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# Integration of the cost-risk phase trajectories method and the method of finding the optimal value of profitability in the system for diagnosing the financial results of an enterprise

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The article presents theoretical and applied approaches to the integration of economic and mathematical methods of cost-risk phase trajectories and finding the optimal profitability of an enterprise. The initial information of the study is that risk can be one of the main identifiers of the state of fluctuations in the financial results of an enterprise, constantly changing in conditions of uncertainty in doing business, and therefore its value can be determined by various signs of inconsistency, which affects the graphical representation of phase trajectories – isoclines on the plane. The method of finding the optimal value of profitability from introduced credit operations at various interest rates and risks in the financial results diagnostic system allows the area of the zone of real profits and losses of the corresponding enterprise to be calculated as accurately as possible in quantitative terms. Together, these methods improve the results of financial diagnostics of the stability and security of the enterprise.

**Keywords:** *economic and mathematical methods; phase trajectories; costs; risk; optimal profitability; profit and loss zone; financial results of the enterprise.*

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## 1. Introduction

Entrepreneurial activity at the present stage requires not only increased responsibility and independence in making management decisions for a number of enterprises and other market entities that are strategically important for the state, but also the necessary applications of mathematical methods and models to monitor the achievement of the level of their efficiency and the corresponding level of economic security [1], whose safety in a full-scale war depends primarily on a quantitatively accurate diagnostic assessment of their financial results. This will have a positive impact on the future actions of investors interested in generating income and acceptable risk of investing their funds in an enterprise and in win-win relationships with creditors and suppliers who want to ensure the necessary solvency of the business entity.

## 2. Theoretical aspects

The choice of effective management solutions at an enterprise can be lengthy and therefore impossible without a comprehensive dynamic analysis of a complex of interrelated factors, identification and comparative assessment of possible alternatives and acceptable action plans for a business entity using economic and mathematical modeling. In this regard, in practice, to assess the procedures for making or not making certain management decisions, an important role is played by the diagnosis of costs and risks, which, together with a quantitative assessment of the optimal value of profitability, will make it possible to carry out the highest quality diagnosis of the state of economic security of the enterprise based on the assessed financial results. It can be periodically subjected to the recommended highly differentiated control using a variety of economic and mathematical methods, which will allow the final financial and economic result of the enterprise to be assessed with absolute accuracy in time regarding compliance with the requirements of economic stability, independence, solvency, profitability, etc. [2].

 

In this regard, there is a need to search, study and integrate various economic and mathematical methods for business entities that directly affect the accurate assessment of its financial stability and allow us to assess such key economic parameters such as: costs, risk, loan size, expected income, probability of successful implementation of an investment project, which can be used as the basis for approaches to developing financial diagnostic systems. An innovative approach in this direction should first of all be based on such a process of forming an economic and mathematical assessment of the financial results of an enterprise, which would cover the assessment of the sphere before its financial and credit activities and would be flexible enough to any changes in production and would give a visual and graphic assessment of the completeness of its activities [3].

Such scientific research was carried out by the following scientists: Zaichkovskyi A. O., Ivanyuta T. M., Kavun S. V., Nizyaeva S. A., Ostankova L. A., Filippova S. V., Shvydanenko H. O., Yaremenko O. F. and many others. The scientific achievements of each of these authors deserve attention in the aspect of a broad approach to assessing the financial results of business entities and their economic security, but at the present stage of development of entrepreneurial activity it requires the applied and effective use of other economic and mathematical methods. In particular, scientists Nizyaeva S. A. and Filippova S. V. [4], used a nontransitive matrix of pairwise comparisons of the enterprise net asset method, real options and methods using a multiplier, however, this approach requires graphical clarification and visualization. Therefore, in particular, the applied use of the "cost–risk" phase trajectories method with the construction of isoclines and with a combination of the method of determining the optimal value of the enterprise's profitability from the introduction of credit operations makes it possible to improve the graphical representation and separation of the real zone of income and losses of the business entities under study.

## 3. Research purpose and methodology

The purpose of the article is to outline the applied aspects of the combination of the "cost–risk" phase trajectories method and the method of determining the optimal amount of profitability from the introduction of credit operations at an enterprise in the system for diagnosing its financial results, allowing one to achieve a comprehensive and accurate quantitative assessment of the performance indicator that falls under the control process during the course of entrepreneurial activity and graphically represent the area of income and losses of an economic entity in real-time, affecting the dynamic assessment of the enterprise's economic security.

### 3.1. Applied methodology for using the phase trajectories method

Experience shows that for a visual analysis of the dynamics of the state of financial and economic activity and its impact on the diagnosed value of the state of economic security of an enterprise, it is better to use a graphical method — the method of phase trajectories. This method is based on depicting changes as the trajectory of a point that is a selected financial and economic parameter that reflects it in phase space. If a second-order system is chosen, that is, a dependence between two parameters, for example, the size of the enterprise's costs on the time interval or the dependence of the costs and risk of the corresponding business entity, then this movement will be presented on the phase plane. As the coordinate axis, it is recommended to select the adjustable cost–risk parameter $y$ and the speed or magnitude of their changes ($\Delta y$) at the enterprise [5]. The nature of parameter changes allows conclusions to be drawn about the enterprise's overall activity. During various economic processes occurring at the enterprise, directly or indirectly affecting the state of its economic security, the point with the indicated coordinates $(y, \Delta y)$, moving along the phase plane, will continuously determine the phase trajectory, which will improve the control result. The set of phase trajectories reflecting the dynamic properties of the system is called the phase portrait of the system. To construct them, the isocline method is most often used [6]. The more times the trajectory crosses the $x$-axis in the zone II and III quadrant of the coordinate system, the less stable the state of the enterprise is considered from the point of view of cost control, if actual costs exceed the planned ones.

Under the continuous linear part, the enterprise's planned expenses for performing work and their change in comparison with the services actually provided or the volume of products sold can be displayed. The initial value of $V$ will be the risk of non-reimbursement of costs.

Transfer function ($W(p)$) of the linear part of the system according to [6, p. 20], can be expressed as:

$$W(p) = \frac{k}{p\,(T+1)}, \tag{1}$$

where $k$ is a constant value equal to 1 if a signal is received about the study of the economic system of the enterprise, and otherwise – 0; $T$ is the specific weight of the enterprise costs; $p$ is the number of partners a certain enterprise has.

The initial information is that financial risk, which can be one of the identifiers of the state of the financial results of an enterprise, is constantly changing, and its value will be determined by the inconsistency sign $+1$ or $-1$. Then the equation of the phase trajectory system is expressed as:

$$T\,y'' + y' - k\,\mathrm{sign}(x-y) = 0, \tag{2}$$

where $x$ and $y$ are the coordinate values of regulated economic parameters that affect the activities of the enterprise.

We will consider an approximate application of this method of phase trajectories using the example of two variable parameters, such as cost–risk in business activity, below. Graphical representation of this method improves the visualization of the impact of costs and risk on the main final financial results of the enterprise.

### 3.2. Systematic methodology for assessing financial results at an enterprise

As you know, the assessment of financial results, in particular of profit or loss, understanding that the financial result is the difference between income and expenses, is an important point of an enterprise.

In general, the limits of financial and economic analysis in the process of diagnosing the state of economic security of an enterprise in modern conditions are still not clearly defined, which makes it possible to constantly modify and transform it, expanding the range and detailing the assessment of the activities of the production and economic structure, achieving positive results in combination with a wide a range of economic and mathematical methods and models.

The essence and content of economic and mathematical modeling acquire special importance in the system of planning financial indicators and in the management of a complex production entity and lies in establishing the relationship between financial indicators and factors influencing the value of these indicators. Mostly, the economic-mathematical model allows one to characterize the structure and patterns of dynamics of this economic phenomenon using mathematical approaches and evaluate any financial result. It is also possible to computerize a number of models, using either the functional or correlation dependence of one indicator on others, with the improvement of future financial results, providing comprehensive control over the financial activities of the enterprise. The simpler the models are, the better and faster they adapt to the real economic situation at the enterprise. A variety of economic and mathematical models are also financial models, the number and efficiency of which is constantly growing, in particular, we will present a model of capital accumulation by an enterprise in the process of commercial activity, which has a great influence on the quality of ensuring the economic stability and security of the enterprise [1]:

$$N^k_{t+1} = \frac{P^1_{t+1}}{P_t}\,(1-\Pi_{e+1})\,(q_t N^k_t + k_t) - (1+r)\,k_t, \tag{3}$$

where $N^k_{t+1}$ is the amount of money accumulated by commercial structures due to credit and transactions of purchase and sale of industrial goods, in hryvnias; $k_t$ is the amount of credit to commercial structures, in hryvnias; $P_t$ is the purchase price of the product, in hryvnias; $P^1_{t+1}$ is the selling price of the product, in hryvnias; $\Pi_{e+1}$ is the sales rate of goods, in relative units; $q_t$ is the rate of accumulation (percentage of money that is directed into turnover), in relative units; $r$ is the interest rate for the loan, in relative units; $t$ is the term of accumulation of cash by commercial structures, years $(0, 1, 2, \ldots)$.

Since capital accumulation is a cyclical process of summing up such a financial result as net income from the sale of finished products or proceeds from purchase and sale transactions of industrial products after deducting taxes and interest on loans, equation (3) is solved by the method of mathematical induction.

Without the use of mathematical models, it would be impossible to accurately quantify the final financial result of the activity of an economic entity, and even more so in the future its profit or loss. Evaluating the activities of enterprises with the participation of quantitative economic and mathematical methods and models and their combination is especially important in the process of cost management when manufacturing innovative products, which increases the overall efficiency of the management process of business structures and stabilizes their economic security in conditions of uncertainty.

Issues related to cost management at an enterprise are quite diverse, and for their rational management, the construction of a decision-making criterion regarding the rationalization of all types of costs at the same time is of particular importance. Costs reflect the cost of resources that the enterprise uses in its activities to produce products and perform various works. As you know, the main goal of a commercial enterprise is to maximize profits through the rational distribution of resources used. Formally, the functional dependence of the profit maximization problem ($\Pi_p$) in a certain time interval has the form:

$$f\left(\Pi_p\right) \rightarrow \max . \tag{4}$$

This formulation of the maximization problem depends on what time period: long-term or short-term precedes the period in which the enterprise maximizes its financial result such as profit. Under long-term conditions, the enterprise can freely choose any cost vector $x=(x_1, x_2)$ from the cost space, provided that $x_1 \geqslant 0$ and $x_2 \geqslant 0$, then the profit maximization function will look like this:

$$p_0 f\left(x_1, x_2\right)-\left(p_1 x_1+p_2 x_2\right)=f\left(\Pi_p\left(x_1, x_2\right)\right) \rightarrow \max, \tag{5}$$

where $p_0$, $p_1$, $p_2$ are market prices for resources.

In the case of a short-term period of activity of the enterprise, restrictions on the volume of resources used shall be taken into account:

$$g\left(x_1, x_2\right) \leqslant r, \tag{6}$$

where $g$ is the functional dependence on the amount of resources used; $r$ is the upper limit of the total use of resources of type $x_1$ and $x_2$.

In practice, the head of the enterprise can make many different decisions on cost management. The specifics of solutions are determined by the specifics of a particular enterprise, its technology, innovative potential, organizational structure, and place on the market.

The criteria for making such decisions are primarily influenced by such factors as supply and demand. But the usual exchange of goods can also be observed. According to [2], the condition of normal exchange is as follows:

$$\sum_n P_n\left(x_{ni}^{\prime}-x_{ni}\right)=0, \quad i=1, \ldots, k, \tag{7}$$

where $P_n$ is the price of manufactured the $n$-th industrial product that entered the market, in hryvnias; $x_{ni}$ is the initial volume of goods, pcs.; $x'_{ni}$ is the volume of the $n$-th product of the $i$-th subject, pcss.; $i$ is the subject; $n$ is the type of industrial product.

Then the cost of goods sold is equal to the cost of purchased goods. That is, ordinary exchange does not give profit. Otherwise, in the case of commercial trade, equation (7) will be as follows:

$$\sum_n P_n\left(x_{ni}^{\prime}-x_{ni}\right)=\Pi_p, \tag{8}$$

where $\Pi_p$ is the amount of profit received in the process of selling goods, in hryvnias.

However, in real conditions, consumers of finished industrial products, when buying a product, are guided by their own utility function, and sellers – by their utility function. Then, using the utility function $K_i$ and the Lagrange method maximization function as follows:

$$\max \sum_i \left( K_i - \lambda_i \sum_n P_n \left( x'_{ni} - x_{ni} \right) \right), \tag{9}$$

the Lagrange multipliers $\lambda_i$, can be calculated as follows:

$$\lambda_i = \frac{1}{P_n} \frac{\partial K_i}{\partial K_{ni}}. \tag{10}$$

They allow to establish a balance between resource prices and profits, maximizing profits and minimizing enterprise costs.

Taking into account the above studies, we can conclude that the most important criterion for fluctuations in costs at an enterprise is making a decision on the volume and structure of product output, which is influenced by production and sales, and then factors influencing the volume and structure of output, insufficient demand for certain types of products, availability of old or destroyed equipment in war conditions, the capacity of which is lower than at other analogous enterprises, scarce materials, semi-finished products, low qualifications of specialists or a shortage of qualified specialists of a certain profile due to forced relocation.

When forming decision-making criteria for cost management at such an enterprise, it is important to maximize the existing marginal profit $M$ that is the amount for all types of products and services:

$$M = \sum_{j=1}^{n} \left( p_j - b_{pj} \right) x_j \to \max, \tag{11}$$

where $p_j$ is the price of industrial production of the $j$-th type, in hryvnias; $b_{pj}$ are planned variable costs of industrial production of type $j$, in hryvnias; $x_j$ is the planned volume of sales of industrial products of type $j$, pcs.

When choosing between creating your own products and purchasing them from outside, the presence of risk situations should be taken into account. If a risk arises as a result of in-house production, then lost profits shall be added to the costs, that is, the possible amount of marginal profit, but that is already lost.

When deciding to set prices for products to be sold, it should be taken into account that in market conditions the price of products depends on supply and demand. Sometimes, due to competitive pressure, an enterprise cannot dictate its terms to the buyer. Therefore, it shall know the minimum price at which it is advisable to produce and sell the relevant products.

Research shows that the lower price limit depends on the following factors: the availability of a traditional range of goods or goods under an additional contract. If the manufacturer of a certain product is a monopolist in the market, then the demand for this product does not grow, but is constant. Then demand, as a function of costly innovation activity, can be expressed by the expression [7]:

$$C = k\, I^{\varepsilon}, \tag{12}$$

where $C$ is the demand for products; $I$ means costs of implementing innovative activities; $\varepsilon$ is the elasticity of demand for innovative activities; $k$ is some constant.

In this case, the profit (or loss) of the enterprise will be represented by the equation:

$$\Pi_p = \left( P_0 - B_{zm.} \right) C - B_{tot.} - B_{post.}, \tag{13}$$

where $\Pi_p$ is the profit (loss) of the enterprise, in hryvnias; $P_0$ is the set price for the product, in hryvnias; $B_{zm.}$, $B_{post.}$ are, respectively, variable and fixed costs, in hryvnias; $B_{tot.}$ is the total amount of costs, in hryvnias.

The maximum profit (or minimum loss) of an enterprise can be obtained by taking the first derivative of equation (13) and equating the left side of the equation to zero, i.e.:

$$\frac{d\Pi_p}{dB_{tot.}} = \left( P_0 - B_{zm.} \right) \frac{dC}{dB_{tot.}} - 1 = 0. \tag{14}$$

Multiplying both parts of the equation by $\frac{B_{tot.}}{P_0 C}$, we obtain the original expression for the condition of optimizing the amount of costs for innovative activities in relation to determining the optimal budget of the enterprise:

$$\frac{P_0 - B_{zmin.}}{P_0} \frac{dC}{dB_{tot.}} \frac{B_{tot.}}{C} - \frac{B_{tot.}}{P_0 C} = 0. \tag{15}$$

After transformations of equation (15), it is possible to determine:

1) percentage of costs for innovative activity

$$\frac{B_{tot.}}{P_0 C} = \varepsilon \frac{P_0 - B_{zmin.}}{P_0}; \tag{16}$$

2) optimal amount of costs for innovative activity ($B_{opt}$):

$$B_{opt} = \left[\varepsilon \left(P_0 - B'\right) a\right]^{\frac{1}{1-\varepsilon}}, \tag{17}$$

where $a = \frac{C}{I^{\varepsilon}}$, $0 < \varepsilon < 1$.

Further, based on the Vidale–Wolfe model [8], it is possible to establish a connection between sales volume or revenue and the costs of an enterprise's innovative activities:

$$\frac{dr}{dt} = g\, I\, (N - V)/N - (1 - \gamma)\, V, \tag{18}$$

where $\frac{dr}{dt}$ is the increase in sales of goods; $g$ is the marginal profit from the implementation of innovative activity (if $V = 0$); $I$ means costs of implementing innovative activities; $N$ is the level of sales saturation; $V$ is the volume of sales of a certain type of product; $\gamma$ is the level of sales inhibition to optimize the pricing mechanism in the market.

However, it is important to combine the method of phase trajectories with the selection of such parameters as: costs and risk, which have the greatest impact on the financial results of any business entity.

## 4. Results and discussions

### 4.1. Application of the 'cost–risk' phase trajectories method in entrepreneurial activity

Knowing the essence of the phase trajectories method [5,6], we use the following points. Taking $x = 0$ and taking into account that $\operatorname{sign}(-y) = -\operatorname{sign}(y)$, we obtain:

$$T\, y'' + y' + k \operatorname{sign} y = 0. \tag{19}$$

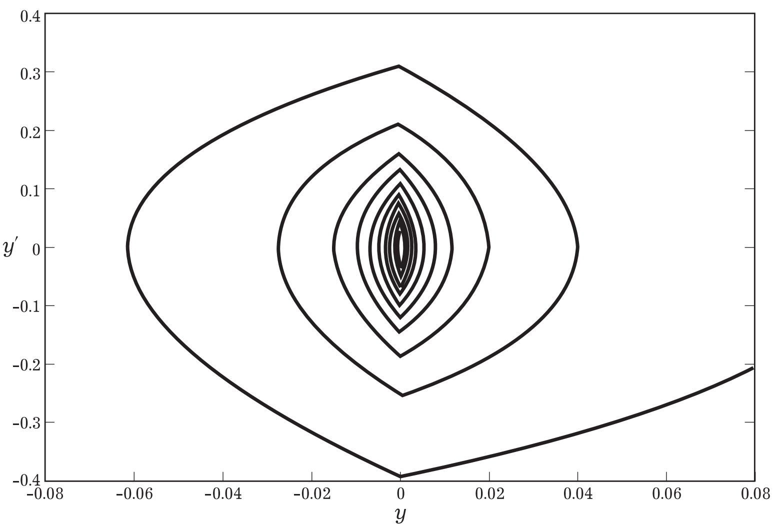


**Fig. 1.** The phase trajectory, which is defined by Eq. (19) ($T = 1$, $k = 1$).

Figure 1 shows the phase trajectory, which is the solution of Eq. (19) ($T = 1$, $k = 1$) under the following initial conditions: $y(0) = 0.1$, $y'(0) = 0.1$. The movement occurs in a clockwise spiral to the origin of the origin of coordinates. It can be seen from Eq. (19) that $y = 0$ there is a change of sign in the third term. This causes non-smoothness of the phase trajectory at $y = 0$ (i.e. the discontinuity of the first derivative), as can be seen from this figure.

In dimensionless coordinates $T = 1$, $k = 1$ and denote $\operatorname{sign} y = -\delta$, therefore, $\delta = 1$ for $\Delta y < 0$ and $\delta = -1$ for $\Delta y > 0$.

Then

$$V' + V = \delta,$$

where

$$y' = V \quad \text{or} \quad \frac{dV}{dy} = \frac{\delta - V}{V}. \tag{20}$$

The equation of the phase trajectories is obtained by integrating this equation under the condition $t = 0$ and marking $V_0 = \Delta y_0$:

$$y = y_0 + \Delta y_0 - V + \delta \ln \Delta y_0 - \delta, \tag{21}$$

where $y$ is the actual cost of the enterprise; $y_0$ is the planned value of the enterprise's costs; $\Delta y_0$ is the change of costs; $V$ is the risk; $\delta$ is the identifier value $+1$ or $-1$.

If the identifier value of $\delta = 1$ is set, the phase trajectory equation will look like:

$$y = y_0 + \Delta y_0 - V + \ln(1 - \Delta y_0). \tag{22}$$

For $\delta = -1$, the phase trajectory equation will be as follows:

$$y = y_0 + \Delta y_0 - V + \ln(1 + \Delta y_0). \tag{23}$$

The transition of the trajectory from $\delta = 1$ to $\delta = -1$ on the phase plane occurs on the $y$ axis. In this case, a transition zone is formed, reflecting the change in the values of the actual costs of the enterprise in comparison with the planned ones, respectively, located sequentially according to a growing trend from the origin of coordinates. The signs on the $V$ axis also change in accordance with the change in $\delta$ values from $+1$ to $-1$ and vice versa.

Let us consider the applied application of the phase trajectories method based on the data given in Table 1 on the dynamics of costs due to the generalized financial and production activities of the simulated enterprise.

**Table 1.** Simulation of the dynamics of enterprise costs.

| Planned cost values of $y_0$, in hryvnias | Actual cost values of $y$, in hryvnias | Setting the identifier of $\delta$ | Calculation formula |
|---|---|---|---|
| 84000 | 82000 | $\delta = 1$ | $y = y_0 + \Delta y_0 - V + \ln(1 - \Delta y_0)$ |
| 12000 | 12800 | $\delta = -1$ | $y = y_0 + \Delta y_0 - V + \ln(1 + \Delta y_0)$ |
| 6800 | 6700 | $\delta = 1$ | $y = y_0 + \Delta y_0 - V + \ln(1 - \Delta y_0)$ |
| 20600 | 19600 | $\delta = 1$ | $y = y_0 + \Delta y_0 - V + \ln(1 - \Delta y_0)$ |
| 34000 | 37500 | $\delta = -1$ | $y = y_0 + \Delta y_0 - V + \ln(1 + \Delta y_0)$ |
| 63100 | 64900 | $\delta = -1$ | $y = y_0 + \Delta y_0 - V + \ln(1 + \Delta y_0)$ |
| 51200 | 48300 | $\delta = 1$ | $y = y_0 + \Delta y_0 - V + \ln(1 - \Delta y_0)$ |

$$\Delta y_0^1 = y^1 - y_0^1 = 82000 - 84000 = -2000 \quad \Rightarrow \quad \text{costs decreased} \quad (\delta = 1);$$
$$\Delta y_0^2 = y^2 - y_0^2 = 12800 - 12000 = 800 \quad \Rightarrow \quad \text{costs increased} \quad (\delta = -1);$$
$$\Delta y_0^3 = y^3 - y_0^3 = 6700 - 6800 = -100 \quad \Rightarrow \quad \text{costs decreased} \quad (\delta = 1);$$
$$\Delta y_0^4 = y^4 - y_0^4 = 19600 - 20500 = -900 \quad \Rightarrow \quad \text{costs decreased} \quad (\delta = 1);$$
$$\Delta y_0^5 = y^5 - y_0^5 = 37500 - 34000 = 3500 \quad \Rightarrow \quad \text{costs increased} \quad (\delta = -1);$$
$$\Delta y_0^6 = y^6 - y_0^6 = 64900 - 63100 = 1800 \quad \Rightarrow \quad \text{costs increased} \quad (\delta = -1);$$
$$\Delta y_0^7 = y^7 - y_0^7 = 48300 - 51200 = -2900 \quad \Rightarrow \quad \text{costs decreased} \quad (\delta = 1).$$

Then

$$82000 = 84000 + (-2000) - V_1 + \ln(1 + 2000) \to V_1 = 7.6\%;$$
$$12800 = 12000 + 800 - V_2 + \ln(1 + 800) \to V_2 = -6.7\%;$$
$$6700 = 6800 + (-100) - V_3 + \ln(1 + 100) \to V_3 = 4.6\%;$$
$$19600 = 20500 + (-900) - V_4 + \ln(1 + 900) \to V_4 = 6.8\%;$$
$$37500 = 34000 + 3500 - V_5 - \ln(1 + 1500) \to V_5 = -7.3\%;$$

$$64900 = 63100 + 1800 - V_6 - \ln(1 + 1800) \to V_6 = -7.5\%;$$
$$48300 = 51200 + (-2900) - V_7 + \ln(1 + 2900) \to V_7 = 8.0\%,$$

where $V_1 - V_7$ are the calculated risks, %.

The results of costs in an enterprise can be considered simultaneously as a result of operating, financial and investment activities or collectively as a result of innovative activities. A graphical interpretation of the phase trajectories of the "cost–risk" system at a simulated enterprise is shown in Figure 2.

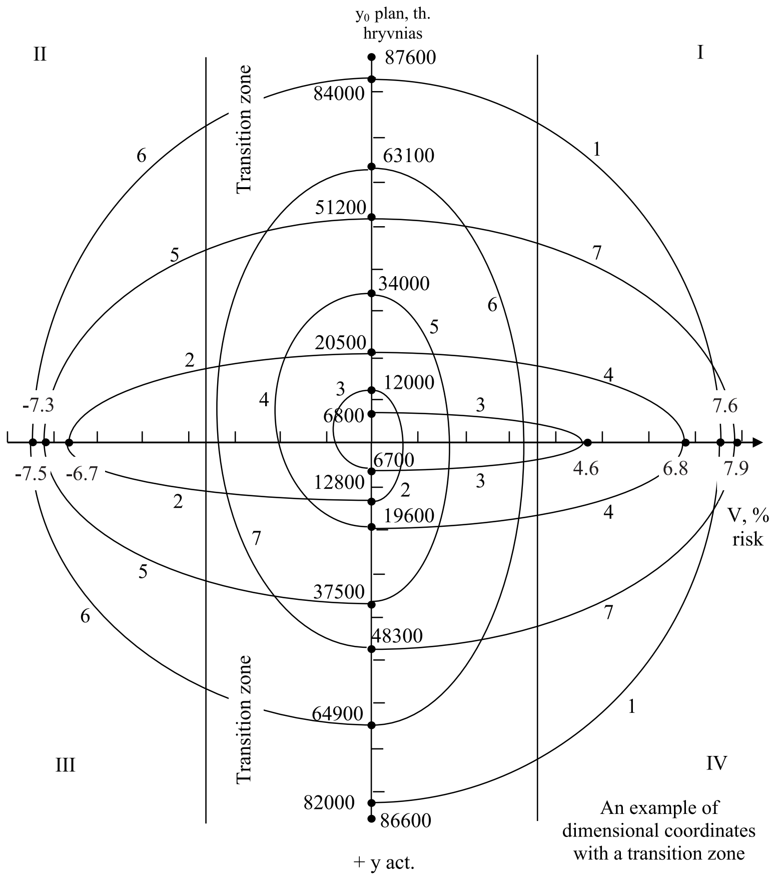


**Fig. 2.** Graphical interpretation of the "cost–risk" method of functional dependencies at a simulated enterprise.

The number of intersections of zones I, IV characterizes the positive dynamics of the reduction of actual costs relative to planned values within the limits of permissible risks. The number of intersections of zones II, II characterizes the negative dynamics of the growth of the actual expenses of the enterprise relative to the planned values, fixing the negative amount of risks. Since the sum of intersections of zones I and IV is equal to 8, and the sum of intersections of zones II and III is 6, i.e. $8 > 6$, the positive trend in the simulated enterprise prevails, i.e. the business entity controls all actual costs, has reserve funds and can lend them to similar enterprises of a certain spheres of industry, already acting as a creditor.

However, this may not always correspond with the dynamics of the expected increase in the level of economic security of a real enterprise, but only if its financial results, such as income of a certain reporting period, exceed actual expenses and in the subsequent time period this trend not only continues, but also has a corresponding forecasted growth rate income according to linear, logarithmic, exponential and polynomial trend dependencies when diagnosing the absolute or at least normal financial stability of a business entity. However, the presence of absolute financial stability of the enterprise at the reporting date also does not always guarantee a sufficiently high level of economic security of the enterprise, in contrast to the opposite statement [9]. By setting the rate of change in the amount of costs at the enterprise, it is possible to achieve the predicted risk value, which will be important for maintaining an appropriate state of economic security.

Therefore, the cost-risk phase trajectories method should be supplemented with a method for finding the optimal value of profitability from the introduction, for example, of credit operations in the system for diagnosing the financial results of an enterprise.

### 4.2. Finding the optimal value of profitability from credit operations of an enterprise in a system for diagnosing financial results

In the system for diagnosing the financial results of a production and economic structure, the following information data is important: existing information that the specified enterprise is a creditor, with reserve available funds as of the current date, which it puts into circulation to make a profit. In such a situation, the level of its economic security should be within acceptable values and its value may further increase to a certain value, which will depend on the borrower enterprises it has with a solvent financial condition for a certain fixed period of time. If the company's receivables are not hopeless, then the level of economic security will be stable and will not fall below the minimum acceptable value.

To further maintain a stable reputation on the market in relations with partners and an acceptable state of stability for the simulated enterprise, which can be, in turn, a creditor, it is recommended to use an additive-multiplicative economic-mathematical model of the dependence between the expected income from introduced credit operations at different interest rates and the risk of implementation of the same class of projects between enterprises from the same industry.

Let us assume that the corresponding creditor – enterprise provides loans to such borrowers as manufacturing companies in a certain region of the country belonging to the same industry. Let $j$ be the serial number of the enterprise, characterized by probability $q_j$, i.e. that is, the successful implementation of the project with an expected income equal to $G_j$. In the opposite case, with a probability equal to $(1 - q_j)$, the return from the project will be low (possibly zero), but the same for all enterprises in the specified industry and is taken as $\bar{G}$. Let us assume that these enterprises form a homogeneous class in terms of risks, which, according to [10, 11], can be written as:

$$q_j G_j + (1 - q_j)\bar{G} = G, \tag{24}$$

where $G$ is the average expected income from the implementation of projects for a number of enterprises; $\bar{G}$ is the average income of the $j$-th enterprise for a certain period; $q_j$ is the planned rate of return of the enterprise; $G_j$ is the gross income of the $j$-th enterprise.

Then let the necessary resources for the implementation of the projects be in the amount of $I$. And the personal financial capabilities of the enterprises are estimated by the same value $K$. In this case, the financial needs for the implementation of the projects of all enterprises will be:

$$I - K = F. \tag{25}$$

If potential consumers of products (services) or customers are enterprises with large authorized capital, then they can act directly as investors or, if they do not have available funds, as guarantors for a powerful creditor enterprise lending to other enterprises at an interest rate of $r$. Guarantees in the case of loans to such enterprises can be bills of exchange, pledge of shares that are quoted on the stock market. In the case of a loan in the amount of $F$ with credit debt $F(1 + r)$, the following inequality shall be satisfied, according to [11]:

$$G_j \geqslant (1 + r)F > \bar{G}, \tag{26}$$

where $F$ is the loan size; $r$ is the interest rate.

If enterprises with a smaller authorized capital know and can evaluate the success of the project, then the creditor enterprise does not know it. In the absence of guarantees, this type of risk, such as lending risks, increases noticeably, and then elements of discrimination appear in relation to high-risk projects and borrowers.

Let us assume that both parties, enterprises acting as borrowers and lenders, are risk neutral, that is, they do not prefer the amount of credit risk. The expected return of funds from the project to investors will be:

$$M(w_j) = q_j[G_j - (1+r)F], \tag{27}$$

where $M(w_j)$ is the expected return of funds to the enterprise as a result of the implementation of its own project.

The expected payments of borrowers to the creditor enterprise will be as follows:

$$M(w_j) = (1+r)F \int_0^q q_j f(q_j)\, dq_j + \bar{G} \int_0^q (1-q_j)\, f(q_j)\, dq_j, \tag{28}$$

where $q$ is the threshold level for the probability with which the borrower turns to the lender.

If expression (27) is given taking into account (24), according to [11], we get:

$$\begin{aligned} M(w_j) &= q_j G_j - q_j(1+r)F = G - (1-q_j)\bar{G} - q_j(1+r)F \\ &= G - \bar{G} + q_j\bar{G} - q_jF - q_jr\,F = G - \bar{G} - q_j[(1+r)F - \bar{G}]. \end{aligned} \tag{29}$$

So, taking into account (29), we agree with the well-known position that investors are willing to pay more for high-risk projects in order to make the necessary loans. Note that if $g$ is the level of guaranteed profitability, that is, the level of a safe form of deposits, then the return on the project shall satisfy the condition:

$$M(w_j) \geqslant (1+g)K, \tag{30}$$

where $K$ is total capital (which includes collateral deposits of enterprises with a larger authorized capital and enterprises with a smaller capital of the specified industry).

Let us introduce a value that will allow the borrower company to count on such optimal profitability using loan funds, and the creditor company to guarantee the return of receivables without deteriorating its economic security:

$$g_j = \frac{M(w_j)}{F}. \tag{31}$$

Let us carry out the calculation using conditional data for an economic entity acting as a lender for two borrowers — enterprise 1 and enterprise 2, belonging to the same industry (Table 2).

**Table 2.** Distribution of the loan amount between borrowing enterprises for a certain reporting period at a given interest rate, thousand conventional units.

| Credit portfolio | Enterprise 1 | Enterprise 2 |
|---|---|---|
| as of 01.01.2023 | 2103.8 | 1101.5 |
| as of 01.01.2024 | 2705.7 | 988.5 |
| $r_j$ | 0.25 | 0.32 |

Lending to enterprise 1 at an interest rate of $r = 0.25$ as of 01.01.2023 at of the borrowing enterprise for the period of 2020–2023 of: 1619.2 thousand conditional units for 2020; 807.0 thousand conditional units for 2021; 1302.8 thousand conditional units for 2022; 5103.8 thousand conditional units for 2023, where $\bar{G} = 2208.2$ thousand conditional units.

Taking into account formula (30), as of 01.01.2023 we have to verify: $G_j = 5103.8 \geqslant (1 + 0.25) \times 2103.8 = 2629.75$ thousand conditional units, which, in turn, corresponds to the inequality: $2208.2 < 2629.75 \leqslant 5103.8$; then the expected value of the return of funds to the creditor enterprise as a result of the implementation of the project with the planned rate of profitability $g = 0.5$, according to formula (31), will be equal to: $M(w_j) = 0.5 \cdot [5103.8 - (1 + 0.25) \cdot 2103.8] = 1237.025$ thousand conditional units; in addition, it is known that $M^{opt}(w_1) \geqslant (1 + g)K_1$, where $K_1$ is the total capital of enterprise 1 (assume that $K_1 = 800$ thousand conditional units), that is, during the interim calculations $(1 + 0.5) \cdot 800 = 1200$ thousand conditional units, condition (26) is fulfilled: $1237.025 \geqslant 1200$.

Therefore, it is possible to calculate the optimal profitability of enterprise 1 as of 01.01.2023 using borrowed funds at an interest rate of 0.25 without losing the permissible state of economic security:

$$g_1 = \frac{M(w_1)}{F} = \frac{1237.025}{2103.8} = 0.59.$$

That is, in this case, with the involvement of loan funds, the rate of return of the relevant enterprise will actually increase by 9%.

Similarly, we will consider lending to enterprise 1 at an interest rate of $r = 0.25$ as of 01.01.2024 at $G_j$ of the borrower enterprise for the period 2021–2024 is: 807.0 thousand conditional units for 2021;

1302.8 thousand conditional units for 2022; 5103.8 thousand conditional units for 2023; 5070.0 thousand conditional units for 2024, where $\bar{G} = 3070.9$ thousand conditional units. The condition of formula (26) is completely fulfilled, that is, during intermediate calculations we have: $(1 + 0.25) \cdot 2705.7 = 3382.125$ thousand conditional units, then in total: $3070.9 < 3382.125 \leqslant 5070.0$.

Accordingly $M(w_2) = 0.5 \cdot [5070 - (1 + 0.25) \cdot 2705.7] = 843.9$ thousand conditional units; at $K_2 = 670$ thousand conditional units $M^{opt}(w_2) = (1 + 0.25) \cdot 670 = 837.5$ thousand conditional units; similarly, according to formula (30), the inequality is satisfied: $843.9 \geqslant 837.5$; then $g = \frac{843.9}{2705.7} \approx 0.31$, that is, we can expect a drop in $g$ from 0.5 to 0.31 by 19% with a partial deterioration in the state of economic security for the borrowing enterprise.

Lending to enterprise 2 at an interest rate of $r = 0.32$ as of 01.01.2023 at $G_j$ for the borrower enterprise for the period of 2020–2023 is: 513.0 thousand conditional units for 2020; 1180.0 thousand conditional units for 2021; 1002.0 thousand conditional units for 2022; 2050.0 thousand conditional units for 2023, where $\bar{G} = 1186.3$ thousand conditional units. Similarly, for intermediate calculations as of 01.01.2023: $(1+0.32) \cdot 1101.5 = 1453.98$ with the general fulfillment of the conditions of formula (26): $1101.5 < 1453.98 \leqslant 2050.0$.

Then, according to the rate of profitability, for example, $g = 0.4$ we get: $M(w_2) = 0.4 \cdot [2050 - (1 + 0.20) \cdot 1101.5] = 238.4$ thousand conditional units.

If $K_1 = 112$ thousand conditional units, then $M(w_1) = (1 + 0.4) \cdot 112 = 156.8$ thousand conditional units; this corresponds to condition (30): $238.4 \geqslant 156.8$; then $g = \frac{238.4}{1101.5} \approx 0.22$. Therefore, the rate of return of the enterprise should be reduced from 0.4 to 0.22 in connection with the stabilization of the amount of credit risk, so that the state of economic security of the borrower enterprise does not deteriorate further.

Lending to enterprises 2 at a nominal interest rate of $r = 0.32$ as of 01.01.2024 at $G_j$ of the borrower enterprise for the period of 2021–2024 is: 1180.0 thousand conditional units for 2021; 1002.0 thousand conditional units for 2022; 2050.0 thousand conditional units for 2023; 1300.0 thousand conditional units for 2024, where $\bar{G} = 1383.0$ thousand conditional units. During intermediate calculations $(1 + 0.32) \cdot 988.5 = 1304.82$ the condition of formula (26) is not satisfied: $1383 < 1304.82 \leqslant 1300$.

This means that taking a loan in the specified amount of 988.5 thousand conventional units for enterprise 2 at an interest rate of $r = 0.32$ with its gross income of 1300.0 thousand conventional units as of the reporting date is a risky business that can lead not only to temporary loss of the company's creditworthiness and will require an urgent search for new ways to increase gross profit, but also to bankruptcy and complete loss of economic security.

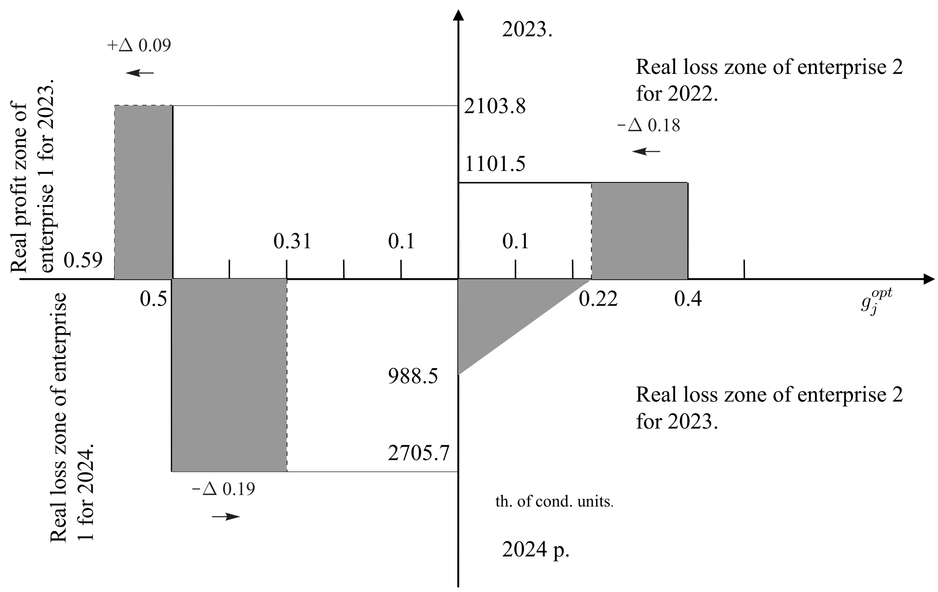


**Fig. 3.** Zones of real profits and losses of enterprises 1 and 2.

Graphically, the positive and negative consequences of servicing the borrowers of enterprise 1 and enterprise 2, that is, the zones of real profits and losses of these enterprises are shown in Figure 3.

That is, the combination of several economic and mathematical methods and models provides a more accurate assessment of the main financial results of a business entity and is one of the innovative approaches not only to the assessment of entrepreneurial activity, strengthening the significance of mathematical methods in the aspect of making economic decisions in the conditions of a market economy regarding the clear gradation of creditor enterprises and borrower enterprises, but also regarding the systematic diagnosis of the financial results of the enterprise at a certain point in time.

## 5. Conclusion

Thus, the integration of the cost-risk phase trajectories method with the method for determining the optimal profitability from credit transactions offers a comprehensive approach for diagnosing financial results. This approach enables precise quantitative calculations of the real profits and losses zone for enterprises, prioritizing key factors such as costs and risks while incorporating profitability calculations.

Furthermore, the dynamic interpretation of financial risk in relation to current costs can serve as a critical indicator of an enterprise's financial stability or vulnerability. This dynamic is represented through the equations of the phase trajectory system, incorporating inconsistency signs ($+1$ or $-1$) to model the regulated economic parameters that influence enterprise activities.

Future research should focus on modeling practical business scenarios, combining diverse economic and mathematical methods to enhance diagnostics and restore acceptable levels of economic security for enterprises, particularly during the war and post-war recovery periods.

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# Інтеграція методу фазових траєкторій витрати-ризик та методу знаходження оптимального значення рентабельності в систему діагностики фінансових результатів підприємства

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У статті представлено теоретичні та прикладні підходи до інтеграції економіко-математичних методів фазових траєкторій витрати–ризик та знаходження оптимальної прибутковості підприємства. Вихідною інформацією дослідження є те, що ризик може бути одним із основних ідентифікаторів стану коливань фінансових результатів підприємства, постійно змінюючись в умовах невизначеності ведення бізнесу, а тому його значення можна визначати за різними ознаками неузгодженість, що позначається на графічному зображенні фазових траєкторій – ізоклін на площині. Методика знаходження оптимального значення прибутковості від запроваджених кредитних операцій при різних процентних ставках і ризиках в системі діагностики фінансових результатів дозволяє максимально точно в кількісному вираженні розрахувати площу зони реальних прибутків і збитків відповідного підприємства. У сукупності ці методи покращують результати фінансової діагностики стійкості та безпеки підприємства.

**Ключові слова:** *економіко-математичні методи; фазові траєкторії; витрати; ризик; оптимальна прибутковість; зона прибутків і збитків; фінансові результати діяльності підприємства.*