SUBSTANTIATION OF THE RESULTS OF THE LASER LOCATION OF THE TRAJECTORY OF THE MOON MOVING AWAY FROM THE EARTH

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Abstract. Progress in astronomical measurements of the trajectories of the movement of celestial bodies reveals new effects that require justification. In particular, it refers to the slight drift of the Moon from the Earth. A solution to this problem is possible only based on an adequate mathematical model. To perform this, it was adapted Newton's law of universal gravitation to the case of moving masses in flat space and physical time. At the same time, the final speed of propagation of the gravitational field can be considered. The differential equations of motion of the three-mass Sun-Earth-Moon cosmic system are obtained. In the heliocentric coordinate system, transient processes of the movement of the named celestial bodies are simulated, which testify to the presence of the Moon moving away from the Earth and the influence of the effects of movement on its course. The significant dependence of the final result on numerical methods of integration, computer calculation tools, and the values of space-velocity initial conditions is shown. The results of the simulation of transient processes are attached.

Key words: Drift of the Moon from the Earth, Euclidean space, Newton's law of gravitation of moving masses, differential equations of motion of the cosmic three-mass system Sun-Earth-Moon.

1. Introduction

The current article is a direct addition to the works published on the pages of this journal [1-2], devoted to the refutation of the anomaly of the movement of the Pioneer spacecraft in the gravitational field of the Sun and the gravitational maneuver of spacecraft in the gravitational field of large celestial bodies [3-4]. A similar solution to the problem of theoretical substantiation of the measured data of the laser location of the trajectory of the Moon's distance from the Earth is proposed here.

The moon is the only natural satellite of the planet Earth. It is the second brightest object in the Earth's sky after the Sun. The first and only extraterrestrial body visited by man. The average distance between the centers of the Earth and the Moon is 384,400 km. Since ancient times, people have tried to describe and explain the movement of the moon with increasingly accurate models. The basis of modern calculations is Brown's theory. It was created at the turn of the 19th and 20th centuries and explained the movement of the moon with the precision of the measuring instruments of that time. At the same time, more than 1,400 factors were used while calculating. In modern science, expressions with tens of thousands of terms are applied to compute them with the accuracy of laser location measurements. There is no limit to the number of terms in the expression if even greater accuracy is required. Such a solution to the problem is not based on a physical theory but on a mathematical approximation of the results of measu-rements.

In the first approximation, it can be assumed that the Moon moves in an elliptical orbit with an eccentricity of 0.0549. The movement is quite complex and for its calculation, it is necessary to consider many factors, at least the influence of the Sun, which attracts the Moon 2.2 times more than the Earth. Such a movement can at least be considered as a combination of several movements [5]:

– rotation around the Earth in an elliptical orbit with a period of 27.32166 days (sidereal month);

– periodic change in the inclination of the lunar orbit to the Earth's ecliptic in the range from 4°59' to 5°19';- periodic changes in the dimensions of the lunar orbit: perigee from 356.41 to 369.96, apogee from 404.18 to 406.74 (thousand km).

– the gradual distance of the Moon from the Earth by 4 cm per year.

The Moon is always turned to the Earth on one side, which is a consequence of tidal blocking, due to which the independent rotation of the Moon around its axis stopped a long time ago! The rotation of the Moon observed in the heliocentric coordinate system is not its independent rotation, but occurs exclusively due to its orbit around the Earth. The physical libration of the Moon is caused by its oscillation around the equilibrium position due to the shifted center of gravity. The Moon does not revolve around the Earth, but both bodies revolve around a common center of inertia located inside the Earth at a distance of 4700 km from its center. The libration has a value of 0.02° in longitude with a period of 1 year and 0.04° in latitude with a period of 6 years.

The Moon moves around the Earth at an average speed of 1.02 km/s in the same direction as the vast majority of other bodies in the Solar System. The semi-major axis of the orbit is 384,400 km. Due to the ellipticity of the orbit and disturbances, the distance to the Moon varies between 356,400 and 406,800 km. Studying the movement of the Moon around the Earth is one of the most difficult tasks of celestial mechanics. Therefore, it is worth considering the movement of the Moon around the Sun and the perturbation of this movement by the Earth, which is proposed in this study.

2. Drawbacks and Problems

It is quite clear that the given task is a task of high precision. The logical path to its solution is related to the integration of nonlinear differential equations of motion by numerical methods, which in turn is related to the predetermined accuracy of integration and the capabilities of available computer technology. But in this work, emphasis is placed on the other side of the problem, which none of the researchers has paid attention to yet - on the influence of the effect of the movement of the involved moving celestial bodies, associated with the finite speed of physical interaction.

3. Goal

To develop on a strictly mathematical basis nonlinear differential equations of motion of the three-mass celestial system Sun-Earth-Moon in heliocentric coordinates considering the finite speed of propagation of the gravitational field and to obtain with the help of dynamic simulation mathematical criteria for substantiating the results of measurements of new effects that appear in the trajectory of the Moon's movement.

4. Studies, Results and Explanation

Consider the transient process of interaction of three cosmic masses: the Sun, the Earth, and the Moon. If the logical assumption is made that the neighboring heavenly bodies do not affect the movement of the considered bodies, which is shown in [6], the balance of inertial forces and gravitational forces in the heliocentric coordinates of the stationary Sun can be written in the form of four vector differential equations:

$$\frac{d\mathbf{v}_2}{dt} = \frac{1}{m_2}(\mathbf{F}_{21} + \mathbf{F}_{23}); \quad \frac{d\mathbf{r}_2}{dt} = \mathbf{v}_2;$$
$$\frac{d\mathbf{v}_3}{dt} = \frac{1}{m_3}(\mathbf{F}_{31} + \mathbf{F}_{32}); \quad \frac{d\mathbf{r}_3}{dt} = \mathbf{v}_3, \qquad (1)$$

where $\mathbf{F}_{21}, \mathbf{F}_{23}$ are the gravitational forces acting on the Earth from the side of the Sun and the Moon; $\mathbf{F}_{31}, \mathbf{F}_{32}$ are gravitational forces acting on the Moon from the side of the Sun and the Earth; $\mathbf{r}_2$ is the radius vector directed from the Sun to the Earth; $\mathbf{r}_3$ is radius vector directed from the Sun to the Moon; $\mathbf{v}_2$ is Earth velocity vector; $\mathbf{v}_3$ is moon velocity vector; t is time. The vectors of the distance between celestial bodies m_2 and m_3 and their mutual velocities are found by the results of integration (1) as the difference:

$$\mathbf{v}_{23} = \mathbf{v}_2 - \mathbf{v}_3; \quad \mathbf{r}_{23} = \mathbf{r}_2 - \mathbf{r}_3. \qquad (2)$$

The vector of gravity between two interacting moving celestial bodies is written in the general form [6]

$$\mathbf{F}_{ik} = G\frac{m_i m_k}{r_{ik}^2}\mathbf{d}_{ik}, \quad \mathbf{d}_{ik} = \left(1 + \frac{v_{ik}^2}{c^2} + 2\frac{v_{ik}}{c}\mathbf{r}_{ik0}\cdot\mathbf{v}_{ik0}\right)\mathbf{r}_{ik0}, \qquad (3)$$

where r_{ik} is the radius of the distance between the masses m_i and m_k; v_{ik} is mutual instantaneous movement speed; G is gravity constant; $\mathbf{r}_{ik0}, \mathbf{v}_{ik0}$ are unit vectors of distance and movement speed; $\mathbf{d}_{ik}$ is speed coefficient [6].

The first term in (4) represents the static Newtonian interaction. The second tax is due to the tangential component of speed. In an electric field, it completely coincides with the Lorentz force, which in classical electrodynamics represents the force effect of a magnetic field or the so-called relativistic effect in an electric field. Being extended to mechanical interaction, this term presents the corresponding gravitomagnetic force [7-9]. The functional dependence on the speed of movement of the third component (3) is higher than that of the second, because under the condition $v \le c$ the multiplier v/c in the second is raised to the second power, and in the third - to the first. It is the third component that closes the hitherto unknown triune essence of gravitational forces and makes it possible to solve the given problem on a strict mathematical basis.

In addition, equation (3) needs an explanation, since in the general case it may be about sublight velocities. Functional dependence $m = m(v)$ s one of the unfortunate misunderstandings of STV, not a mathematical one, but an incorrect physical interpretation. The fact is that the Lorentz coefficient refers to the force interaction of masses, not the masses themselves! This is crystallized in the process of taking into account the finite velocity of gravity propagation.

Differential equations (1) in Cartesian heliocentric coordinates take the form:

$$\frac{dv_{2i}}{dt} = \frac{1}{m_2}(F_{21i} + F_{23i}); \quad \frac{dr_{2i}}{dt} = v_{2i};$$
$$\frac{dv_i}{dt} = \frac{1}{m_3}(F_{31i} + F_{32i}); \quad \frac{dr_{3i}}{dt} = v_{3i}, \qquad (4)$$
$$i = x, y, z.$$

We write the projections of gravitational forces according to (3):

$$F_{21i}=-\frac{Gm_1m_2r_{21i}}{r_{21}^3}\times\left(1+\frac{v_{2i}^2}{c^2}+2\frac{r_{21x}v_{2x}+r_{21y}v_{2y}+r_{21z}v_{2z}}{cr_{21}^2}\right);$$

$$F_{31i}=-\frac{Gm_1m_3r_{31i}}{r_{31}^3}\times\left(1+\frac{v_{3i}^2}{c^2}+2\frac{r_{31x}v_{3x}+r_{31y}v_{3y}+r_{31z}v_{3z}}{cr_{31}^2}\right);\quad i=x,y,z. \tag{5}$$

$$F_{23i}=-\frac{Gm_2m_3r_{23i}}{r_{23}^3}\times\left(1+\frac{v_{23i}^2}{c^2}+2\frac{r_{23x}v_{23x}+r_{23y}v_{23y}+r_{23z}v_{23z}}{cr_{23}^2}\right);$$

where

$$r_{23i}=r_{21i}-r_{31i};\quad v_{23i}=v_{21i}-v_{31i},\quad i=x,y,z. \tag{6}$$

$$r_i=\sqrt{r_{ix}^2+r_{iy}^2+r_{iz}^2},\quad i=21,31,23;$$
$$v_i=\sqrt{v_{ix}^2+v_{iy}^2+v_{iz}^2},\quad i=2,3,23. \tag{7}$$

Expressions (4)–(7) form a complete system of algebraic differential equations for the analysis of transient processes in the Sun-Earth-Moon cosmic system in physical 3D space. To obtain the desired unique solution, it is necessary to set constant parameters G, m_1, m_2 and space-velocity initial conditions:

$$r_{2i}(0), r_{3i}(0);\quad v_{2i}(0), v_{3i}(0),\quad i=x,y. \tag{8}$$

As for the mass m_3 of the Moon, according to (5), (6) it does not take part in the calculations, because it is shorting. Which is consistent with the principle of equivalence [9].

Simplified practical analysis

Since this work does not deal with high-precision calculations, we allow ourselves to simplify the further practical analysis by accepting two unprincipled assumptions from the point of view of the problem to be solved:

1. Let's narrow down the problem to 2D space, considering the process in the ecliptic plane of the Earth;
2. We assume that the mass of the Moon (m_3) does not affect the movement of the Earth (m_2), although in fact both celestial bodies revolve around a common center, which is at a tiny cosmic distance from the center of the Earth.

Assuming the planet's orbit is close to circular, the mechanical characteristics of its motion can be made dependent on the angle of the orbital motion:

$$\alpha=\alpha_0-\omega t, \tag{9}$$

where ω is the orbital angular velocity of the planet, α_0 is the initial value α, as

$$r_{2x}=r_{12}\cos\alpha;\quad r_{2y}=r_{12}\sin\alpha;$$
$$v_{2x}=\omega r_{12}\cos(\alpha+\pi/2); \tag{10}$$
$$v_{2y}=\omega r_{12}\sin(\alpha+\pi/2).$$

The second assumption (9), (10) makes it possible to halve the system of differential equations (4)−(7)

$$\frac{dv_{3x}}{dt}=\frac{1}{m_3}(F_{31x}+F_{32x});\quad \frac{dr_{3x}}{dt}=v_{3x};$$
$$\frac{dv_{3y}}{dt}=\frac{1}{m_3}(F_{31y}+F_{32y});\quad \frac{dr_{3y}}{dt}=v_{3y}, \tag{11}$$

provided there are no z-th components. To obtain the required one-to-one solution of differential equations (11), it is necessary to set constant parameters $G, m_1, m_2, \omega, r_{12}$ and space-velocity initial conditions:

$$\alpha_0(0),\ r_{3i}(0),\ v_{3i}(0),\quad i=x,y. \tag{12}$$

Simulation results

The results of the compatible solving of (5)-(11) by the numerical method are shown in Fig. 1 - fig. 7 with constant parameters:

Gm_1=13.274935144·10^{19},

Gm_2=39.8574405096·10^{13}, ,

m_3=7.3477·10^{22}, ω=0.19910638·10^{6}

$r_{12}=1.495978707\times10^{11}$, corresponding to the Sun, the Earth, and the Moon. All dimensions in the simulation results are in SI. Our numerical results are only close to being practically justified. Because we just want to confirm by means of computer simulation the presence of certain gravitational effects and the workability of the theoretical results of mathematical support. The calculated data were obtained under spatial-velocity initial conditions:

$$\alpha(0)=\pi/2;\ v_{3x}(0)=30810.22;$$
$$v_{3y}(0)=0;\ r_{3x}(0)=0; \tag{13}$$
$$r_{3y}(0)=1.499822727\cdot10^{11}.$$

Fig. 1 shows the time dependence of the hodograph of the spatial radius-vector of the distance of the Moon from the Sun $r_3(t)$, in heliocentric Cartesian coordinates. The duration of the transition process corresponds to one sidereal (stellar) month $(t=2360448\text{ s})$.

Fig. 2 shows the same trajectory as in fig. 1, only in geocentric coordinates.

Fig. 3 shows the same trajectory as in fig. 2, but the duration of the transition process is increased four times $(t=9441792\text{ s})$. This was done in order to theoretically confirm the existence of a physical effect of the Moon moving away from the Earth, caused precisely by the forces of gravity due to the real masses of all three involved celestial bodies and their space-velocity initial conditions. The nature of the shown trajectory clearly tends to expand over time. While, as shown in [6,10], solar planets-satellites of the terrestrial type have the opposite property - to approach the Sun with the passage of time. Thus, our solar system is still far from a stationary state. It is in a transitional state. This clearly confirms the discontinuity of the trajectories of the planets and their satellites!

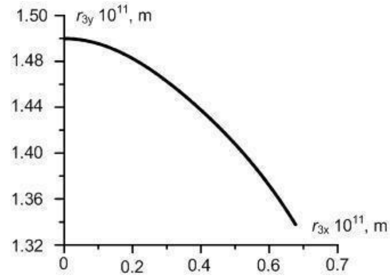


Fig. 1. One-month sidereal trajectory of the Moon $r_3(t)$ in heliocentric coordinates

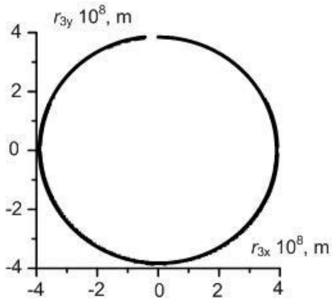


Fig. 2. One-month sidereal trajectory of the Moon $r_3(t)$ in geocentric coordinates

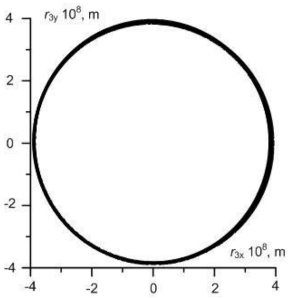


Fig. 3. Four-month $(t = 9441792\ \text{s})$ trajectory of the Moon's $r_3(t)$ movement in geocentric coordinates

Fluctuations of the lunar trajectory are shown in Fig. 4-5. On the first of them, we can see the space-time dependences of the fluctuation of the distance of the Moon from the Sun, and on the second - the speed-time dependences of the fluctuation of the linear orbital velocity in heliocentric coordinates during two sidereal months $(t = 4720896\ \text{s})$. As you can see, the spatial fluctuations are within the diameter of the Moon's circular orbit around the Earth, and the high-speed ones are again related to the Earth's orbit, provided that the angular velocity of the Earth's orbit around the Sun is constant. It is clear that we are talking about both idealized orbits around the Sun and around the Earth.

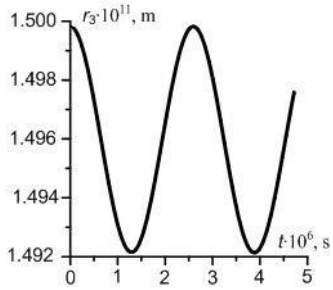


Fig. 4. Space-time dependence of the distance of the Moon from the Sun $r_{32}(t)$ in heliocentric coordinates

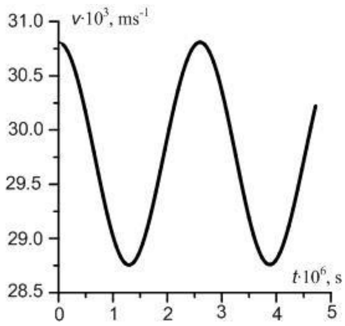
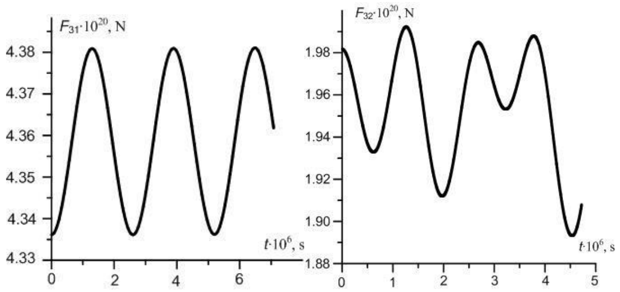


Fig. 5. Time dependence of the transient speed of the Moon $v_3(t)$ in heliocentric coordinates

Fig. 6 shows the force characteristics of the transition process depicted in Fig. 4 and Fig. 5 in heliocentric coordinates, but not on the time interval of two sidereal months, but three $(t = 7081344\ \text{s})$.

Finally, we came to the main thing - identifying the influence of the speed effects of the movement of the planet and its satellite on the trajectory of the latter. At first glance, it would seem that there is a lot to talk about, because we are talking about the tiny orbital linear velocities of the Earth around the Sun (29785.13 ms^{-1}) and the Moon around the Earth (1023.16 ms^{-1}), which are far, far from sublight (relativistic). But let's not forget that we are talking about a much lower speed, namely, the distance of the Moon from the Earth (0.04 m year^{-1}, or $0.13 \cdot 10^{-8}$ ms^{-1}). Figure 7 shows the time dependences of the speed coefficients (3) of the trajectory of the Moon around the Sun ($d_{31}(t)$) and the Moon around the

Earth ($d_{32}(t)$). As we can see from the course of bothcurves, they affect, respectively, the sixth and fourth decimal places against the tenth decimal place of



the annual drift distance of the Moon from the Earth. Which is not so little!

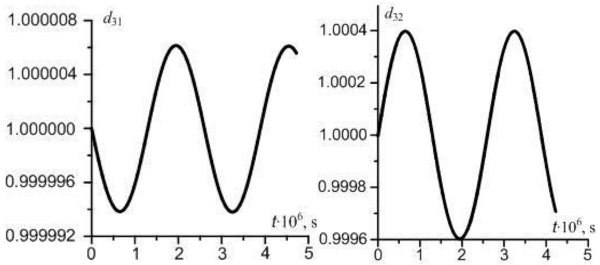


Fig. 6. Strength characteristics of the gravitational fields of the Sun and the Moon (left), the Earth and the Moon (right) in the transitional process shown in Fig. 4-5

Fig. 7. Time dependences of the speed coefficients of the Moon's movement around the Sun ($d_{31}(t)$) (left) and around

the Earth ($d_{32}(t)$) (right) in the transition process shown in Fig. 4 and fig. 5

We strictly mathematically confirmed the pre-sence of the gravitational phenomenon of the Moon moving away from the Earth in the gravitational field of the massive celestial bodies of the Sun, the Earth, and the Moon. Moreover, in previous studies we have shown that in the group of terrestrial solar planets [1] it is observed that on the contrary, the planets are approaching their star. And is it possible to get a reliable result in space practice? - You can. But at the same time, it is necessary to know not only the exact initial space- velocity conditions and constant parameters of the space system, but also to use sophisticated computer systems that would ensure the necessary accuracy of the simulation results.

5. Conclusions

The presented results of calculations of model transient processes of the "Sun-Earth-Moon" gravi-tational system illustrate the presence of the Moon mo-ving away from the Earth, detected by the laser location method, and the influence of motion effects on its co-urse. The phenomenon that we studied earlier[6,10] is the opposite in the movement of the solar planets of the terrestrial group that is the drift of the planets towards the star. This convinces that the solar system is still in a transitional state and is far from stable.

Despite the relatively small cosmic speeds of the studied celestial bodies, in the tasks of an in-depth analysis of their motion, neglecting speed effects is unacceptable.

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