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REFLECTION OF THE 3q±1 PROBLEM ON THE JACOBSTHAL MAP

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Abstract. The work shows that the task is the problem $C_{3q\pm1} = 3q \pm 1$ conjecture positive integers $q \geq 1$ in the reverse direction $n \to 0$ of the branching of the Jacobsthal tree, according to the rules of transformations of recurrent Jacobsthal numbers. For the first time, the Collatz problem is analyzed from the point of view of the increase in information entropy after the passage of the so-called fusion points (nodes) on the polynomials $\theta \cdot 2^n$ by the Collatz trajectories. For the first time, the Collatz problem is considered from the point of view of Shannon-Hartley information entropy behavior. It is also shown for the first time that the Collatz trajectory is a one-dimensional graph on a kind of two-dimensional lattice of recurring Jacobsthal numbers.

Keywords: recurrence sequence, Jacobsthal numbers, Collatz conjecture, information entropy
2020 Mathematics Subject Classification: 37P99;11Y16; 11A51; 11-xx; 11Y50

Introduction and problem statement

It is known [1-9] that the classical Collatz problem is formulated from two arithmetic operations on an arbitrary integer $q \geq 1$: if the number is even, it is divided by two q/2, and if it is odd, it is transformed as $3q+1$:

$$C_{3q+1} = \quad if \quad q \equiv 0 \mod 2 \quad then \quad \frac{q}{2} \quad else \quad C^{+}_{3q+1} = 3q+1 \ . \tag{1}$$

The transformation is formulated similarly

$$C_{3q-1} = \quad if \quad q \equiv 0 \mod 2 \quad then \quad C_{q/2}\frac{q}{2} \quad else \quad C^{-}_{3q-1} = 3q-1 \ , \tag{2}$$

the regularities of which are fundamentally different from (1) [10-11]. The famous Collatz conjecture states that the so-called Collatz trajectories (CT_{3q+1}) lead to unity for all transformable numbers from the semi bounded set of positive integers q ϵ [1,+∞] Collatz's hypothesis cannot be verified, so they try to prove it in the form of a theorem.

Both types of number transformations q :

$$C_{3q\pm1} = \quad if \quad q \equiv 0 \mod 2 \quad then \quad C_{q/2} \quad else \quad C^{\pm}_{3q\pm1} \ , \tag{3}$$

built on the properties of the set of numbers with a binary base:

$$1\cdot2^0, 1\cdot2^1, 1\cdot2^2, 1\cdot2^3, 1\cdot2^4, 1\cdot2^5, 1\cdot2^6, 1\cdot2^7, .., 1\cdot2^n, ... \ n \in \mathrm{N} \ \mathrm{U}\{0\}. \tag{4}$$

which is generated by the binomial theorem for arbitrary n :

$$(x+y)^n = \sum_{i=0}^{n}\binom{n}{i}x^i y^{n-i} \quad \Rightarrow If \;\; x=y=1 \quad then \quad (1+1)^n = \sum_{i=0}^{n}\binom{n}{i} = 2^n . \tag{5}$$

On the properties of numbers (4), the well-known [12] recurrent Jacobsthal numbers are constructed. In this work is to show that the problem $C_{3q\pm1} = 3q \pm 1$ is a problem of converting positive integers $q \geq 1$ in the reverse direction $n \to 0$ branching of the Jacobsthal tree, according to the rules of transformations of recurrent Jacobsthal numbers. Such studies were started by the author back in [13-17]. For the first time, the Collatz problem is analyzed from the point of view of the increase in information entropy after passing the CT_{3q+1} so-called fusion points (nodes) on the polynomials $\theta \cdot 2^n$. It is also shown for the first time that the Collatz trajectory is a one-dimensional graph on a kind of two-dimensional lattice of recurring Jacobsthal numbers. The model of recurrent Jacobsthal numbers in the Collatz problem is also used in [8].

Main Material Presentation

Consider a set of parameterized $\theta \geq 1$ power polynomials $\theta \cdot 2^n$ with the first two adjacent terms:

$$\theta \cdot 2^0 - 1, \theta \cdot 2^0, \theta \cdot 2^0 + 1, \quad \theta \cdot 2^1 - 1, \theta \cdot 2^1, \theta \cdot 2^1 + 1, \quad \theta \cdot 2^3 - 1, \theta \cdot 2^3, \theta \cdot 2^3 + 1, \quad ..., \tag{6}$$

which we structure in the form of:

$$\begin{matrix} \theta \cdot 2^0 + 1 & \theta \cdot 2^1 - 1 & \theta \cdot 2^2 + 1 & \theta \cdot 2^3 - 1 & \theta \cdot 2^4 + 1 & \theta \cdot 2^5 = 1 & \\ \theta \cdot 2^0 \;\; , & \theta \cdot 2^1 \;\; , & \theta \cdot 2^2 \;\; , & \theta \cdot 2^3 \;\; , & \theta \cdot 2^4 \;\; , & \theta \cdot 2^5 \;\; , & ..., \\ \theta \cdot 2^0 - 1 & \theta \cdot 2^1 + 1 & \theta \cdot 2^2 - 1 & \theta \cdot 2^3 + 1 & \theta \cdot 2^4 - 1 & \theta \cdot 2^5 + 1 & \end{matrix} \tag{7}$$

Then $\theta = 1$ we get the following sequence of numbers:

$$\begin{matrix} k: & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & & \\ J_{1,k}: & 2 & 1 & 5 & 7 & 17 & 31 & 65 & 127 & 257 & & A014551 \\ & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & . \\ & 2^0, & 2^1, & 2^2, & 2^3, & 2^4, & 2^5, & 2^6, & 2^7, & 2^8, & ... & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ \Im_{1,k}: & 0 & 3 & 3 & 9 & 15 & 33 & 63 & 129 & 255 & & A062510 \end{matrix} \tag{8}$$

Multiple three-m numbers $J_{1,k}$ are known as recurrent Lucka-Jacobsthal numbers, and multiples of three $\Im_{1,k}$ are known as recurrent Jacobsthal $\Im_{1,k}$ numbers [12].

Now consider recurring numbers $J(\Im)_{\theta,k}$ with the other three index values $\theta = 3,5,7$:

$$\begin{matrix} k & : & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \Im_{3,k} & : & 4, & 5, & 13, & 23, & 49, & 95, & 193, & 383, & 769,... \\ J_{3,k} & : & 2, & 7, & 11, & 25, & 47, & 97, & 191, & 385, & 767,... \\ \Im_{5,k} & : & 6, & 9, & 21, & 39, & 81, & 159, & 321, & 639 & 1281,... \\ J_{5,k} & : & 4, & 11, & 19, & 41, & 79, & 161, & 319, & 641, & 1279,... \\ J_{7,k} & : & 8, & 13, & 29, & 55, & 113, & 223, & 449, & 895, & 1793,... \\ \Im_{7,k} & : & 6, & 15, & 27, & 57, & 111, & 225, & 447, & 897, & 1791,... \end{matrix} . \tag{9}$$

So, if they are θ multiples of three, then both numbers are multiples of three $J(\Im)_{\theta,k}$, otherwise one of them is multiples of three. In general, for the number (9), the recurrence relations are true:

$$J(\Im)_{\theta,k+2} = J(\Im)_{\theta,k+1} + 2J(\Im)_{\theta,k}\,, \tag{10}$$

with generalized and parameterized θ initial conditions:

$$\begin{aligned} J_{\theta,0} &= \theta+1, \quad and \quad J_{\theta,1} = \theta-1, \\ \Im_{\theta,0} &= 2\theta-1, \quad and \quad \Im_{\theta,1} = 2\theta+1, \end{aligned} \tag{11}$$

with an even prime number. Therefore, with the index θ (see also [15]), the numbers $J(\Im)_{\theta,k}$, can still be structured as follows:

$$\begin{aligned} &\frac{\theta-1}{3} = even \quad then \quad \theta=\theta_1 = 1+6i, \quad and \quad \Im_\theta = 3\cdot \text{int}\,eger, \quad J_\theta \neq 3\cdot I \quad (a) \\ &\frac{\theta+1}{3} = even \quad then \quad \theta=\theta_5 = 5+6i, \quad and \quad \Im_\theta \neq 3\cdot \text{int}\,eger, \quad J_\theta = 3\cdot I \quad (b), \\ &\frac{\theta}{3} \neq even \quad then \quad \theta=\theta_3 = 3+6i, \quad and \quad \Im_\theta \neq 3\cdot \text{int}\,eger, \quad J_\theta \neq 3\cdot I \quad (c) \end{aligned} \tag{12}$$

where $\mathrm{I} = \text{integer}$ - is an integer, $i = 0,1,2,3,4,5,...$ Rules (12) express the connection of recurrent numbers $J(\Im)_{\theta,k}$ with the parameter θ and will be the basis of the formation of the Jacobsthal tree.

A Jacobsthal tree is a branching of parameters parameterized by index θ graphs of multiples of three integers $J(\Im)_{\theta,k}$ for which the rules hold:

$$J(\Im)_{\theta,k+1} = 4J(\Im)_{\theta,k} \mp 3 \tag{13}$$

or

$$\frac{J(\Im)_{\theta,r(s)+1}}{3} = 4\frac{J(\Im)_{\theta,r(s)}}{3} \pm 1. \tag{14}$$

We summarize the selection of integers from (14) $J(\Im)_{\theta,k}$ in the form of numbers:

$$\mathrm{K}^{\pm}_{\theta,n} = \frac{\theta\cdot 2^n \pm (-1)^n}{3}, \tag{15}$$

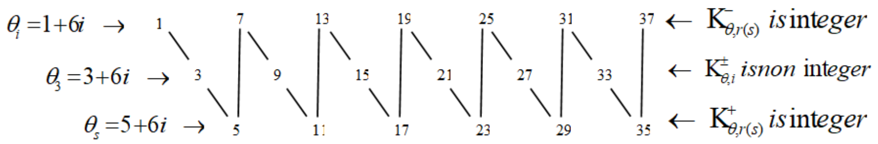
with parameterized θ initial conditions

$$If \quad \frac{\theta\pm 1}{3} = \text{int}\,eger \;\; then \;\; \mathrm{K}^{\pm}_{\theta,0} = \frac{\theta \pm 1}{3} \;\; and \;\; \mathrm{K}^{\pm}_{\theta,1} = \theta - \mathrm{K}^{\pm}_{\theta,0}. \tag{16}$$

So for θ=49, therefore $\mathrm{K}^{-}_{\theta,0} = 16$ and $\mathrm{K}^{-}_{\theta,1} = 49-16 = 33$. For numbers (15), the rule holds true:

$$\mathrm{K}^{\pm}_{\theta,n+1} = 4\mathrm{K}^{\pm}_{\theta,n} \pm 1. \tag{17}$$

However, rule (17) is unambiguous only in the direction $n \to \infty$ and ambiguous in the reverse $n \to 0$ direction, that is, in the direction of iteration CT_{3q+1}. The fact that (17) is not correctly applied, which can lead to a false conclusion, including regarding Collatz's hypothesis, was drawn attention to in [15]. The structuring of the parameter and recurrent Jacobsthal numbers is related as follows:



(18)

If $\theta = 1$, then from (15) we have the known [12] Binnet formula:

$$K^{-}_{1,n} = J_n = \frac{1}{3}\left[\theta \cdot 2^n - (-1)^n\right] = \text{integer},\ n \in \mathbb{N} \cup \{0\}. \quad (19)$$

If the initial numbers are $J_0 = 0$ and $J_1 = 1$, then we have Jacobsthal numbers calculated by the formula $J_{n+2} = J_{n+1} + 2J_n$, otherwise we obtain the Jacobsthal-Luckas sequence $j_{n+2} = j_{n+1} + 2j_n$ if $j_0 = 2$ and $j_1 = 1$.

From the point of view of the Collatz problem, the positive integers highlighted in black and red in (Table 1) and (Table 2) are of interest $K^{-}_{\theta_1,n}$

Table 1.

r (s)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*OEIS*
θ_1=1	[0]	1	1	[3]	5	11	[21]	43	85	[171]	A001045
θ_1=7	2	5	[9]	19	37	[75]	149	299	[597]	1195	A062092
θ_1=13	4	[9]	17	35	[69]	139	277	[555]	1109	2219	Unknown
θ_1=19	[6]	13	25	[51]	101	203	[405]	811	2389	[3243]	Unknown
θ_1=25	8	17	[33]	67	133	[267]	533	1067	[2133]	4267	Unknown
θ_1=31	10	[21]	41	83	[165]	331	661	[1323]	2645	5291	Unknown
θ_1=37	[12]	25	49	[99]	197	395	[789]	1579	3157	[6315]	Unknown
θ_1=43	14	29	[57]	115	229	[459]	917	1835	[3669]	7339	Unknown
θ_1=49	16	[33]	65	131	[261]	523	1045	[2091]	4181	8363	Unknown

and positive integers $K^{+}_{\theta_5,n}$

Table 2.

r (s)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*OEIS*
θ_5=5	2	[3]	7	13	[27]	53	107	[213]	427	853	A0485573
θ_5=11	4	7	[15]	29	59	[117]	235	469	[939]	1877	Unknown
θ_5=17	[6]	11	23	[45]	91	181	[363]	725	1451	[2901]	Unknown
θ_5=23	8	[15]	31	61	[123]	245	491	[981]	1963	3925	Unknown
θ_5=29	10	19	[39]	77	155	[309]	619	1237	[2475]	4949	Unknown
θ_5=35	[12]	23	47	[93]	187	373	[747]	1493	2987	[5973]	Unknown
θ_5=41	14	[27]	55	109	[219]	437	875	[1749]	3499	6997	Unknown
θ_5=47	16	31	[63]	125	251	[501]	1003	2005	[4011]	8021	Unknown

For both, highlighted by different colors of numbers $K^{\pm}_{\theta,n}$, the recurrence relation holds true:

$$K^{\pm}_{\theta,n+1} = 2K^{\pm}_{\theta,n} \mp (-1)^n \quad (20)$$

For each θ, every third number is a $K^{\pm}_{\theta_3,n}$ multiple of three (separated by square brackets), between which two multiples of three numbers are formed $K^{\pm}_{\theta_{1,5},n}$. As shown in (Table 1) and (Table 2), as the number $K^{\pm}_{\theta_3,n}$ increases θ, mirror-symmetric directions relative to the horizontal line are formed, the first number of which is an even number. For each θ, adjacent numbers $K^{\pm}_{\theta_3,n}$ are connected by the ratio:

$$K^{\pm}_{\theta_3,n+1} = 64K^{\pm}_{\theta_3,n} \pm 21. \tag{21}$$

Thus, in black and red, the numbers (Table 1) and (Table 2) are recurrent of the second order of the Jacobsthal type, in which the first number is even, and the second is odd so that the sum is equal to their index θ(18). The indices θ belong to the subset of odd natural numbers, therefore the odd Jacobsthal numbers also belong to the semi-bounded subset of odd natural numbers $[1,+odd_{\infty})$.

Mark the numbers highlighted in black as $m_{\theta_{1,5},r(s)}$. Equality is realized for them

$$\theta_{1,5} \cdot 2^{r(s)} = 3m_{\theta_{1,5},r(s)} + 1. \tag{22}$$

The numbers $p_{\theta_{1,5},r(s)}$ are highlighted in red. For them, equality (22) has the form:

$$\theta_{1,5} \cdot 2^{r(s)} = 3p_{\theta_{1,5},r(s)} - 1. \tag{23}$$

where $r = 0,2,4,...$, and $s = 1,3,5,...$ Numbers $m_{\theta_{1,5},r(s)}$ are calculated as

$$m_{\theta_{1,5},r(s)+1} = 4m_{\theta_{1,5},r(s)} + 1. \tag{24}$$

and the difference between adjacent ones is equal

$$m_{\theta_1,k+1} - m_{\theta_1,k} = 4^{k-1}(3m_{\theta_1,0(1)} + 1). \tag{25}$$

Analogous relations are true for numbers $p_{\theta_{1,5},r(s)}$:

$$p_{\theta_{1,5},r(s)+1} = 4p_{\theta_{1,5},r(s)} - 1 \;\;\Rightarrow\;\; p_{\theta_1,k+1} - p_{\theta_1,k} = 4^{k-1}(3p_{\theta_1,0(1)} - 1). \tag{26}$$

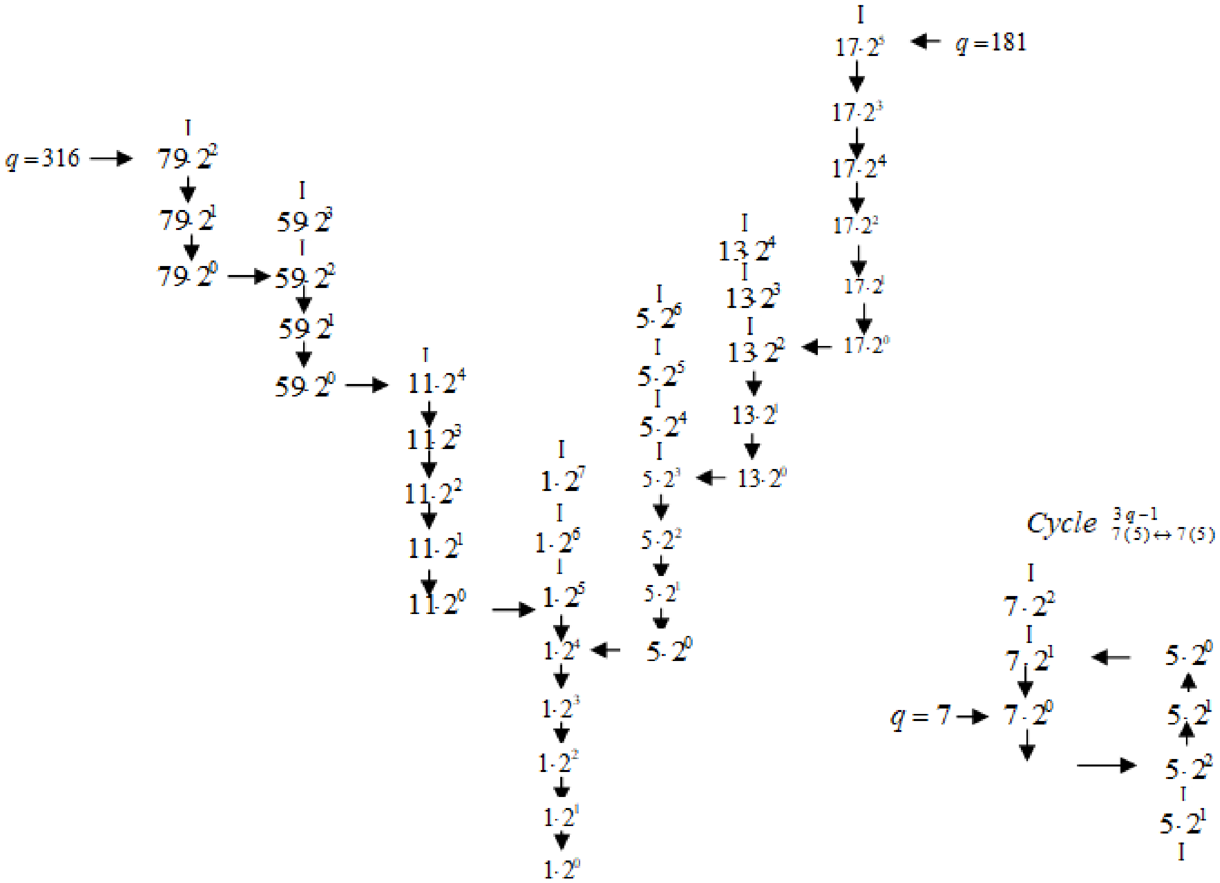
The right-hand sides of equations (22) and (23) coincide with the known rules (1) and (2) for the transformation of odd numbers. Therefore, using (22) and (2=), we construct the so-called Jacobsthal tree, which we reconstruct $CT_{3q\pm1}$ in the reverse direction $n \to 0$. To do this, we will create modules of the type

$$\begin{pmatrix} & p_{\theta_1,1} & & p_{\theta_{,1}3} & & p_{\theta_1,5} & & \\ \theta_1 2^0 & \theta_1 2^1 & \theta_1 2^2 & \theta_1 2^3 & \theta_1 2^4 & \theta_1 2^5 & \theta_1 2^6 & ... \\ m_{\theta_1,0} & & m_{\theta_1,2} & & m_{\theta_1,4} & & m_{\theta_1,6} & \end{pmatrix}, \tag{27}$$

and

$$\begin{pmatrix} p_{\theta_5,0} & & p_{\theta_5,2} & & p_{\theta_5,4} & & p_{\theta_5,6} \\ \theta_5 2^0 & \theta_5 2^1 & \theta_5 2^2 & \theta_5 2^3 & \theta_5 2^4 & \theta_5 2^5 & \theta_5 2^6 ... \\ & m_{\theta_5,1} & & m_{\theta_5,3} & & m_{\theta_5,5} & \end{pmatrix}. \tag{28}$$

In modules (27) and (28), numbers $m(p)_{\theta_{1,5},k}$ form so-called nodes, through which they $CT_{3q\pm1}$ merge with the next module $\varpi \cdot 2^k$. In modules $\theta_3 \cdot 2^n$, there are no nodes, so modules with nodes and be considered active, and modules without nodes can be considered inactive. Members of inactive modules are formed from duplicated values $\theta_3 \cdot 2^n$.



a b

Fig.1. Fragment of a Jacobsthal tree (a) and isolated cycle graph $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$ (b)

An illustration of the branching of the Jacobsthal tree from modules (27) and (28) is shown in Figure 1a. To the right of the module $1 \cdot 2^n$, the tree branches using equality (22), and to the left of the module $1 \cdot 2^n$, the tree branches using equality (23).

Equalities (22) and (23) are true for numbers $m(p)_{\theta_3,r(s)}$ with an arbitrary value of the parameter $\theta_{1.5}$, therefore, in this direction $n \to \infty$, the Jacobsthal tree can branch without restriction.

In the direction $n \to \infty$ of equality (22) and (23) form branching points (bifurcations) with the equally probable implementation of both trajectories on the tree:

$$m(p)_{\theta_3,r(s)} \to \begin{cases} m(p)_{\theta_3,r(s)} \cdot 2^n, \\ 3m(p)_{\theta_3,r(s)} \pm 1. \end{cases} \tag{29}$$

At the main module $1\cdot 2^n$, the Jacobs tree starts branching from active nodes $m_{1,4}=5$ and $p_{1,5}=11$ inactive nodes $p_{1,3}=3$. It is this asymmetry between the nodes with the Jacobsthal numbers that form the cardinal differences between the $CT_{3q\pm1}$ numbers q. Consider this.

The $CT_{3q\pm1}$ arbitrarily chosen numbers q =181 and q =316 with arrows on the Jacobsthal tree (Figure 1a). We see that in the reverse direction $n\to 0$, to a single (or other) value, they $CT_{3q\pm1}$ go through the branching points (29). If the initial number is q odd (even division by 2 always reduces to odd), then according to the rules (22) or (23), it turns into an even number as:

$$3q\pm1=\theta_{1,5}\cdot 2^{r(s)}, \tag{30}$$

which is again reduced to the next odd number $\theta_{1,5}\cdot 2^0$ by division by 2, as shown in Figure 2a.

However, as can be seen from (Table 1) and (Table 2), the Jacobsthal numbers $\mathrm{K}^{\pm}_{\theta_{1,5},n}$ can be structured in the form of cells of Jacobsthal in which the active nodes $\theta_1=m(p)_{\theta_1,r(s)}$ and $\theta_5=m(p)_{\theta_5,r(s)}$ in the direction of increasing numbers $m(p)_{\theta_{1,5},r(s)}$ from θ_1 to θ_5 are surrounded by two multiples of three numbers $[m(p)_{\theta_3,r(s)}]$. For example, the first cell of the polynomial $5\cdot 2^n$ looks like this:

$$\theta_5=5:\ \begin{cases} m_{5,1} \quad m_{5,3} \quad m_{5,5} \quad m_{5,7} \\ [3] - 13 - 53 - [213]- \end{cases}. \tag{31}$$

On modules $1\cdot 2^n$, the first cell

$$\theta_1=1:\ \begin{cases} m_{1,0} \quad m_{1,2} \quad m_{1,4} \quad m_{1,6} \\ [0] - 1 - 5 - [21]- \end{cases}, \tag{32}$$

is formed by the first four Jacobsthal numbers conjecture C^{+}_{3q+1} :

$$0,\ 1,\ 5,\ 21,\ 85,\ 341,\ 1024,\ 2048,\ 4096,\ \dots. \tag{33}$$

and has only one active node $m_{1,4}=5$, which CT_{3q+1} leads to the unit value.

For the sign-symmetric conjecture C^{-}_{3q-1}, the first is formed by the first four Jacobsthal numbers conjecture C^{-}_{3q-1}:

$$1,\ 3,\ 11,\ 43,\ 171,\ 683,\ 2731,\ 10923,\ \dots. \tag{34}$$

therefore, has two active nodes $p_{1,3}=11$ and $p_{1,5}=43$

$$\theta_1=1:\ \begin{cases} p_{1,1} \quad p_{1,3} \quad p_{1,3} \quad p_{1,5} \\ [3] - 11 - 43 - [171]- \end{cases}. \tag{35}$$

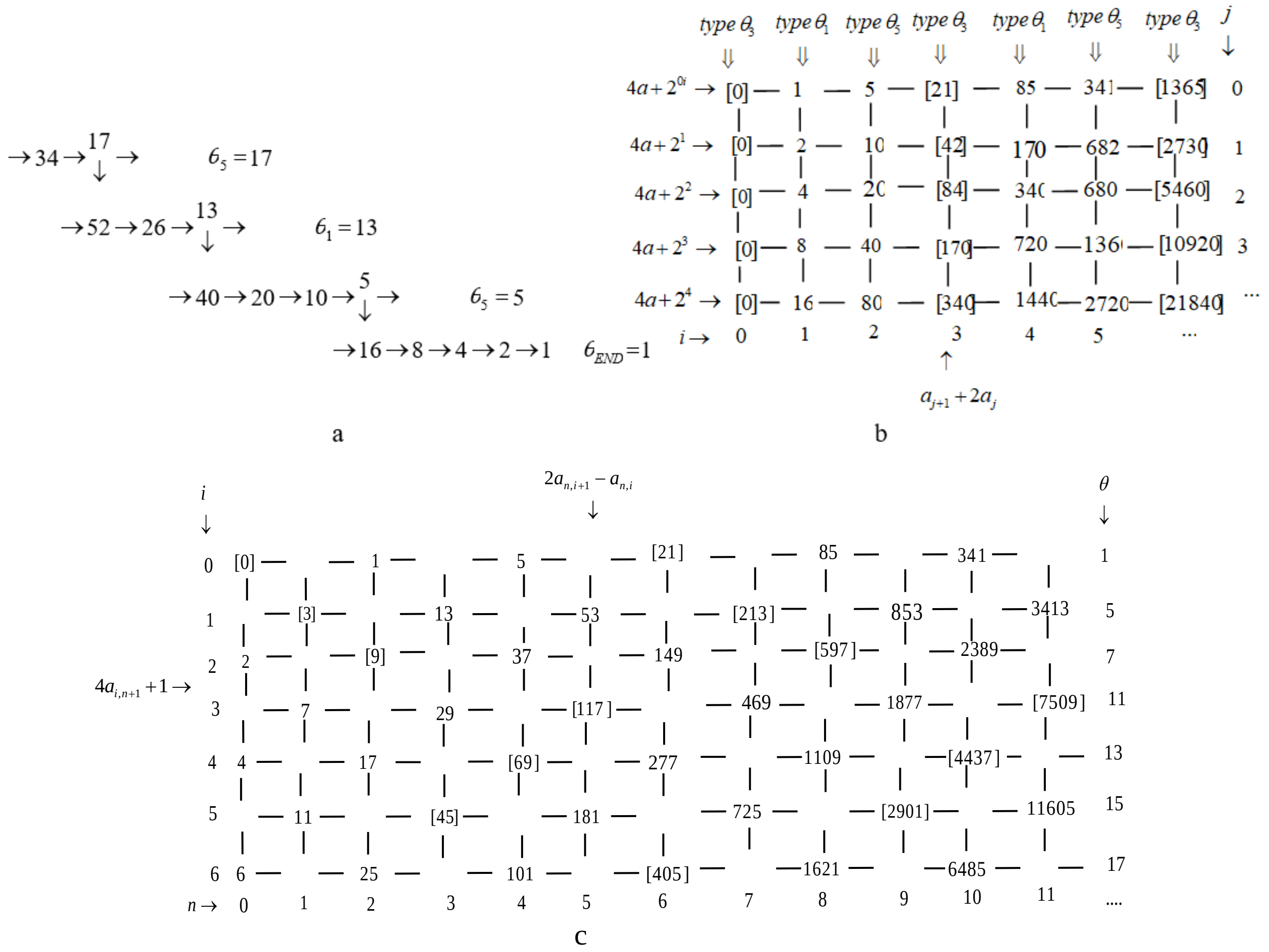


Fig. 2. Collatz conjecture (a) and Jacobsthal lattice (b) and (c)

Figure 2b shows the so-called flat Jacobsthal lattice, the nodes of which are formed by transformations $4a_{i,j} \pm 2^j$ between adjacent members in rows and $a_{i,j+1} + 2a_{i,j}$ between adjacent members in columns. Analogous lattices hold true for Jacobsthal numbers $K^{\pm}_{\theta_5,n}$ with arbitrary indices in (20) and (21) θ (Figure 2c). In three-dimensional space ($4a_{i,j} \pm 2^j, a_{i,j+1} + 2a_{i,j}, \theta$), the lattice structure will have the form of a cube of layers ($4a_{i,j} \pm 2^j, a_{i,j+1} + 2a_{i,j}$).

Consider circular models of periodic cycles $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$, $cycle^{3q-1}_{1\leftrightarrow 1}$ and $cycle^{3q-1}_{17\leftrightarrow 17}$ for the numbers $5, 7$ and 17. As shown in Fig. 3, in the form of closed cycles $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$ and $cycle^{3q-1}_{17\leftrightarrow 17}$. We see that on the cycle $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$ there are two branching points with the numbers 20 and 14, through which other branches are multiplied. On the cycle $cycle^{3q-1}_{17\leftrightarrow 17}$ there are 7 branching points with the numbers 68, 272, 182, 122, 164, 74 and 50. The arrows show the directions of development CT_{3q-1} for the numbers 5, 7 and 17. So, if the Jacobsthal tree branches in the direction of $n \to \infty$, then CT_{3q-1} develop in the reverse direction $n \to 0$. The circle of the cycle $cycle^{3q-1}_{1\leftrightarrow 1}$ has one branching point with the number 2.

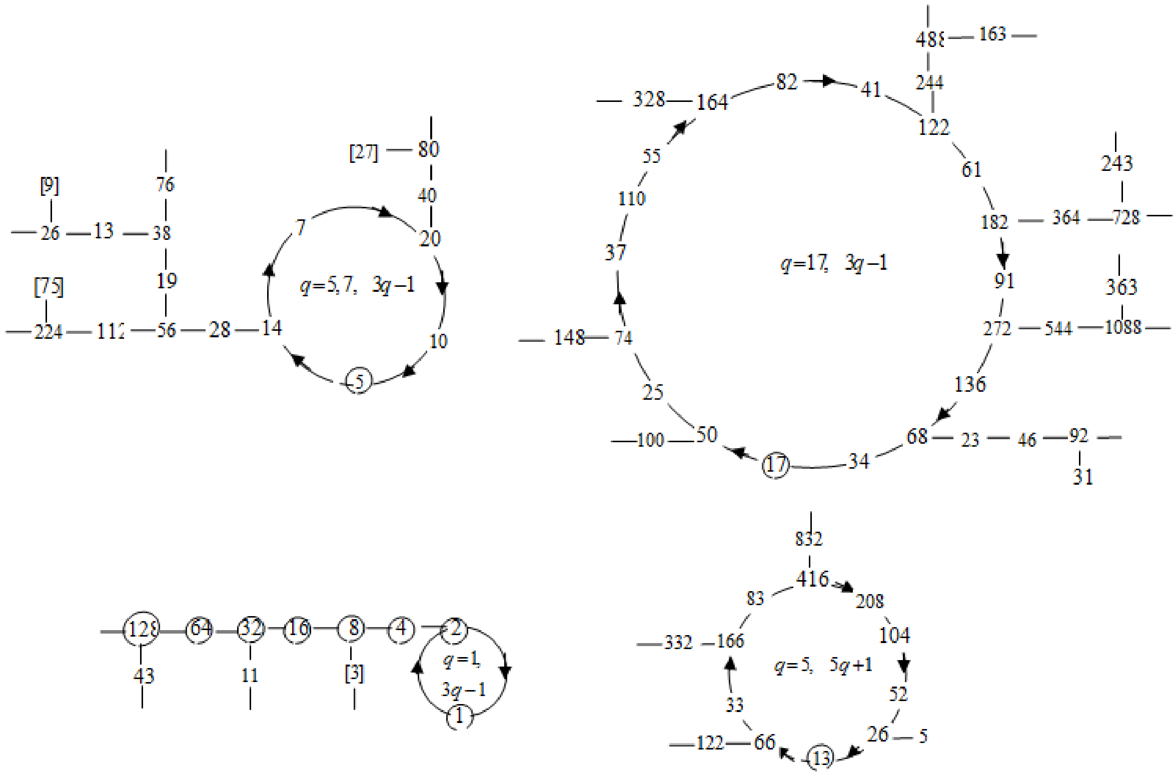


Fig. 3. Circular model of cycles $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$, $cycle^{3q-1}_{1\leftrightarrow 1}$ and $cycle^{3q-1}_{17\leftrightarrow 17}$

Therefore CT_{3q-1} ends with oscillations with minimum amplitudes $q_{\min} = 1, 5, 17$. Analysis of the frequency $\nu = \dfrac{Number_{cycle}}{Number_{full}}$ of the distribution of numbers by completion cycles shows a trend of uniform distribution: $cycle^{3q-1}_{1\leftrightarrow 1}$ (32.4%), and $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$ (32.2%), and $cycle^{3q-1}_{17\leftrightarrow 17}$ (35.4%).Therefore, from the point of view of the Collatz process, the cycles $cycle^{3q-1}_{1\leftrightarrow 1}$, $cycle^{3q-1}_{5(7)\leftrightarrow 5(7)}$ are $cycle^{3q-1}_{17\leftrightarrow 17}$ equal to each other. This is a strong argument in favor of Collatz's hypothesis about the existence of a single final cycle $cycle^{3q+1}_{1\leftrightarrow 1}$.

This statistical conclusion was confirmed for 1000 numbers [3]. The analysis of other transformations showed that similar isolated cycles are formed in the transformation C_{5q+1} with the minimum value of oscillations $q_{odd,\min} = 13,17$ and with an anomalously long period of the order of 19645 iterations with $q_{odd,\min} = 7$, which, as in the case of C_{5q+1}, are caused by periodic cycles of Jacobsthal transformations.

Let's investigate the conditions under which periodic cycles are formed. A loop occurs when an intermediate value of the converted number is repeated. If the period of the cycle is formed from one transformation $C_{3q\pm 1}$ and k iterations $q/2$, then the equality is

$$\frac{2^k q_{k,odd} \mp 1}{3} = q_{k,odd} = integer. \tag{36}$$

For the transformation C_{3q-1} from (36), we have that $q_{1,odd} = 1$ at $k = 1$, that is, the cycle $cycle^{3q-1}_{1\leftrightarrow 1}$ is formed by one iteration of $q/2$. For the transformation C_{3q+1}, the value of $q_{2,odd} = 1$ is an intermediate value of the transformed number, that is, the cycle is formed from two iterations of $q/2$. At $k>2$, $q_{k,\min} \neq I$.

Cycle $cycle_{5\leftrightarrow 5}^{3q-1}$ consists of two stages (34). In this case, the equality holds true [18]:

$$\frac{\frac{2^k q_{,mk,odd} \mp 1}{3} 2^m \mp 1}{3} = q_{k,m,odd} = integer \Rightarrow q_{k,m,odd} = \frac{\pm(3+2^m)}{2^{k+m}-3^2} . \tag{37}$$

Therefore, according to (37), for the transformation C_{3q+1} we have one cycle with the parameter $q_{2,2,odd} = 1$ at k =2 and m =2, while for the transformation C_{3q-1} we have three cycles with the parameters $q_{1,2,odd} = 7$ at k =1, m =1, $q_{2,1,odd} = 5$ at k =2, m =1, and $q_{1,2,odd} = 7$ at k =1, m =2.

For the transformation C_{3q-1}, a cycle with the minimum value of an odd number $q = 17$ is known. The cycle $cycle_{17\leftrightarrow 17}^{3q-1}$ is formed from seven stages (24), for which the equality holds:

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{2^k q_{k,\dots,f,odd} \pm 1}{3} 2^m \pm 1}{3} 2^s \pm 1}{3} 2^r \pm 1}{3} 2^d \pm 1}{3} 2^f \pm 1}{3} 2^t \pm 1}{3} = q_{k,\dots,f,odd} \Rightarrow \tag{38}$$

$$q_{k,\dots,f,odd} = \frac{\pm(3^6 + 3^5 \cdot 2^t + 3^4 \cdot 2^{t+f} + 3^3 \cdot 2^{d+f+t} + 3^2 \cdot 2^{r+d+f+t} + 3^1 \cdot 2^{s+r+d+f+t} + 2^{m+s+r+d+f+t})}{-3^7 + 2^{k+m+s+r+t+d+f}},$$

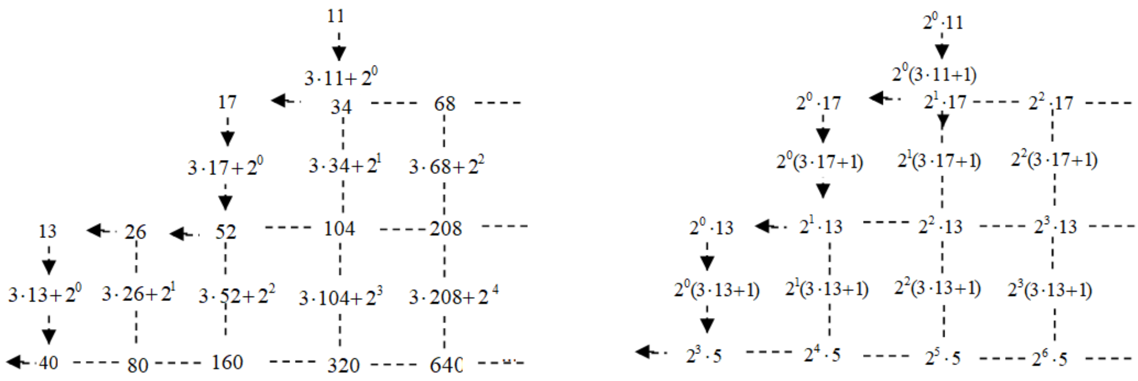
According to (38), the parameter $q_{4,1,1,1,1,1,1,1,odd} = \frac{2363}{139} = 17$. For other integer values of k, $q_{k,\dots,f,odd} \neq I$ or negative. For general conversion

$$C(q) = \quad if \quad q \quad is\,odd \quad then \quad C_{\kappa q \pm 1} = \kappa q \pm 1 \quad (a) \quad else \quad q/2 \quad (b), \kappa = 1, 3, 5, ... \tag{39}$$

formula (28) has the form

$$\tau \begin{cases} \frac{\frac{\frac{\frac{2^k q_{k,\dots,f,odd} - 1}{\kappa} 2^m - 1}{\kappa} 2^s - 1}{\kappa} 2^r - 1}{\dots} 2^f - = q_{k,\dots,f,odd} \\ \left. q_{k,\dots,f,odd} \right|_{\kappa q \pm 1} = \frac{\kappa^{\tau-1} + \kappa^{\tau-2} \cdot 2^f + \kappa^{\tau-3} \cdot 2^{r+\dots} + \dots + \kappa \cdot 2^{s+r+\dots} + \kappa \cdot 2^{m+s+r+\dots+f}}{\kappa^\tau - 2^{k+m+s+r+\dots+f}} \end{cases} . \tag{40}$$

In formula (40), indices of powers of $2^i, i = k, m, s, \dots$ reflect the number of iterations $q/2$ at each i -th stage (37).



a b

Fig. 4. Collatz lattice

Figure 4 shows the so-called flat Collatz lattice, the nodes of which are formed by transformations $\theta \cdot 2^n$ between adjacent members in rows and $2^m(3q \pm 1)$ between adjacent members in columns. The lattice is semi-bounded, the right side of which is bounded CT_{3q+1} (the trajectory is shown by arrows in the figure), and the opposite side is multiplied to infinity. The other two opposite sides are formed by initial $\theta \cdot 2^n$ and final $\Omega \cdot 2^m$ polynomials. The grid in Figure 4 shows that classical trajectories CT_{3q+1} are the optimal algorithm for achieving a single value by the Collatz sequence.

Conclusions

In the paper, a generalized Jacobsthal model of the transformation of numbers $q \in N$ in the direction of increasing power $n \to \infty$ of the polynomial $\theta \cdot 2^n$ is developed. It is shown that nodes are formed on the polynomial t under the condition $\frac{\theta \cdot 2^n \mp 1}{3}$ = *Integer*, from which the graphs are in the form of a tree (Jacobsthal tree). It is shown that in the reverse direction $n \to 0$ on the Jacobsthal tree, trajectories of transformations $3q \pm 1$ of numbers $q \in N$ are formed. It is shown that the transformation $3q \pm 1$ is one type of problem of the bisection of the polynomial $\theta \cdot 2^n$ by powers. It is shown that the periodic cycles of the Collatz transformations $3q \pm 1$ isolated from the polynomial $1 \cdot 2^n$ are due to the formation of isolated clusters with periodically oscillating graphs on the Jacobsthal tree. Reasoned analytical relations that allow you to calculate the parameters of the formation of periodic cycles. It is shown that the transformations of Jacobsthal and Collatz numbers develop in mutually opposite directions.

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ВІДОБРАЖЕННЯ ЗАДАЧІ 3Q±1 НА КАРТІ ЯКОБСТАЛЯ

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Анотація. У роботі показано, що актуальним завданням є вирішення задачі $C_{3q\pm1} = 3q \pm 1$ припущення натуральних чисел $q \geq 1$ у зворотньому напрямку $n \to 0$ розгалуження дерева Якобсталя, згідно з правилами перетворень рекурентних чисел Якобсталя. Вперше задачу Коллатца проаналізовано з точки зору зростання інформаційної ентропії після проходження так званих точок злиття (вузлів) на поліномах $\theta \cdot 2^n$ траєкторіями Коллатца. Вперше проблема Коллатца розглядається з точки зору поведінки інформаційної ентропії Шеннона-Хартлі. Також вперше показано, що траєкторія Коллатца є одновимірним графіком на своєрідній двовимірній решітці повторюваних чисел Якобсталя.

Ключові слова: рекуррентна послідовність, числа Якобсталя, гіпотеза Коллатца, інформаційна ентропія 2020 математична предметна класифікація: 37P99;11Y16; 11A51; 11-xx; 11Y50