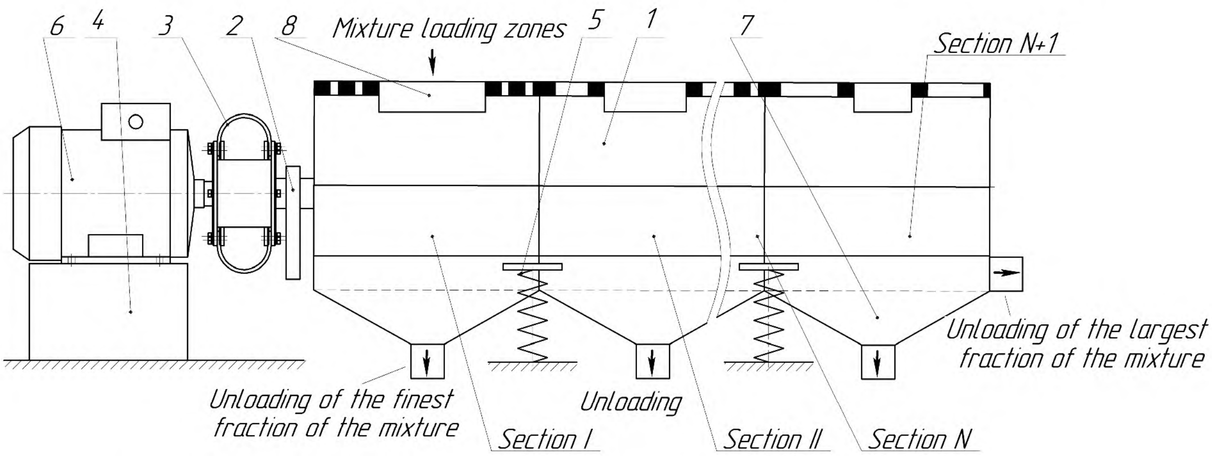

Fig. 1. Schematic diagram of the vibratory separator with sequential screen placement, unbalanced drive, and spring suspension

Given the presence of a set of parameters influencing the intensity of operation of a vibratory separator with sequential screen placement, it is necessary to determine the magnitude and significance of these parameters. It is also important to consider which parameters can be adjusted, for example, during the operation of the separator, and which ones should be selected and incorporated into the separator design at the design stage. Based on practical experience, the intensity of separation is determined by the level of interaction between the mixture and the screen – the level of mixing of the mixture on the screen, its throwing, and movement on the screens of the separator. The key factor here is the amplitude of oscillations and the frequency of the screen's oscillatory motion. The latter can be set as an input parameter of the drive. Accordingly, the mathematical model describing the motion of the vibratory separator should reflect the influence of separator parameters (including frequency) on the amplitude and nature of the oscillatory motion of the sequential screen system.

The vibratory separator with sequential screen placement undergoes complex motion. The key component is the planar-parallel movement of the screens in planes parallel to the planes of rotation of the separator drive unbalances. This oscillatory motion determines the intensity of separation by disturbing the separator screens. To simplify the model, we will limit ourselves to the planar motion of the separator, assuming that it oscillates only in the planes of the unbalanced motion. The transverse stiffness of the suspension is high, so we disregard longitudinal vibrations of the separator (along the axis of the cylindrical surfaces of the screen system). According to these considerations, the schematic diagram of the investigated separator will take the form shown in Fig. 2. In other words, Fig. 2 represents the schematic diagram of the oscillatory motion of the cross-section of any screen of the separator at any given moment of its operation.

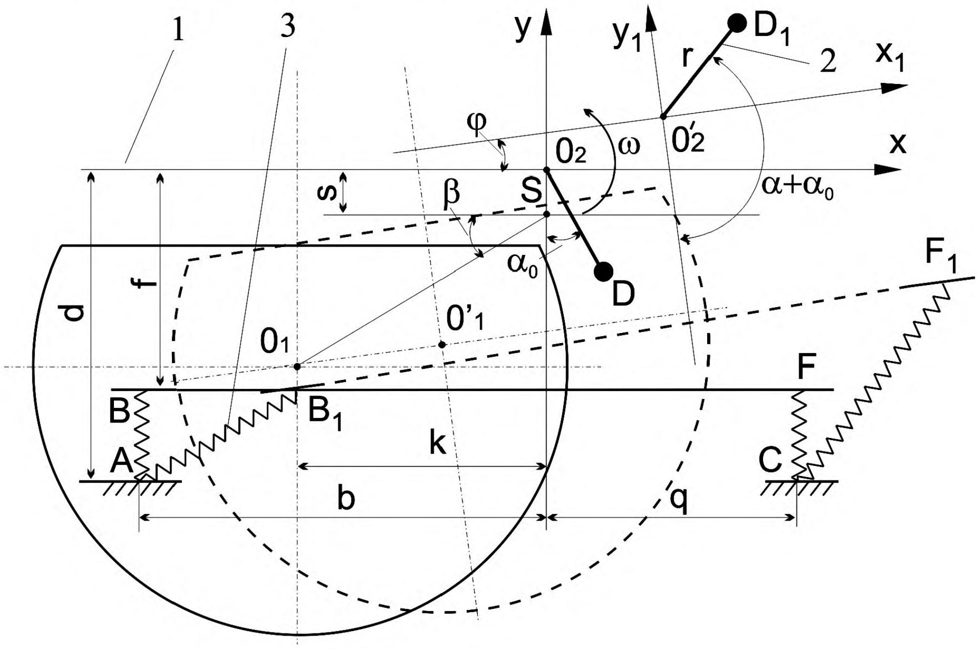

Fig. 2. Schematic diagram of the vibratory separator with sequential screen placement, unbalanced drive, and spring suspension

In Fig. 2, the following are indicated: 1 – circular cross-section of the screen, 2 – drive unbalance (vibration generator), the center of rotation of which can be located at any point of the cross-section, 3 – spring support (similarly can be located in any zone of the cross-section).

Also, the following notations are introduced for the construction of the model:

– M_1 – oscillating mass (includes cylindrical screens, loose mixture);

– XO_2Y – a coordinate system related to the frame of the separator (stationary);

– $X_1 O_2' Y_1$ – a moving coordinate system attached to the oscillating mass of the separator;

– φ – the angle of rotation of the separator relative to its rest state during oscillatory motion (the angle of rotation of the coordinate system $X_1 O_2' Y_1$ relative to the system XO_2Y, when the separator is off, the coordinate systems coincide and $\varphi = 0$);

– β and O_1S – polar coordinate of the center of mass of the loaded screen of the separator;

– $r = O_2D$ – displacement of the center of mass of the unbalance relative to its center of rotation (radius of the unbalance);

– Md – unbalanced mass of the unbalance (material point D);

– ω – angular velocity of the rotational motion of the unbalance;

– α_0 – phase of the unbalanced position when the drive is off;

– $\alpha = \omega t$ – value of the angle of rotation of the unbalanced mass during separation;

– C – stiffness of the separator suspension – the sum of the left C1 and right C2 components;

– $L_{np} = AB = CF$ – length of the suspension springs when the separator is off;

– AB_1 and CF_1 – suspension lengths during separation;

– b and q – horizontal coordinates of the suspension relative to the separator screen;

– d and f – vertical coordinates of the suspension relative to the separator screen.

Results and discussion

The mathematical model of the vibration separator with sequential arrangement of screens, unbalanced drive, and spring suspension is constructed using the laws of mechanics of a rigid body in plane motion, Lagrange's equations of the second kind, and asymptotic methods of nonlinear mechanics [8]. It represents a set of analytical expressions with all the parameters of the separator determining its motion, in symbolic format. To construct it, the main point of the investigated vibration system is chosen – the model describes the oscillatory motion of this point. In this case, point O_2' – the center of the moving coordinate system $X_1 O_2' Y_1$ is selected. This point, according to the above notation, will also be the center of rotation of the unbalanced mass of the separator drive. The system of expressions describing the motion of all other points of the separator will be as follows:

$$
\begin{aligned}
x'_{i2}(t) &= x_{02}(t) + x_i \cos\varphi(t) - y_i \sin\varphi(t), \\
y'_{i2}(t) &= y_{02}(t) + x_i \sin\varphi(t) + y_i \cos\varphi(t),
\end{aligned} \tag{1}
$$

where x'_{i2}, y'_{i2} are the coordinates of the required point of the vibrating mass of the separator relative to the stationary coordinate system XO_2Y, and x_i, y_i are the coordinates of the required point of the vibrating mass of the separator relative to the moving coordinate system $X_1 O_2' Y_1$.

The task was to find the laws of motion $x_{02}(t)$, $y_{02}(t)$, O_2' and the rotation angle of the separator during its operation $\varphi(t)$. These expressions will allow us to: a) describe the horizontal and vertical oscillatory motion of any point of the separator, which, when combined, gives its planar motion; b) plot the trajectory of the separator's screen motion over the investigated time interval of separation; c) determine the influence of all separator parameters on the amplitude and character of screen oscillations, which determine the intensity of separation.

The Lagrangian equation of the second kind takes the form:

$$
\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0 \tag{2}
$$

where $L = T - (\Pi + \Pi_p)$ is the Lagrangian (a mathematical function of a certain form), $T = T_k + T_Д$ is the kinetic energy of the separator (which includes the kinetic energy of the vibrating mass of the separator and separately the kinetic energy of the drive unbalances), Π is the potential energy of the vibrating part of the separator, Π_p is the potential energy of the separator suspension, and q_j are generalized coordinates. Accordingly: $q_1 = x_{0_2}$, $q_2 = y_{0_2}$, $q_3 = \varphi$, generalized speeds – $\dot{q}_1 = \dot{x}_{0_2}$, $\dot{q}_2 = \dot{y}_{0_2}$, $\dot{q}_3 = \dot{\varphi}$.

The Lagrangian of the mechanical system "vibratory separator with mixture, spring suspension, and unbalances" will be represented as:

$$
\begin{aligned}
L = {} & \frac{M_1}{2}((\dot{x}_{02} + r\cos(\omega t + \varphi + \alpha_0)(\omega + \dot{\varphi}))^2 + (\dot{y}_{02} + r\sin(\omega t + \varphi + \alpha_0)(\omega + \dot{\varphi}))^2) + \\
& + \frac{M_1}{2}((\dot{x}_{02} + \dot{y}k\sin\varphi + \dot{\varphi}(ktg\beta + S)\cos\varphi)^2 + (\dot{y}_{02} - \dot{\varphi}k\cos\varphi + \dot{\varphi}(ktg\beta + S)\sin\varphi)^2) + \\
& + \frac{1}{2} j\dot{\varphi}^2 - \left[\begin{array}{l} \frac{C_1}{2}((x_{02} - b\cos\varphi + d\sin\varphi + b)^2 + (y_{02} - b\sin\varphi - \\ - d\cos\varphi + d)^2 - (d - f)^2) + \frac{C_2}{2}((x_{02} + q\cos\varphi + f\sin\varphi - q)^2 + \\ + (y_{02} + q\sin\varphi - \cos\varphi + d)^2 - (d - f)^2 \end{array} \right] +
\end{aligned} \tag{3}
$$

$$+\begin{bmatrix} M_1 g(y_{02} - k\sin\varphi - \cos\varphi(ktg\beta + S) - (ktg\beta + S)) + \\ + M_Д g(r\cos\alpha_0 + y_{02} - r\cos(\omega t + \varphi + \alpha_0)) \end{bmatrix}.$$

From the obtained expression (3), the partial derivatives of the generalized coordinates x_{02}, y_{o2}, φ and the generalized velocities $\dot{x}_{02}$, $\dot{y}_{02}$ та $\dot{\varphi}$ were found, and their expressions were substituted into the dependence (2). Further, after certain mathematical transformations, the system of equations was obtained:

$$\begin{cases} \ddot{x}_{02} + \omega_1{}^2 x_c = \varepsilon f_x(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0); \\ \ddot{y}_{02} + \omega_1{}^2 y_c = \varepsilon f_y(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0); \\ \ddot{\varphi} + \omega_\varphi^2(t)\varphi = \varepsilon' f_\varphi(\varphi, \dot{\varphi}, \ddot{x}_{02}, \ddot{y}_{02}). \end{cases} \quad (4)$$

In the right-hand side of the system (4), the generalized coordinate of the rotation angle and the values of all parameters of the vibrating separator are included in symbolic form. In (4), $\varepsilon = 1/M_1 << 1$ represents the coefficient, $\omega_1 = \sqrt{C/M_1}$ – denotes the natural frequency of the vibrating part of the separator, and $\omega_\varphi(t)$ represents the "frequency" of the torsional vibrations of the separator's screens during its planar motion.

In general, the system of equations (4) represents a mathematical model describing the motion of the separator with sequential arrangement of screens in a differential form. For practical applications, it should be solved by presenting the model with analytical solutions. To achieve this, equation (2) is expanded for each generalized coordinate with the prior substitution of their corresponding derivative values. The solution to (4) is carried out by applying asymptotic methods of nonlinear mechanics [8] in the form of:

$$\begin{gathered} x_{02}(t) = x_0 \sin(\sqrt{\frac{C}{M_1}}t + \alpha_x) + \varepsilon\int_0^t f_x(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0)\sin\left(\sqrt{\frac{C}{M_1}}(t-u)\right)du, \\ y_{02}(t) = y_0 \sin(\sqrt{\frac{C}{M_1}}t + \alpha_y) + \varepsilon\int_0^t f_y(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0)\sin\left(\sqrt{\frac{C}{M_1}}(t-u)\right)du, \\ \varphi(t) = L_1 \sin k1t + L_2\cos k1t, \end{gathered} \quad (5)$$

where $f_x(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0)$ and $f_y(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega t + \alpha_0)$ are the values of the right-hand side function of the equations system (4).

The complete expression of the analytical expressions system (5) considering all the separator parameters in the first approximation will be as follows:

$$x_{02}(t) = x_0 \sin\left(\sqrt{\frac{C}{M_1}}t + \alpha_x\right) +$$

$$+\varepsilon\int_0^t \begin{bmatrix} k^2(L_1\sin k1t + L_2\cos k1t)(M_Д r\cos(\omega t + L_1\sin k1t + L_2\cos k1t + \alpha_0) + \\ + M_1\begin{bmatrix} K\sin(L_1\sin k1t + L_2\cos k1t) + \\ + (Ktg\beta + S)\cos(L_1\sin k1t + L_2\cos k1t) \end{bmatrix} - \\ - (L_1 k\cos k1t - L_2 k\sin k1t)^2 \times \\ \times\begin{bmatrix} M_1(K\cos(L_1\sin k1t + L_2\cos k1t) - (Ktg\beta + \\ + S)\sin(L_1\sin k1t + L_2\cos k1t)) \end{bmatrix} + \\ + M_Д r(\omega + L_1 k\cos k1t - L_2 k\sin k1t)(-C_2 q + C_1 b) - \\ - \sin(L_1\sin k1t + L_2\cos k1t)(C_1 d - C_2 f) + C_2 q - C_1 b \end{bmatrix} \times \sin\left(\sqrt{\frac{C}{M_1}}(t-u)\right)du, \quad (6)$$

$$y_{02}(t) = y_0 \sin\left(\sqrt{\frac{C}{M_1}}t + \alpha_y\right) +$$

$$+\varepsilon\int_0^t \begin{bmatrix} k^2(L_1 \sin k1t + L_2 \cos k1t)(M_Д(r\sin(\omega t + L_1 \sin k1t + \\ + L_2 \cos k1t + \alpha_0) + M_1((ktg\beta + S)\sin(L_1 \sin k1t + \\ + L_2 \cos k1t) - K\cos(L_1 \sin k1t + L_2 \cos k1t)) - \\ -(L_1 k \cos k1t - L_2 k \sin k1t)^2 \times \\ \times (M_1(k\sin(L_1 \sin k1t + L_2 \cos k1t) + (ktg\beta + \\ + S)\cos(L_1 \sin k1t + L_2 \cos k1t)) - \\ -M_Д r(\omega + L_1 k \cos k1t - L_2 k \sin k1t)^2 \cos(\omega t + L_1 \sin k1t + \\ + L_2 \cos k1t + \alpha_0) + \cos(L_1 \sin k1t + L_2 \cos k1t)(C_1 d + C_2 f) + \\ + \sin(L_1 \sin k1t + L_2 \cos k1t)(C_1 b - C_2 q) - \\ - d(C_1 + C_2) + (M_1 + M_Д)g \end{bmatrix} \times \sin\left(\sqrt{\frac{C}{M_1}}(t-u)\right)du,$$

$$\varphi(t) = L_1 \sin k1t + L_2 \cos k1t\ .$$

The obtained analytical system (6) will be the final mathematical model for describing and studying the dynamics of the vibrating separator with sequential arrangement of screens, unbalanced drive, and spring suspension.

Conclusions

The chosen mathematical model of the vibrating separator with sequential screen arrangement offers several advantages compared to other separators: simplicity in construction and maintenance, low sensitivity to the properties of the separated mixture, versatility in vibration patterns, automated discharge of separated fractions, and the potential for process automation.

The developed model of the separator with sequential screen arrangement, vibratory drive, and spring suspension allows for investigating the influence of key parameters on the amplitude of its vibrations, including parameters that can be easily varied during practical use. It is suitable for designing the separator and determining the required operating modes.

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