

BIOMEDICAL MEASUREMENTS AND DEVICES

STUDY OF THE EFFECTIVE RADII OF ULTRASONIC TRANSDUCERS

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Abstract. The study on the effective radii of ultrasonic transducers for hydrophone calibration at the National State Primary Standard of the Unit of Ultrasonic Pressure in Water (NDETU AUV-02-2018) is presented and the appropriate method is described. The impact of the effective radius on measurement distance and diffraction loss coefficients is evaluated. The uncertainty calculation of the effective radius measurement of ultrasonic transducers is provided, and its influence on the accuracy of hydrophone calibration is assessed. Considerable attention is given to estimating the measurement distance between the hydrophone and the ultrasonic transducer, which significantly affects the accuracy of hydrophone calibration.

Key words: Ultrasonic Pressure, Hydrophone, Ultrasonic Transducer, Effective Radius, Uncertainty

1. Introduction

Ultrasonic transducers (UTs) are crucial components in ultrasonic measurement systems, as they both generate and receive ultrasonic waves. These transducers convert high-frequency electrical energy into mechanical energy. There are several basic types of ultrasonic transducers, classified according to their principle of operation and design: piezoelectric, capacitive, and magnetoelastic. Various types of UTs are employed in different applications based on specific requirements and operating conditions. They are applied in fields such as medicine (medical imaging [1,2], ultrasonic therapy [3]), distance measurement (in air) [4], sonar [5], non-destructive testing [6,7], flow monitoring [8,9], etc. Ultrasonic transducers are required to reproduce the unit of ultrasonic pressure, Pascal (Pa), via the two-transducer reciprocity calibration method, which forms the basis of the National State Primary Standard of the Ultrasonic Pressure Unit in Water (NDETU AUV-02-2018) [10].

MANA Instruments E1025-SU, E2312-SU, E3512-, and E9906-SU auxiliary transducers were selected for hydrophone calibration on the NDETU AUV-02-2018 standard. These transducers provide the required levels of ultrasonic pressure in water in the frequency range from 1 MHz to 10 MHz and ensure a signal-to-noise ratio of at least 20 dB during hydrophone calibration. The frequencies at which the transducers are applicable, along with the geometric radii according to the manufacturer's technical specifications, are given in Table 1.

According to the calibration method, the hydrophone transfer function or sensitivity $M_{\square}(f)$, is defined as:

$$M_{\square}(f) = \frac{U_2(f)}{P_0(f) \cdot G_2} = \frac{U_2(f)}{G_2} \cdot \frac{1}{\frac{k \cdot G_1 \cdot e^{-\alpha d}}{U_1(f) \cdot K_R(f)} \cdot \frac{2 \cdot A_{ER}}{I_1(f) \cdot \rho \cdot c}} \quad (1)$$

where $U_2(f)$ is the voltage at the hydrophone, V; $U_1(f)$ is the voltage for the auxiliary transducer in receiving mode, V; $I_1(f)$ is the excitation current through the auxiliary transducer, A; ρ is the density of the measurement liquid (water), kg/m³; c is the speed of sound in water, m/s, $K_R f$ is the correction factor for power amplifier input impedance; α , is the attenuation coefficient of sound wave in water, Hz⁻²·m⁻¹; k is the reflection coefficient of the reflector; G_2 is the correction factor for diffraction loss with the auxiliary transducer and hydrophone; G_1 is the correction factor for diffraction loss with the auxiliary transducer alone; A_{ER} is the effective area of the UT, m²; d is the measurement distance, m.

The measurement distance d is the total length of the sound path from the transducer to the hydrophone (or from the transducer back to the transducer via the reflector) during the hydrophone calibration procedures. It is an important parameter for determination of the hydrophone transfer function $M_{\square}(f)$; it can be lie between 1 and 3 times the near-field distance N for the particular auxiliary transducer [11]. The near-field distance, N , in meters, for the auxiliary UT is calculated as:

$$N = \frac{a_{tr}^2}{\lambda} = \frac{f \cdot a_{tr}^2}{c} \quad (2)$$

where λ is the ultrasonic wavelength in water, m; a_{tr} is the effective radius of the auxiliary transducer at the measuring frequency, m; c is the speed of sound in water, m/s, f is the signal frequency, Hz.

Table 1. Auxiliary ultrasonic transducers applied in the standard NDETU AUV-02-2018

UT type	Frequency, MHz	Geometric radius, mm
E1025-SU	1.0	12.5
E2312-SU	2.0	6.0
E3512-SU	3.0	6.0
	4.0	6.0
E9906-SU	5.0	3.0
	6.0	3.0
	7.0	3.0
	8.0	3.0
	9.0	3.0
	10.0	3.0

The near-field distance depends on the frequency of the emitted ultrasonic signal and the effective radius of the auxiliary transducer. In addition, the measurement distance d needs to be sufficient to avoid the influence of the near-field, which has a complicated interference structure. Therefore, a hydrophone and a transducer must be calibrated in the free field, i.e., in a sound field where the influence of reflected signals is negligible.

According to the definition [11], the effective radius of a non-focusing ultrasonic transducer is the radius of an equivalent piston-like source for which the spatial distribution of the ultrasonic pressure amplitude in the far field is close to the distribution of the ultrasonic pressure amplitude from the transducer itself. The manufacturer of the UT specifies only geometric radius in the technical specification. If the effective radius of the transducer is not available, it is recommended to replace the effective radius with the geometric radius of the UT [11]. Analyzing the impact of the effective radius a_{tr} on the near-field length N according to [12], from equation (2), and taking the partial derivative ∂N of the input parameter ∂a_{tr} , we obtain a sensitivity coefficient of 2. Therefore, if the effective radius of the UT differs from the geometric radius by, e.g., 5%, this discrepancy causes a 10% change in distance N .

2. Drawbacks

Substituting the geometric radius of the transducer instead of the effective radius affects the calculated value of the near-field length N . This may significantly impact the computed values of the measurement distance and diffraction loss coefficients. Since N , G_1 , and G_2 are parameters of the mathematical model for determining hydrophone sensitivity (via the two-transducer reciprocity calibration method), replacing the geometric radius with the effective radius affects the accuracy of the hydrophone calibration.

3. Goal

To evaluate the impact on the calculation of measurement distance and diffraction loss coefficients of substituting the effective radius of the ultrasonic transducer instead of the geometric radius, as well as to determine the values of the effective radii of auxiliary ultrasonic transducers and evaluate the measurement uncertainty.

4. Determination of the effective radius of the ultrasonic transducer

The effective radius, a_{tr} , of the auxiliary transducer is determined from the distance to the last minimum of the ultrasonic field of the transducer, d_{min} , obtained from the plot of the ultrasonic pressure amplitude versus distance along the acoustic axis of the UT. During the effective radius measurement, the UT can be excited by a continuous sinusoidal or a tone burst signal. If a tone burst signal is applied, it is essential to ensure that the duration of the first one is sufficient to establish steady-state conditions in the ultrasonic field and to allow accurate measurement of the RMS voltage at the hydrophone output with an oscilloscope.

The theoretical pressure distribution for a piston-like UT can be calculated from the equation [11]:

$$\frac{P_i}{P_0} = 2 \cdot \sin \frac{\pi}{\lambda} (d_i^2 + a_{tr}^2 - d_i) \cdot e^{-\alpha \cdot d_i}, \quad (3)$$

where P_0 is the ultrasonic pressure at the radiating surface of the transducer; P_i is the ultrasonic pressure at a distance d_i ; λ is the ultrasonic wavelength in water; α is the attenuation coefficient of the sound wave in water.

Fig. 1 shows the pressure distribution of the ultrasonic transducer calculated according to (3). This allows us to make a preliminary estimate of the distance to the last minimum of ultrasonic pressure generated by the UT and to optimize the number of points (steps) for linear scanning to make this process more time-efficient.

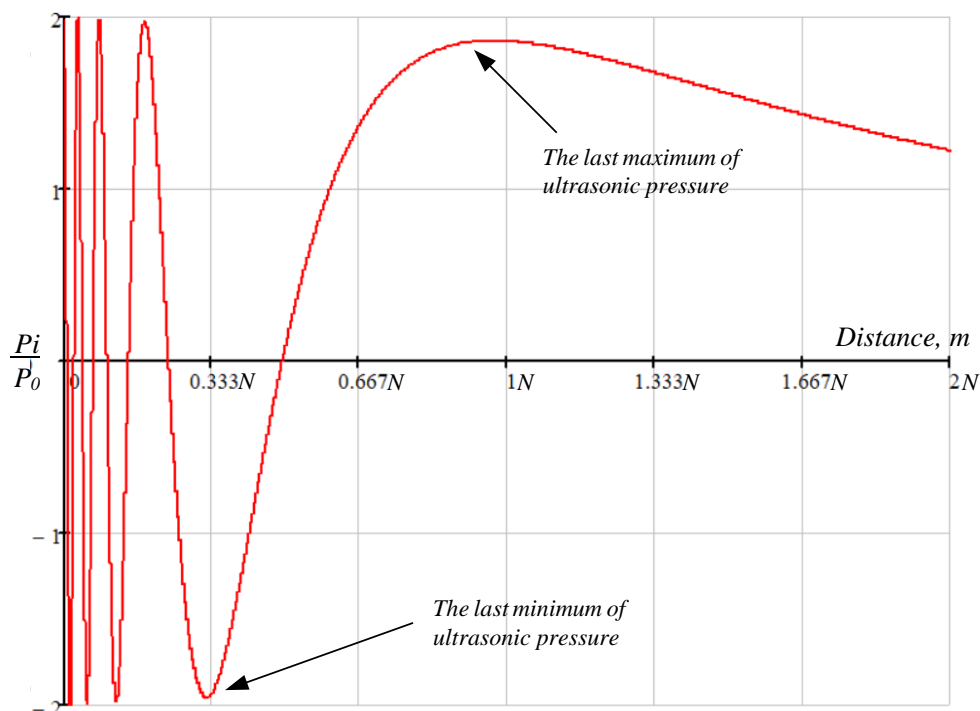


Fig. 1. Theoretical pressure distribution in the ultrasonic field of the transducer along the Z axis.

To determine the distance to the last minimum d_{min} of the transducer ultrasonic field, the raster scanning system of the standard NDETU AUV-02-2018 was implemented [14]. It consists of the following components:

- hydrophone positioning system (HPS);
- measuring equipment: tone burst generator, power amplifier, oscilloscope, hydrophone;
- the water tank of sufficient size for acoustic measurements in the frequency range from 1 MHz to 10 MHz (985 mm × 385 mm × 490 mm, volume 150 l);
- a personal computer with “Acoustic etalon” software.

The distance to the last minimum was determined by linear scanning of the UT ultrasonic field with a 0.5 mm diameter needle hydrophone in discrete steps along the symmetry axis (Z) of the UT ultrasonic beam (Fig. 2), according to the following procedure:

- before the measurement, the UT and the hydrophone were aligned with the HPS so that the geometric center of the UT and the hydrophone were coaxial;
- N , in meter was calculated based on the UT geometric radius according to (2);
- a hydrophone was placed along the axis (Z) at a distance $d = N$ from the UT;
- the acoustic axes of the hydrophone and the transducer were aligned with the HPS by detecting the

maximum voltage level from the hydrophone on the oscilloscope;

- automatic linear scanning was performed by moving the hydrophone from point N towards the transducer surface with a discrete step, and measuring the voltage value on the hydrophone with an oscilloscope; the step is dependent on the excitation frequency of the transducer, e.g., for a frequency of 1 MHz, the step is 1.0 mm, and for a frequency of 10 MHz – 0.2 mm;

- the distance at which the lowest voltage value was recorded on the hydrophone is equal to d_{min} .

The “Acoustic Etalon” software automatically performs a linear scanning by moving the hydrophone along the Z -axis at specified step. At each scan point, the software records the corresponding voltage values at the hydrophone output with the TDS2024 oscilloscope connected to the software via the USB interface (see Fig. 3). The step value (mm) is entered in the “Step” field and the required number of steps along the scan axis is entered in the “Z” field. The time delay (s) between steps is entered in the “Time” field, which is necessary to stabilize the ultrasonic field after moving the hydrophone and reading the RMS voltage on the oscilloscope. The results of measuring the distance d_i and the RMS voltage during the scan are recorded in a .txt file, from which the d_{min} and the effective radius of the transducer a_{tr} are calculated.

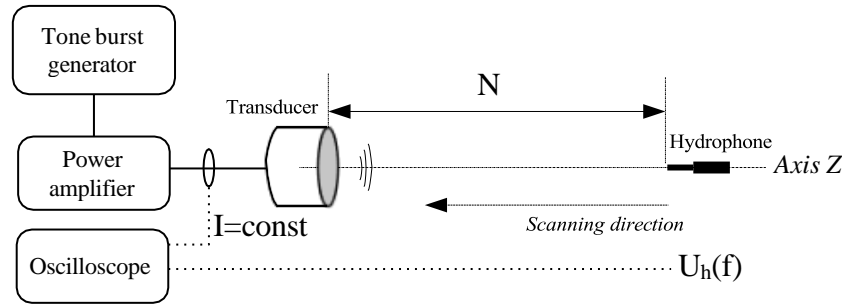
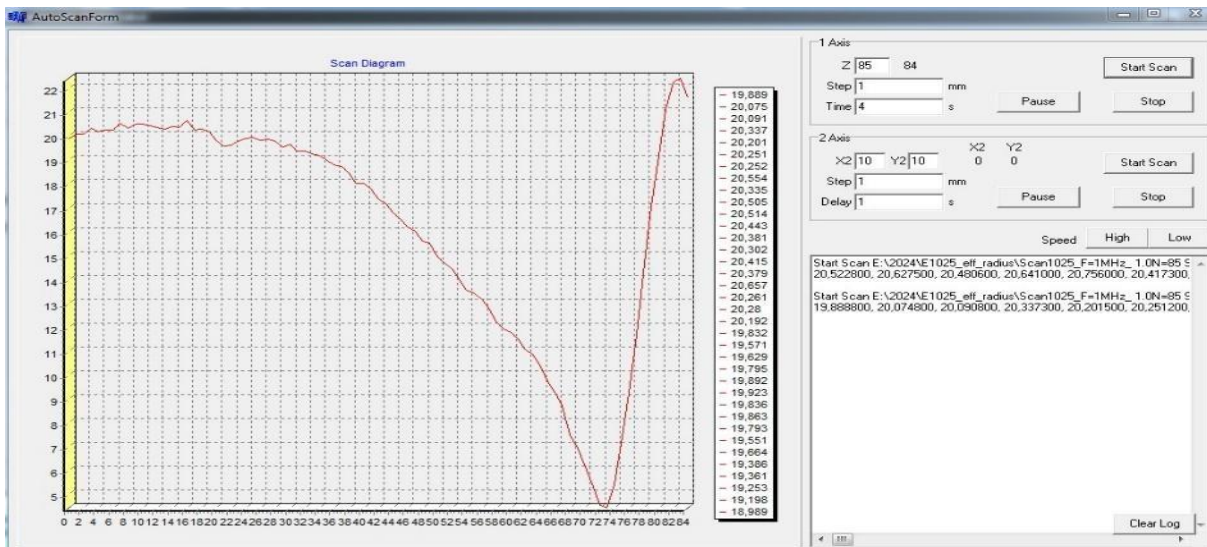

 Fig. 2. Functional scheme of linear scanning of the UT ultrasonic field for determination d_{min}


Fig. 3. "Acoustic etalon" software interface for linear scanning

After receiving the measured data and calculating d_{min} , the UT effective radius, a_{tr} , m, was determined according to the equation [15]:

$$a_{tr} = \sqrt{2\lambda d_{min} + \lambda^2}, \quad (4)$$

where d_{min} is the distance from the ultrasonic transducer surface to the last minimum of the pressure amplitude on the ultrasonic beam symmetry axis, m.

5. Results and discussion

The effective radius was measured for some auxiliary UTs at the frequencies from Table 1. An example of the results obtained when measuring the distance to the last minimum d_{min} for the E2312-SU (2 MHz) and the E9906-SU (8 MHz) is shown in Fig. 4.

The results of the auxiliary UT effective radius calculation, including the combined standard uncertainty and the calculated deviation of the effective radius from the geometric radius, are presented in Table 2. From the obtained results, the near-field length N and the diffraction loss coefficients G_1 , G_2 were calculated for both the effective and geometric radii of the UT. The

difference between the values of the near-field length N and the diffraction loss coefficients G_1 and G_2 obtained with the UT effective radius and the geometric radius is also shown in Table 2 (the diffraction loss coefficients G_1 and G_2 are calculated for the measuring distance $d = N$).

As we can see from Table 2, the difference between the geometric and effective radius of the E1025 is -21% , which may be evidence of its defect, significantly affecting the near-field length of the E1025 and the diffraction loss coefficient G_2 . According to [11], the E1025 needs to be replaced.

Furthermore, a comparison of the theoretically calculated and measured ultrasonic pressure distributions of ultrasonic transducers, e.g., type E3512-SU at an excitation frequency of 4 MHz, shows their significant difference. As we can see from Fig. 1, the theoretical distance to the last minimum of the UT ultrasonic field is $0.33N$ which does not agree with the measured distance ($0.5N$) shown in Fig. 5. These results confirm the importance of an experimental evaluation of the ultrasonic pressure distribution of auxiliary transducers.

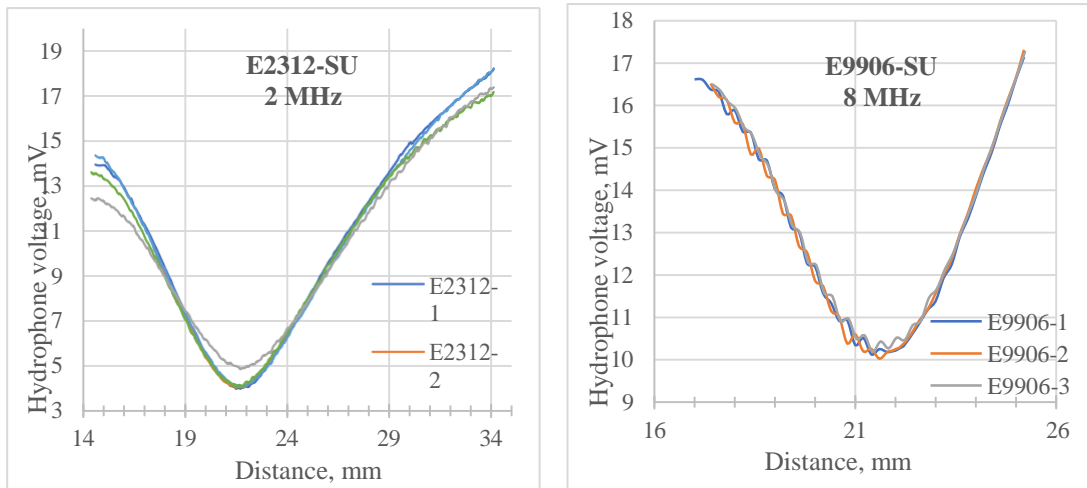


Fig. 4. The measurement results of the distance to the last minimum d_{min}

Table 2. The effective radii of the UTs.

UT type	E1025	E2312	E3512	E3512	E9906	E9906	E9906	E9906	E9906	E9906
Frequency, MHz	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Geometric radius, mm	12.5	6.0	6.0	6.0	3.0	3.0	3.0	3.0	3.0	3.0
Effective radius a_{tr} , mm	9.88	5.75	6.05	6.08	2.93	2.91	2.90	2.83	2.76	2.78
$u_{c(a_{tr})}$, %	0.88	1.73	0.83	0.71	0.91	1.05	0.84	0.91	0.47	0.78
Deviation of a_{tr} from the geometric radius, %	-21.0	-4.2	+0.8	+1.3	-2.3	-3.0	-3.3	-5.7	-8.0	-7.3
Deviation of the near field distance N , %	-37.5	-8.2	+1.7	+2.7	-4.6	-5.9	-6.6	-11.0	-15.4	-14.1
Deviation for the coefficient G_1 , %	-3.3	0.7	-0.1	-0.1	0.4	0.5	0.6	0.8	0.5	0.6
Deviation for the coefficient G_2 , %	69.9	0.9	0.0	0.1	0.5	0.7	0.9	2.3	4.6	4.6

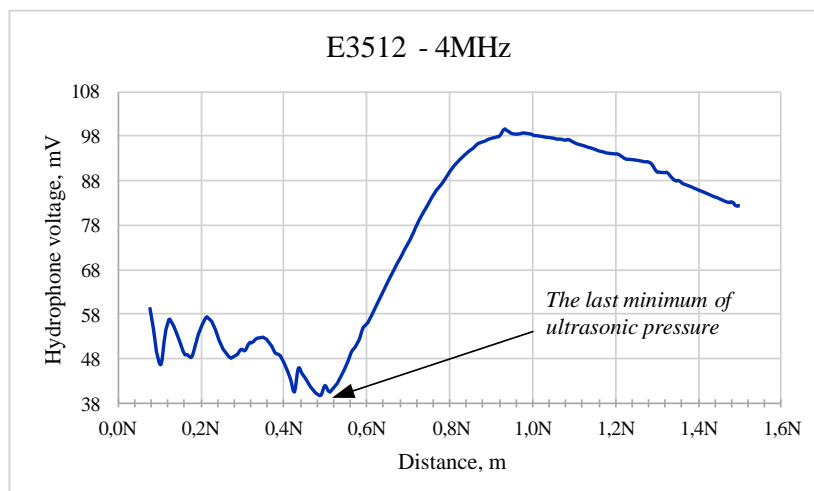


Fig. 5. Measured pressure distribution in the UT ultrasonic field along the Z axis

It was demonstrated in experimental studies that the distribution of the ultrasonic field is unique to each transducer. However, the ultrasonic field reaches the free field conditions at a distance greater than $1N$ (see Fig. 5) for all tested transducers, i.e., beyond the limit of the last ultrasonic pressure maximum of the transducer, where the pressure decrease with distance is close to the exponential law. We, therefore, consider it reasonable to measure the sensitivity of hydrophones with the auxiliary transducers listed in Table 1 at a measurement distance greater than $1.2N$.

6. Estimation of the measurement uncertainty of UT effective radius

The sensitivity coefficients related to the parameters d_{min} and λ for the mathematical model of the effective radius, a_{tr} , were calculated according to [12]. By applying the partial differentiation of equation (4) by the corresponding parameter, were obtained the following coefficients:

$$c_{dmin} = \frac{\partial a_{tr}}{\partial d_{min}} = 0.5; c_{\lambda} = \frac{\partial a_{tr}}{\partial \lambda} = 0.5 \quad (5)$$

The sources of uncertainty for the parameter d_{min} are:

- the scan step; e.g., 1.0 mm, refers to the type B uncertainty, which is estimated as half the value of the scan step with the rectangular distribution law, i.e., $u_{step B} = 1.0/2\sqrt{3}$ (mm);

- scan length along the Z-axis - according to the manufacturer of the linear guideway incorporated in the HPS for linear movement, the accumulated movement error over a distance of 10 mm is 0.0017 mm. Therefore, as an example, for a linear displacement of 100 mm, assuming a rectangular distribution law, the standard uncertainty is $u_{add_scan B} = 100.0 \cdot 0.0017/\sqrt{3}$ (mm).

- the RMS voltage at the hydrophone output is measured with an oscilloscope, however, while measuring the distance to the last minimum of the UT, the absolute value of the ultrasonic pressure is not required as we are only interested in its relative change. Therefore, the sensitivity of the hydrophone and the accuracy of the voltage measurement by the oscilloscope can be neglected, and the contribution from these sources of uncertainty can be accepted $u_{hydr B} = 0$.

The Type B standard uncertainty for linear scanning $u_{dmin B}$ was defined as:

$$u_{dmin B} = \sqrt{u_{step(B)}^2 + u_{add_scan(B)}^2 + u_{hydr(B)}^2} \quad (6)$$

The Type B standard uncertainty of the parameter λ , $u_{\lambda B}$, depends on the accuracy of the water temperature measurement. The water temperature was measured with a Hanna Check Temp1 instrument with an expanded uncertainty of 0.25% according to the calibration certificate with a normal distribution law, thus $u_{\lambda B} = 0.25/2$ (%).

Table 3. Uncertainty budget for the a_{tr} of the UT E2312 at the frequency of 2 MHz

Quantity	Estimation	Probability distribution	Standard uncertainty, u_i	Sensitivity coefficient, c_{xi}	Uncertainty contribution, %	$u_{c(a_{tr})}$, %
Last minimum, d_{min} , mm	21.95	normal	0.035	0.5	0.08	1.73
$u_{step B}$, mm	0.1	rectangular	0.029	-	-	
$u_{add_scan B}$, mm	0.0017	rectangular	0.020			
$u_{hydr B}$	0	normal	0			
Ultrasonic wavelength, λ , mm	0.741	normal	0.0009265	0.5	0.06	
$u_{\lambda B}$, %	0.250	normal	0.125	-	-	
$u_{a_{tr} A}$, %	-	normal	1.72	1	1.72	

The Type A standard uncertainty of the effective radius of the UT, $u_{a_{tr} A}$, was determined from a series of 10 independent observations according to [12]. For each measurement of d_{min} , a separate mounting and adjustment of the UT and hydrophone was performed to take into account all possible errors due to inaccurate positioning of the UT and hydrophone, mounting mechanisms, and operator. The combined standard uncertainty of the effective radius of the UT, $u_{c(a_{tr})}$, was calculated according to [12]:

$$u_{c a_{tr}} =$$

$$= \sqrt{u_{dmin B} \cdot c_{dmin}^2 + u_{\lambda B} \cdot c_{\lambda}^2 + u_{a_{tr}(A)}^2} \quad (7)$$

An example of the estimation of the uncertainty budget for a_{tr} of the E2312 at a frequency of 2 MHz (scanning step 0.1 mm, displacement along the Z-axis – 20 mm) is shown in Table 3.

7. Conclusions

1. The study of the effective radii of the auxiliary ultrasonic transducers employed in the measurement standard NDETU AUV-02-2018 showed that the transducers' effective radii differ significantly

from the transducers' geometric radii depending on the frequency and affect the accuracy of determining the sensitivity of needle hydrophones.

The studies have revealed that the free field length and diffraction coefficients obtained with the effective and geometric radii of the auxiliary transducers are distinct. These results confirm the significance of consideration of the effective radii of ultrasonic transducers to improve the hydrophone calibration accuracy.

The analysis of the ultrasonic field distribution of auxiliary ultrasonic transducers from the set of NDETU AUV-02-2018 showed that the hydrophones sensitivity measurement should be performed at a measurement distance greater than $1,2N$, where N is the near-field distance.

It is necessary to consider the estimated combined standard uncertainty of the transducer effective radius measurement as a component of the standard uncertainty of the measuring distance d when calculating the hydrophone sensitivity uncertainty since the measuring distance is directly determined by the transducer effective radius.

8. Gratitude

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9. Conflict of Interest

The authors state that there are no financial or other potential conflicts regarding this work.

10. References

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