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ON THE MATHEMATICAL MODEL OF THE TRANSFORMATION OF NATURAL NUMBERS BY A FUNCTION OF A SPLIT TYPE

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Abstract. In this work justified incorrectness of the algorithm proposed in the publication "M. Remer.[A Comparative Analysis of the New -3(-n) - 1 Remer Conjecture and a Proof of the 3n + 1 Collatz Conjecture. Journal of Applied Mathematics and Physics. Vol.11 No.8, August 2023"] in terms of the Collatz conjecture**.** And also that the transformation -3(-n) - 1 is not equivalent to Collatz's conjecture on the natural numbers 3n + 1. The obtained results can be used in further studies of the Collatz hypothesis

Key words: Recurrence sequence, Jacobsthal numbers, Collatz conjecture, natural numbers, conjecture

Introduction and Problem Statement

Recently, a work [1] was published, in which the author proposed, in his opinion, a new algorithm for the formation of Collatz sequences, which are relevant from the point of view of designing generators of pseudorandom numbers [5], cryptology, etc. Therefore, the proposal [1] attracted attention and deserves a detailed analysis, which is why this comment is dedicated.

Collatz's conjecture has been studied for many years, mostly by foreign authors [2]. In Ukraine, the last works were published more than ten years ago [3-4]. However, after the publication of the work [5], interest in Collatz's conjecture increased again and such research was also started at the Computer Aided Design Systems Department of the Lviv Polytechnic National University [6-9]. It was shown that the basic regularities of the Collatz algorithm can be described within the framework of the transformation model of Jakobstal recurrent numbers, well-known in discrete mathematics [10]. In addition to popularizing [7], the author introduced this approach to specialists of the I.Franko Lviv National University, an international seminar on fractal analysis of the Institute of Mathematics (Kyiv), thanks to which other approaches became known. Historically, a significant contribution to the research of this problem was made by a graduate of the Lviv Polytechnic National University [11], a co-author of the well-known method of probabilistic modeling in mathematics, known today as the Monte Carlo method[12]. So, accepting the relevance of Collatz's problem, the question arose of a detailed analysis of the number conversion algorithm $\mathrm{R_q} = -3(-n) - 1$ from the point of view of model $3n+1$ equivalence [1].

Main Material Presentation

The mathematical model of number conversion according to the Kollatatz algorithm is formulated in the form (1).

$$C_q^+ = \begin{cases} q/2 \ \ if \ q \ is \ even \\ 3q+1 \ \ if \ q \ is \ odd \end{cases} . \quad (1)$$

Here, the function $3q+1$ turns an odd q_{odd} number into an even number q_{even}, and by division $q/2$, an even number turns into an odd q_{odd} one again:

$$\mathrm{q}_{\mathrm{even}} = \theta \cdot 2^n \rightarrow \frac{q_{even}}{2^1} = \theta \cdot 2^{n-2^1} \rightarrow \frac{q_{even}}{2^2} = \theta \cdot 2^{n-2^2} \rightarrow \frac{q_{even}}{2^k} = \theta \cdot 2^{n-2^k}, \quad (2)$$

for which a binary-based sequence $\mathrm{q}_{\mathrm{even}} = \theta \cdot 2^n$ is used, where $q \in q_{even} \cup q_{odd} \in N\{0\}$, $\theta \in q_{odd}$.

In the title of the article [1], the author announces the transformation of the view $R_q = -3(-q)-1$. This means that it can be reduced to the form $R_{3q-1} = 3q-1$. As is known [13], in contrast to the transformation (1), which ends with a periodic trivial cycle $\ldots \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow \ldots$, the transformation

$$C_q^- = \begin{cases} q/2 \ \ if\ q\ is\ even \\ 3q-1 \ \ if\ q\ is\ odd \end{cases}, \quad (3)$$

already has three periodic trivial cycles isolated from one another $\ldots \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow \ldots$, $\ldots \rightarrow 5 \rightarrow \ldots \rightarrow 7 \rightarrow \ldots \rightarrow 5 \rightarrow \ldots$ i $\ldots \rightarrow 17 \rightarrow \ldots \rightarrow 17 \rightarrow \ldots$. Therefore, the function C_q^- has three root sequences $\{1 \cdot 2^n\}_{n=0}^{\infty}$, $\{5 \cdot 2^n\}_{n=0}^{\infty}$ i $\{17 \cdot 2^n\}_{n=0}^{\infty}$ with attractors 1,5 i 1 in the form of fixed points on the phase plane.

So, to avoid the ambiguity of the function $R_q = -3(-q)-1$ and preserve the recurrence of intermediate transformations, in [1], a change of sign is artificially introduced

$$+q \rightarrow -q, \quad (4)$$

which the author confirms with a concrete example

$$\begin{aligned} &-3(-1)-1 = 2 \\ &-3(-2)-1 = 5 \\ &-3(-5)-1 = 14 \\ &-3(-14)-1 = 41 \\ &-3(-41)-1 = 122 \\ &-3(-122)-1 = 365. \end{aligned} \quad (5)$$

In (4), each subsequent number is greater than the previous one, which is interpreted as an argument in support of Collatz's hypothesis.

However, similar results to (5) are obtained by classical transformation:

$$\begin{aligned} &2 = +3(+1)-1 \\ &5 = +3(+2)-1 \\ &14 = +3(+5)-1 \\ &41 = +3(+14)-1 \\ &122 = +3(+41)-1 \\ &365 = +3(+122)-1, \end{aligned} \quad (6)$$

without applying an additional sign transformation (4), which contradicts transformations (1) and (3).

In fact, the sequences (5) and (6) are well known in the literature [13], and are not equivalent to the Collatz sequences formed by the transformations (1) and (3). Moreover, in contrast to the existing point of view regarding (1) and (3), we will show that the functions of transformations of odd numbers of the type $C_q^{\pm} = 3q \pm 1$, are actually derivatives of the equations describing the transformation of recurrent Jacobstal numbers with the participation of sequences $\{\theta \cdot 2^n\}_{n=0}^{\infty}$ powers of two 2^n, which are calculated according to the formula in closed form

$$J^{\pm}_{\theta,n} = \frac{1}{3}\left[\theta\cdot 2^n \pm (-1)^n\right] = Integer,\ \ \theta \in N_{odd},\ \ N_{odd}N = N_{odd} \cup N_3 \cup N_{even}, \tag{7}$$

where in $\mathrm{N}_{\mathrm{odd}}$ is the set of odd integers not multiples of three, N_3 is the set of odd integers that are multiples of three, N_{even} is the set of even positive integers.

Numbers (7) are the numbers of points on the sequences $\{\theta\cdot 2^n\}_{n=0}^{\infty}$ (nodes), through which in the direction $n\to\infty$ branch out the graphs of the so-called Jacobstal tree, which is shown on the left in Fig.1.

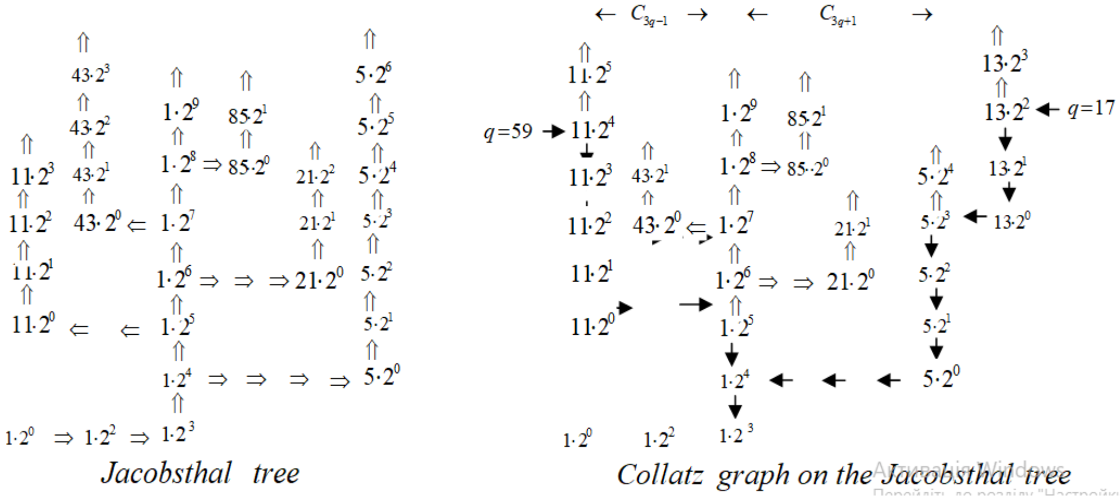

Fig.1. The Jakobstal tree (left) and, built on its basis, Collatz graph (right).

In the direction $\mathrm{n}\to\infty$, members of sequences $\{\theta\cdot 2^n\}_{n=0}^{\infty}$ are doubled, i.e. grow until, according to (7), one of the conditions (8) is fulfilled:

$$\frac{1}{3}\left[\theta\cdot 2^{s(r)} - 1\right] = m_{3,\theta,r(s)} \text{ for (a) and } \frac{1}{3}\left[\theta\cdot 2^{r(s)} + 1\right] = p_{3,\theta,s(r)} \text{ for (b),} \tag{8}$$

that is

$$\theta\cdot 2^{s(r)} = m_{1,\theta,s(r)} - 1 \text{ for (a) and } \theta\cdot 2^{r(s)} = p_{3,\theta,s(r)} + 1 \text{ for (b)} \tag{9}$$

where bisection by powers is applied

$$\mathrm{n} = \mathrm{r(even)} + \mathrm{s(odd)}. \tag{10}$$

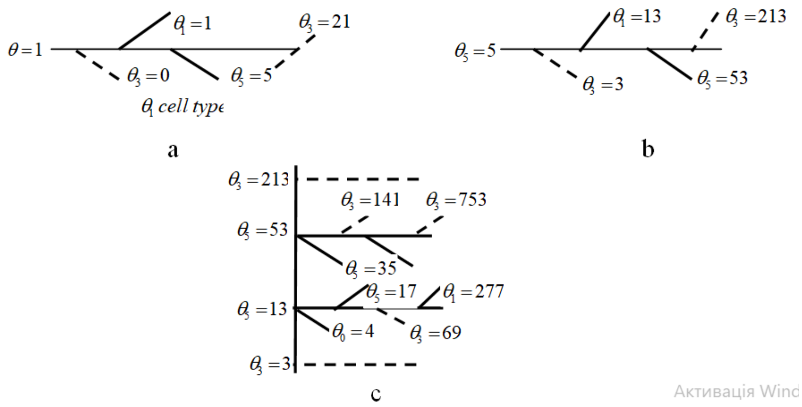

Fig.2. Cells of two nodes with numbers θ_3, nodes with numbers are formed between them $\theta_{1,5}$.(a,b) and branching of sequence graphs $\{\theta_{1,5}\cdot 2^{\mathrm{n}}\}_{n=0}^{\infty}$ (c)

Numbers $m_{a,\theta,r(s)}$ and $p_{a,\theta,r(s)}$ introduced because the recurring numbers are of the same type.

$$\begin{cases}\theta\cdot 2^r = J^-_{1,\theta,r}-1,\\ J^-_{1,\theta,r+2}=2K^-_{1,\theta,r}-3,\end{cases}(a)\quad \begin{cases}\theta\cdot 2^s = j^+_{1,\theta,s}-1,\\ J^+_{1,\theta,s+2}=2J^+_{1,\theta,s}-3,\end{cases}(c)$$
$$\text{and}\qquad \Rightarrow\begin{cases}J^+_{1,\theta,r}=J^-_{1,\theta,s}\\ J^-_{1,\theta,r}=J^+_{1,\theta,s}\end{cases}(f) \tag{11}$$
$$\begin{cases}\theta\cdot 2^s = J^-_{1,\theta,s}+1,\\ J^-_{1,\theta,s+1}=2J^-_{1,\theta,s}+3,\end{cases}(b)\quad \begin{cases}\theta\cdot 2^r = J^+_{1,\theta,r}+1,\\ J^+_{1,\theta,r+2}=2J^+_{1,\theta,r}+3,\end{cases}(d)$$

In order to distinguish between sequences $\{\theta\cdot 2^n\}_{n=0}^{\infty}$, they are parameterized with an odd number θ. Therefore, for transformation (9a) we equate

$$\theta = m_{a,\theta,r(s)}, \tag{12}$$

and for transformation (9b)

$$\theta = p_{a,\theta,r(s)}. \tag{13}$$

According to the parameter θ, the sequences are divided into three types:

$$N_{odd} = N_1 \cup N_5 \cup N_3, \tag{14}$$

where is $\theta_1 \in N_1$, $\theta_5 \in N_5$, $\theta_3 \in N_3$ and for numbers $\theta_{1,5}$ conditions are met:

$$\frac{\theta-1}{3} = Integer\ for\,\theta_1\ and\ \frac{\theta+1}{3} = Integer\ for\,\theta_5. \tag{15}$$

Result and Discussion

Figures 2a and 2b show the so-called cells of two nodes with numbers θ_3, nodes with numbers are formed between them $\theta_{1,5}$. Fig. 2c shows an illustration of the branching of sequence graphs $\{\theta_{1,5}\cdot 2^n\}_{n=0}^{\infty}$, where inactive branches with parameters θ_3, shown by a dotted line.

Numbers of sequence nodes $\{\theta_1\cdot 2^n\}_{n=0}^{\infty}$ form sequences of recurring Jacobstal numbers, the first member of which is an even number. Numbers of sequence nodes $\{\theta_5\cdot 2^n\}_{n=0}^{\infty}$ form sequences of recurring Jacobstal numbers, the first term of which is an odd number. Nodes with numbers of sequences $\{\theta_3\cdot 2^n\}_{n=0}^{\infty}$ are not active, have fractional values, so no other sequences are generated from them. The Collatz transformation process (1,3), which was reversible to the transformation of numbers with the participation of Jacobstal numbers, is shown on the right in Fig. 1 for two numbers 59 by the function $C_q^{=}$ and 17 by the function C_q^{+}. We see that the Kollats problem is a partial problem of the reverse transformation of numbers in the Jacobstal model. In general, the corresponding algorithm is fundamentally different from the one proposed in [1], therefore, a comparative analysis of the results of research of these approaches with each other from the point of view of their similarity is fundamentally wrong.

In conclusion, we note the following. The approach described above is based on the recursion model of Jacobstal numbers [6-9], which correlates with periodic cycles of the completion of the process of relaxation of the dynamic system to an equilibrium state, depicted, as an example, in Fig. 3 for root sequences with a specifically defined attractor, in this case equal to 17. In this figure, the arrows show the process of converting numbers at the nodes of the sequences $\{\theta\cdot 2^n\}_{n=0}^{\infty}$ with Jacobstal numbers. In the opposite direction of this process, a Collatz transformation is formed.

Fig. 4 shows a block diagram of the structuring of the initial set N according to the Jakobstal-Collatz processes, which develop in opposite directions. In the direction $n\to\infty$ the set N is partitioned into three subsets N_1, N_5, N_3. For each of them, the process of branching (non-branching) of graphs in sequence nodes $\{\theta \cdot 2^n\}_{n=0}^{\infty}$ is subject to appropriate conditions, which is clearly reflected in the structuring of odd numbers with a numerical period of 6. The model [1] does not reflect these regularities.

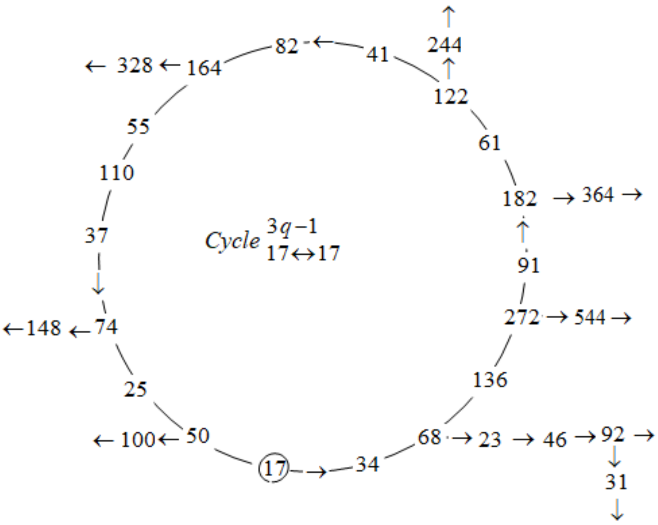

Fig.3. The process of relaxation of a dynamic system into an equilibrium state, for root sequences with attractor 17

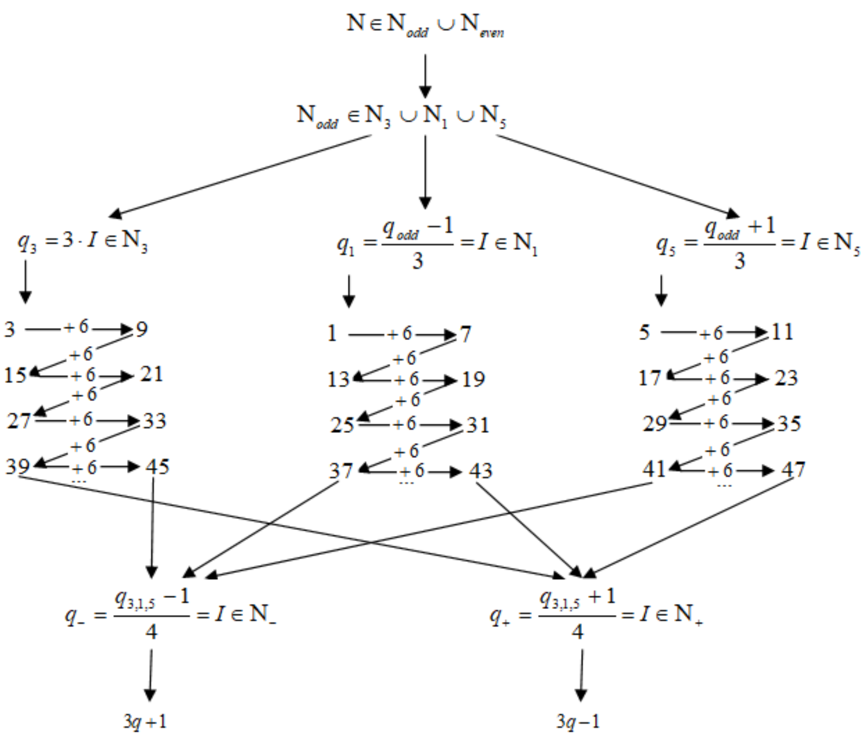

Fig.4. Block diagram of the structuring of the initial set N according to Jakobstal-Kollatz processes developing in opposite directions.

On sequences $\{\theta \cdot 2^n\}_{n=0}^{\infty}$ nodes $m(p)_{\theta,n}$ are formed periodically in powers with the period $T_n = 2^2$. Therefore, in the direction $n \to 0$ the values $m(p)_{\theta,n}$ are decreasing. For numbers $m(p)_{\theta,n}$ for which the relations are true

$$\frac{m_{\theta,r(s)}-1}{4}=I_m \text{ for (a) and } \frac{p_{\theta,s(r)}+1}{4}=I_p \text{ for (b),} \tag{16}$$

after three iterations ($N_{\min}=3$) inequalities (17) are true.

$$m_{\theta,r(s)}<q, \text{ where } q=m_{\theta,r(s)}-I_m \text{ and } p_{\theta,s(r)}<q, \text{ where } q=p_{\theta,s(r)}-I_p \tag{17}$$

Example:

- for $m_{\theta,n}=27$ comes true $\frac{27-1}{4}\neq Integer_m$, so $N_{\min}>3$.
- for $m_{\theta,n}=113$, comes true $\frac{113-1}{4}=28$, so after $N_{\min}=3$ iterations $q=113-28=85$ and the inequality (16a) come true: 85<113 (113→340→170→85→….).
- for $p_{\theta,n}=27$, comes true $\frac{27+1}{4}=7$, so after three iterations $q=27-7=20$ and the inequality (16a) come true: 20<27 (27→80→40→20→…).

Conclusions

Table 1 illustrates the correlation between relations (16) and inequalities (17), which is reflected by the values of the so-called complete stop time of the Collatz transformation (st_{Ter} ,*tst*) for Jacobstal numbers, where $N_{\min}=st_{Ter}$. For numbers $m(p)_{\theta,n}$ with exponents n>4 , the minimum number of iterations of the implementation of inequalities (17) is stably equal to $N_{\min}=3$, if one of the conditions is met (3.4), or $N_{\min}=8$ in the other case. When $n\leq 4$, as follows from table 2.1, the value $N_{\min}$ for numbers $p_{\theta,n}$ is changing.

Table 1

Calculated numbers $J^{\pm}_{\theta,n}$, $m(p)_{\theta,n}$ ***and*** st_{Ter} ***for*** $\theta_1=1$

θ/n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	$J^{\pm}_{\theta,n}$,	(16a)
1	0	1	1	3(6,3)	5/3	11(8,3)	21(3	43(8,3)	85(3,8)	171(8,3)	$J^{-}_{1,n}$,	+ m1,n - p1,n

From the point of view of the Collatz conjecture, inequalities (17) are known as Terrace's inequality [15,16]

$$q_{odd},\ N=0>q_N, \tag{18}$$

which shows that through N iterations, the intermediate value of the number q_N is less than the initial value. This number of iterations st_{Ter} is called the Terrace stopping time $N_{\min}=st_{Ter}$, and its minimum value is equal to $st_{Ter}=3$ for odd numbers for which one of the conditions is met (16), else $st_{Ter}>3$, as shown in Table 2.1 for $m(p)_{\theta,n}(st_{Ter,3.4a}, st_{Ter,3.4b})$ for conversion C^{+}_{q}.

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ПРО МАТЕМАТИЧНУ МОДЕЛЬ ПЕРЕТВОРЕННЯ НАТУРАЛЬНИХ ЧИСЕЛ ФУНКЦІЄЮ РОЗДІЛЕНОГО ТИПУ

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Анотація. У цій роботі обґрунтована некоректність алгоритму, запропонованого в публікації "M. Remer.[A Comparative Analysis of the New -3(-n) - 1 Remer Conjecture and a Proof of the 3n + 1 Collatz Conjecture. Journal of Applied Mathematics and Physics. Vol.11 No.8, August 2023"] в термінах гіпотези Коллатца. А також те, що перетворення -3(-n) - 1 не еквівалентне гіпотезі Коллатца про натуральні числа 3n + 1. Отримані результати можуть бути використані в подальших дослідженнях.

Ключові слова: рекурентна послідовність, числа Якобсталя, гіпотеза Коллатца, натуральні числа, гіпотеза