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Influence of friction characteristics of a bulk material on its outflow from a conical hole

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Taking into account the adhesive interaction of the product with the walls of the hopper, a mathematical model of the fine-grained bulk material stress state in a conical hopper hole under vibration is proposed. A discrete environment model was used for analysis. This model was based on considering the equilibrium of an elementary volume with an infinitesimal thickness. An analytical dependency was established, allowing an estimate of the influence of frictional factors on the fine-grained bulk material stress state, thereby enabling its modification by adjusting certain parameters, such as the dosing process and the discharge hopper geometry.

Keywords: *fine-grained bulk material; stress state; unloading hole; hopper; vibration parameters; friction characteristics; leakage conditions.*

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1. Introduction

The vast diversity of bulk materials (BM), which have permeated virtually all industries today, led to increased attention to these materials and deeper analysis of their behavior. Particularly noteworthy are fine-grained materials with particle sizes smaller than 50 microns. Such BMs include powdered sugar, flour, cement, certain types of pharmaceutical powders. Processing, transportation, mixing and storage of the fine-grained BM give rise to issues such as clumping, uneven flow and product adhesion to the technological equipment, which is in direct contact with the material. These problems stem from the instability of the physical and mechanical material properties and a myriad of factors, both predictable and random, significantly influencing the BM behavior [1]. The rapid development of food, construction, pharmaceutical and other industries closely associated with bulk materials continually raises higher demands regarding packaging accuracy and the quality of both packaging and the products themselves.

One of the most critical characteristics of the BM affecting its behavior during transportation is its flowability. The BM flow can conventionally be divided into static and dynamic categories [2]. Static flow occurs during discharging from a hopper under the force of gravity without any additional applied force. Dynamic flow of the BM is largely determined by external forces that activate the material movement. Analysis of these types of flow involves a detailed examination of frictional product properties, both between individual particles and also between particles and walls of the container where the product is located.

Modern scientists have conducted a significant amount of theoretical and experimental research about BM behavior and analyzed the factors that determine it. Numerous studies have emphasized the significant influence of the material particle size and bulk density of the material [3–7], aeration and cohesive properties of the product [8,9] and the conditions of vibration [10–13] on the behavior of particles in a fluidized bed. In the research [14] considerable attention was devoted to the vibratory fluidized bed of the BM and the impact of vibration intensity on the aggregation of individual phases for multi-component materials.

In the article [1], significant attention was given to the factors determining the BM flowability and, thus, the effectiveness of processes such as pseudo-fluidization, mixing, crushing, granulation, tableting, and discharging from containers. It was established that an important aspect in the study of the BM movement is the product friction against the walls of the hopper and discharge chute, which is determined by the adhesive properties of these elements.

Many studies [15–17] are based on the analysis of the influence of the discharge hopper geometry and the BM particle size on flow particularities, including productivity, depending on the shape of the chutes, and help in selecting the most optimal one. Based on the data analyses, the dependence of mass flow on the discharge hopper size and the BM particle size was confirmed.

A significant area in the field of the BM behavior research is occupied by works dedicated to the analysis of stress distribution that occurs in the material and is transmitted to the bottom and sidewalls of the hopper [18–20]. It is known that friction between particles and between particles and the walls of the discharge hopper contributes to uneven stress distribution in the BM. However, these researches allow us to predict stresses that occur in the BM when it is pressed, rather than during free flow from the hopper or during vibrating disturbance.

The BM can have different flowabilities depending on the material properties and equipment design. Material flow problems are well-known in the processing industry and become particularly noticeable when dealing with fine-grained and cohesive powders. Therefore, it is crucial to characterize the flowability of cohesive materials for better process control.

2. Presenting main material

To date, it is necessary to consider the fact that the BM vast variety, as well as the peculiarities of product behavior during technological operations, often conditioned by specific equipment operating modes, complicates the development of universal models that would allow us to predict and thus control of the BM movement. Therefore, there still remains an unresolved need for a detailed analysis of the stresses occurring in the BM during its discharging from the hopper. Relying on the fact that the BM movement pattern is primarily determined by the shape of the hopper cross-section and the discharge chute, as well as the physical and mechanical properties of the product, a mathematical model of the BM stress state in a conical hopper was developed [15]. However, this model does not account for the influence of friction of the fine-grained bulk product on the container walls. Thus, that research was complemented by factors allowing prediction of the material behavior considering its frictional properties. Several assumptions were taken into account when selecting input parameters for modeling:

- the most unfavorable discharge conditions occur in the conical discharge hopper, as its shape promotes material compaction, agglomeration and adhesion to the container walls;
- most problems arising during hopper discharging are related to ensuring continuous and uniform flow of the BM through the hopper;
- when studying the stress state of the BM, the most informative indicator is radial stress because of the fact that there is transition from overall stress fields to radial ones in the hopper;
- significant attention is paid to analysis the forces of interaction between the BM and the hopper walls due to the cohesive properties of the fine-grained BM, which promote layer adhesion to the container walls and periodic disruption, impacting the overall flow of the BM.

When modeling the influence of adhesive forces on the behavior of the fine-grained BM during its discharging from the conical hopper, a previously developed model was used, allowing the analysis of the stress state of the bulk product. However, it does not account for the influence of cohesive forces to container walls. This model allows us to investigate of processes occurring in the BM during discharge from hoppers, as it considers the linear dependence of tangential stresses on the radial coordinate and discards the assumption of constant radial stresses in the cross-section. Material movement is considered as a continuous process of formation and disruption of strong structures of the free-fall arch over the outlets of the unloading hopper holes. Additionally, this model allows us to evaluate the pressure on the bottom and walls of the loading hopper due to changes in the physical and mechanical characteristics of the BM.

For investigating the BM stress state in the conical hopper, a mathematical model based on the equilibrium of an elementary volume $V_{e.o}$ with an infinitesimal thickness dh was developed Figure 1 [15]. When analyzing of the influence of force factors determining the behavior of the fine-grained BM, the equilibrium of the material's elementary volume was considered. To analyze the adhesive properties of the product and their influence on the stress distribution in the BM, the mathematical model [15] was supplemented. The equation of the force balance influencing the material movement, projected on the axis Ox, took the following form:

$$F_{\sigma_{h0}} + F_{\sigma_\nu} + F_{\tau^w_{lh0}} + F_{\sigma_{l_0}} = F_{\sigma'_{h0}} + F_G + F_s \tag{1}$$

with $F_{\sigma_{h0}}$ is the force gain caused by the vertical stress acting on the upper part of the selected elementary volume; F_G is the weight gain of the material; F_{σ_ν} is the inertial force gain, that occurs in the BM under the vibration action; $F_{\sigma'_{h0}}$ is the force gain caused by the vertical stress acting on the lower part of the selected elementary volume; $F_{\tau^w_{lh0}}$ is the force gain caused by the tangential stress near the hopper hole walls; $F_{\sigma_{l_0}}$ is the force gain caused by the radial stress; F_s is the force gain caused by friction of the material against the hopper walls.

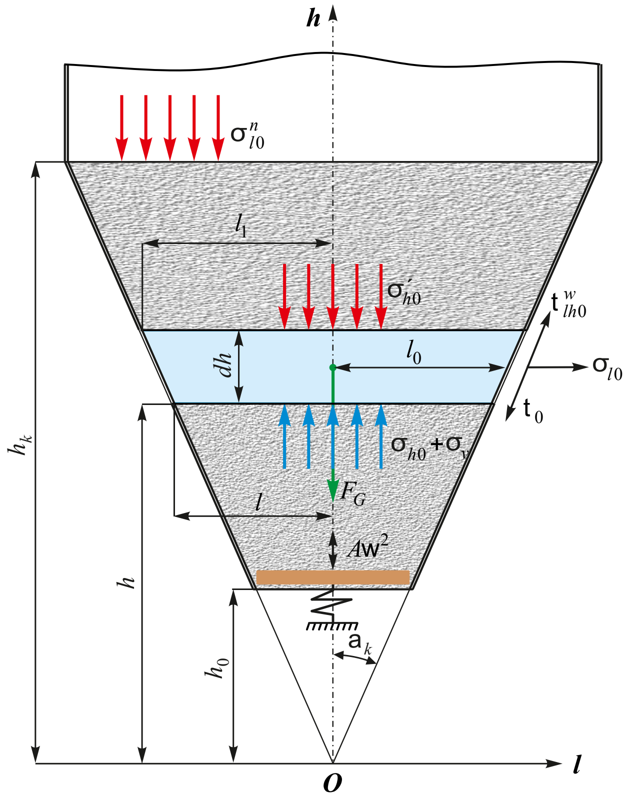


Fig. 1. Scheme for estimating the BM stresses in the unloading conical hopper hole.

Since the frictional force direction is opposite to the vibrating disturbance current direction the projection of this force is taken with a positive sign in equation (1).

Formulas for determining the forces $F_{\sigma_{h0}}$, F_G, F_{σ_ν}, $F_{\sigma'_{h0}}$, $F_{\tau^w_{lh0}}$, $F_{\sigma_{l_0}}$ were derived from the modeling [15]

$$F_{\sigma_{h0}} = \pi\, l_0^2\, \sigma_{l0}\, C_{1,2}, \quad F_G = \pi\, \rho\, g\, l_0{}^2\, dh, \quad F_{\sigma_\nu} = \pi\, A\, \omega^2\, \rho\, l_0^2\, dh, \tag{2}$$

$$F_{\sigma'_{h0}} = \pi\, l_0^2\, C_{1,2}\, (\sigma_{l0} + d\sigma_{l0}), \quad F_{\tau^w_{lh0}} = \pi\, \sigma_{l0}\, f_w\, \frac{2\, l_0\, dh}{\cos\alpha_k}, \quad F_{\sigma_{l0}} = \pi\, \sigma_{l0} \frac{2\, l_0\, dh \sin\alpha_k}{\cos\alpha_k} \tag{3}$$

with l_0 is the radius of the hopper hole on the vertical section middle line of the elementary volume, σ_{l0} is the radial stress in the selected section,

$$C_{1,2} = 1 + 2f^2 \pm \frac{\sqrt{[(1+2f^2)^2 - 1]^3 - \sqrt{[(1+2f^2)^2 - 1 - 4f_w^2(1+f^2)]^3}}}{6f_w^2(1+f^2)}$$

is the active and passive stress states coefficients, respectively, ρ is the BM bulk density, g is the gravitational acceleration, A is the vibration amplitude, ω is the angular frequency, f_w is the external friction coefficient of the BM to the hopper walls, α_k is the angle inclination of unloading hopper walls to the vertical.

Adhesive properties of the material sometimes have a decisive influence on the choice of method and conditions for transporting and dosing the BM. These properties can not be ignored in the designing and development of the equipment, as they contribute to product clumping and its adherence to contacting equipment elements. BM particles form a stationary layer on the container walls, which increases during operation and negatively affects the uniformity of the BM flow and the operation of the equipment.

The adhesive interaction of the BM is determined by properties of the contacting bodies, as well as the environmental conditions. Therefore, the frictional force of BM particles against the walls of the discharge hopper can be calculated by the formula

$$F_s = \tau_0 \, S$$

with τ_0 is the initial coefficient of friction; $S = \frac{2\pi l_0 \, dh}{\cos\alpha_k}$ is the cross-sectional area of the selected elementary volume of the free-fall arch.

Using the equation of the force balance (1) acting on the bulk material within the selected volume, formulas (2), (3), the formula for determining the interaction force of the product with the hopper walls (2) and the developed methodology [2], we obtained the expression:

$$-\frac{d\sigma_{l0}}{dh} + \frac{2(f_w + \sin\alpha_k)}{C_{1,2}\tan\alpha_k} \cdot \frac{l\,\sigma_{l0}}{l_0} = -\frac{A\,\omega^2\,\rho}{C_{1,2}} + \frac{\rho\, g}{C_{1,2}} + \frac{2\,\tau_0\,\cos\alpha_k}{C_{1,2}\,\sin\alpha_k}. \tag{4}$$

By solving the differential equation (4) with boundary conditions $h = h_k$, $\sigma_{l0} = \sigma_{l0}^n$, we obtained the formula for determining the BM radial stress in the discharge hopper:

$$\sigma_{l0} = \frac{a_3 h}{a_1 - 1}\left[1 - \left[\frac{h}{h_k}\right]^{a_1-1}\right] + \frac{a_2}{a_1}\left[1 - \left[\frac{h}{h_k}\right]^{a_1-1}\right] + \sigma_{l0}^n\left[\frac{h}{h_k}\right]^{a_1} \tag{5}$$

with a_1, a_2, a_3 are the coefficients established during modeling.

Taking into account the solution of the mathematical model (5), the coefficients a_1, a_2, a_3 have the following form:

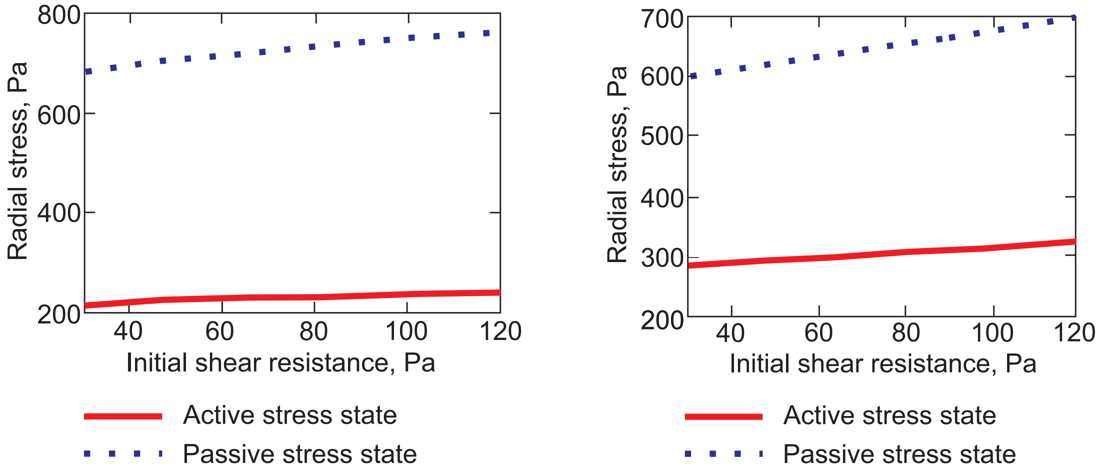
$$a_1 = \frac{2\,(f_w + \sin\alpha_k)\cos\alpha_k}{C_{1,2}\tan\alpha_k}, \quad a_2 = \frac{2\,\tau_0\,\cos\alpha_k}{C_{1,2}\sin\alpha_k}, \quad a_3 = \frac{\rho\left(g - A\,\omega^2\right)}{C_{1,2}}. \tag{6}$$

3. Main results

Using the obtained dependencies (5) and (6), modeling of stress states of the fine-grained BM was conducted with following input parameters: $f = 0.7$, $A = 0.2\,\text{mm}$, $\omega = 125\,\text{rad/s}$, $f_w = 0.55$, $\rho = 0.48 \cdot 10^3\,\text{kg/m}^3$, $\alpha_k = 45^\circ$, $\sigma_{l0}^n = 0$.

The initial shear resistance of the BM depends on several factors, including product properties, a moisture content which also affects the material differently, a particle size and shape and a pressure applied to the material. Another important factor influencing the variation of the BM shear resistance is specific discharge conditions. Temperature, loading rate, atmospheric pressure and other processing conditions can affect the value of the BM shear resistance. Therefore, to generalize this factor, an analysis (Figure 2) was conducted, showing how radial stress, which directly determine the material movement during discharging, depends on the initial shear resistance. For fine-grained bulk materials, this indicator can vary from 30 to 120 Pa [21].

Based on the graphical dependencies (Figure 2), it can be concluded that with an increase in the BM initial shear resistance, the material radial stress which is near the discharge outlet increases, leading to more difficult discharge conditions and increasing potential for the formation of the free-fall arch over the outlet. Using the developed mathematical model (5), the active and passive stress states were analyzed. The active stress state is characterized by movement continuous of the product through

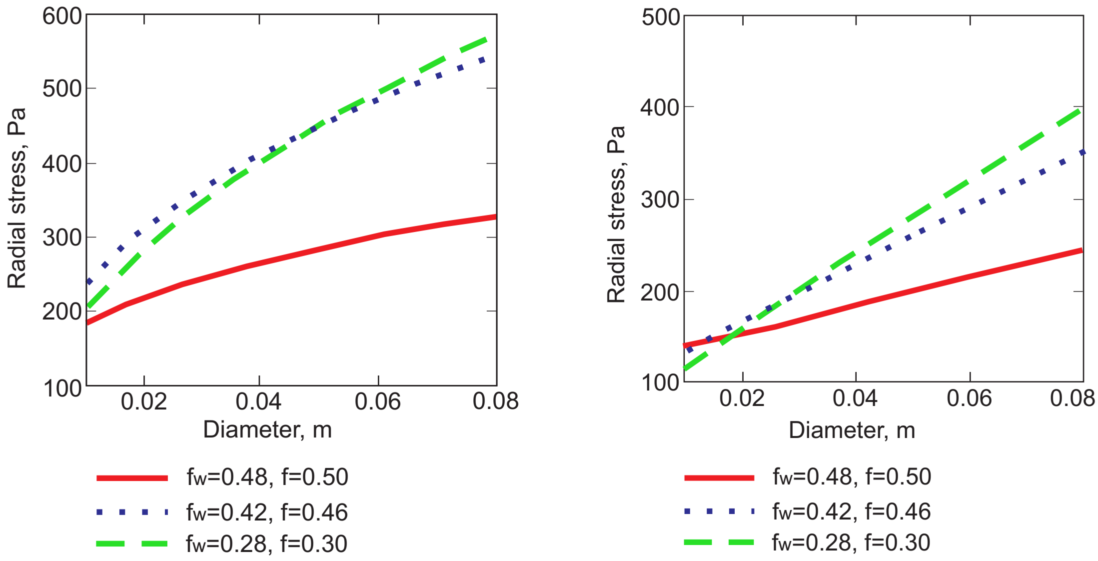


a ***b***

Fig. 2. Dependence of the BM radial stress on the initial shear resistance for (***a***) $\frac{h}{h_k} = 0.6$; (***b***) $\frac{h}{h_k} = 0.4$.

the discharge outlet; however, the flow is carried out up to a certain critical value of the radial stress. Once this critical value is reached, the material transitions to a passive stress state characterized by forming of the free-fall arch and uneven flow. The graphical dependencies indicate that the initial shear resistance predominantly affects the radial stress of the bulk material in the passive stress state, to a lesser extent in the active state. Also, the radial stress depends on the coordinate h. Therefore, analyzing the data from the graphs, it can be seen that the radial stress is slightly higher with a ratio of $\frac{h}{h_k} = 0.6$; for both active and passive stress states (Figure 2a) than in the case of $\frac{h}{h_k} = 0.4$ (Figure 2b).

Additionally, based on the developed mathematical model, an analysis of the influence of other material frictional characteristics on the discharge process was conducted (Figure 3). The effect of the internal friction coefficient of the BM f and the external friction coefficient of the BM to the hopper hole walls f_w on the material stress state was investigated. Active (Figure 3a) and passive (Figure 3b) stress states of the BM were separately considered.



a ***b***

Fig. 3. Dependence of the BM radial stress on the outlet diameter for (***a***) active stress state; (***b***) passive stress state.

Analyzing the modeling graphs, it can be observed that the nature of the curves of radial stresses varies significantly with different frictional characteristics of the product. When analyzing the active stress state (Figure 3a) for curves 2 (blue colored) and 3 (green colored), it is noticeable that with smaller outlet diameters $0.01 \leqslant D \leqslant 0.05$ m, the radial stress increases with higher friction coefficient values, and, conversely, it decreases with $D \geqslant 0.05$. When investigating the passive stress state, the tendency persists, but the critical value of the outlet diameter at which the influence of coefficients f and f_w on the radial stress changes is $D \approx 0.025$ m. It particularity can be explained by the fact that increasing the size of the outlet diameter while keeping other parameters of the discharge hopper geometry unchanged increases the volume of the material inside the hopper and significantly increases the radial stress, making the influence of the frictional characteristics on the stress less significant. Also the inverse relation between radial stress and friction coefficients is observed when studying BM with high frictional characteristics. In Figure 3, this particularity is presented in the example for curve 1 (red colored). The layer of material adheres to the contacting walls of the container for material with high frictional characteristics. Thus, an increase in friction coefficients does not lead to an increase in radial stress in the product.

4. Conclusions

Typically, dosing equipment is designed based on pre-defined physical and mechanical properties of the MB. Therefore, there is not much freedom in choosing factors that affect the magnitude of the radial stress for the engineer. Since the radial stress is a function of many variables, namely $\sigma_{l0} = f(f_w, f, h, h_k, \rho, \alpha_k, \tau_0)$, and the values of ρ, τ_0, f_w, f characterize the product state, prevention of the existence of a passive state can be achieved by the correct selection of the geometry of the discharge hopper and by using the analysis of mathematical models describing the product movement during the discharge process. The developed mathematical model shows that frictional characteristics affect the radial stress in the hopper and ensure the continuous discharge of the dosing product.

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Вплив фрикційних характеристик сипкого матеріалу на витікання із конічної лунки

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Розроблено математичну модель напруженого стану сипкого матеріалу при його витіканні під дією вібрації з урахуванням адгезійної взаємодії продукту із стінками ємності. Для аналізу використано модель дискретного середовища, що ґрунтується на розгляді рівноваги елементарного об'єму склепіння нескінченно малої товщини. Встановлена аналітична залежність, яка дозволяє оцінити вплив фрикційних факторів на напружений стан сипкого матеріалу, а також змінювати його шляхом регулювання певних параметрів, таких як процес дозування та геометрія розвантажувальної лунки.

Ключові слова: *дрібнодисперсний сипкий матеріал; напружений стан; розвантажувальна лунка; бункер; параметри вібрації; фрикційні характеристики; умови витікання.*