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SIMULATION OF THE GEARS STRESS-STRAIN STATE OF ELBOW ORTHOSIS PLANETARY GEARBOX

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Abstract. Gear transmissions of various sizes are used in various fields of mechanical engineering, automotive, marine, and aerospace industries according to the needs. This paper presents the modeling and analysis of the modal and harmonic characteristics of a planetary gear transmission for an elbow orthosis. Orthoses are used during the rehabilitation of patients in the postoperative period or during the restoration of lost limb functions. When modeling the planetary gear, the connection elements are made of 45 steel and PLA polylactide. Using ANSYS Workbench, the planetary gear of the gearbox was analyzed for modal and harmonic characteristics at three different torque values: 1732 N-mm, 3464 N-mm, 5196 N-mm. The modal and harmonic analysis of the stress-strain state of the planetary gear is carried out, a comparative analysis is performed, and the vibration characteristics, including natural frequencies, mode shapes, and harmonic response, are discussed. Also, the stress-strain state of the sun-satellite contact problem made of steel 45 and PLA polylactide is calculated and the corresponding analytical calculation of equivalent contact stresses is performed according to Hertz’s theory.

Keywords: Gearbox, stress, strain, deformation, vibration, frequency, contact, ANSYS.

Introduction

Gearboxes play a pivotal role in diverse mechanical engineering applications, offering a range of sizes, shapes, and weights tailored to specific technical requirements and operational needs. In the realm of traumatology, stringent criteria govern the selection of mechanisms for patient rehabilitation, emphasizing qualities such as compactness, ease of use, reliability, and patient safety. Among these mechanisms, planetary gearboxes stand out for their compact design and reliable performance. With components like shafts and spur gears known for their high efficiency and gear ratios, planetary gearboxes present an optimal solution. Thus, the key focus in refining such gearboxes is to ensure they meet the standards of user-friendly operation and utmost patient safety.

Problem Statement

Gears often fail during operation for a variety of reasons, but one of the main causes is fracture due to harmonic vibrations during operation. On the other hand, the use of additional technologies, such as products created with 3D printers, has become increasingly common in recent years. The advantage of such models is their affordability, lightness, and ease of manufacture, which allows for the creation of parts even in environments such as military operations. However, one of the disadvantages of this material is its relatively rapid destruction due to material fatigue. In addition, since these are polymeric materials, fracture can be both ductile and brittle. Thus, it is important to consider the safety of using polymeric materials in gearboxes under variable load and overload to ensure the reliability of the connection.

This is a dynamic problem where the natural frequencies of the mechanisms are critical parameters. When designing gearbox parts, it is necessary to ensure their resonance-free operation to prevent possible failure. Additionally, contact stresses between the gears during operation must be taken into account to ensure optimal system life.

Review of Modern Information Sources on the Subject of the Paper

Gear design and analysis are crucial aspects of mechanical engineering, impacting various industries including automotive, aerospace, and industrial machinery. Research [1] delves into the realm of non-involute gears, proposing a machine learning-based model for nominal root stress calculation. This innovation offers a practical alternative to Finite Element Method (FEM) simulations, particularly beneficial for nonstandard gear designs lacking established guidelines.

Contact stress analysis is a fundamental area of gear research, as highlighted by studies such as [2, 3, 4, 5] and [6]. These investigations emphasize the importance of understanding contact stress distribution, especially in spur, helical, and helical gear pairs, and its impact on gear performance and longevity.

Finite Element Analysis (FEA) plays a pivotal role in gear research, facilitating detailed numerical simulations to analyze stress distribution and deformation. Papers by [4, 7, 8], and [9] exemplify this approach, demonstrating the use of FEA to study modal characteristics, harmonic response, and contact behavior in spur, composite, and helical gears, respectively, under different operating conditions.

Furthermore, the advancement of analytical methods for stress analysis is evident in studies like [10, 11, 12] and [13]. These works propose analytical models for stress analysis in gear drives, validated through comparisons with FEA results. Such models offer efficient alternatives for predicting stress distribution and critical points in gear systems.

Innovations in gear design, including high-contact ratio spur gears [14] and non-involute gear profiles [1], aim to enhance performance, durability, and efficiency. These designs address challenges such as sliding contact, interference, and fatigue, leading to more robust gear systems.

Experimental validation of theoretical and numerical analyses is essential for ensuring the accuracy and reliability of gear designs. Studies by [4, 15], and [16] demonstrate experimental measurement techniques for evaluating strain, stress, and contact behavior in helical and asymmetric spur gear pairs, corroborating findings from numerical simulations.

Interdisciplinary research efforts, such as the integration of gear systems into exoskeleton designs [17], showcase the diverse applications of gear technology beyond traditional mechanical systems. Custom actuation systems, like cycloidal drives, offer cost-effective solutions for improving performance and ergonomics in assistive devices.

Overall, the collective body of research presented by these authors contributes significantly to the advancement of gear technology, providing insights into design methodologies, stress analysis techniques, and practical applications across various industries. This interdisciplinary collaboration underscores the importance of continuous innovation in addressing evolving challenges and requirements in mechanical engineering.

Objectives and Problems of Research

To develop a reliable structure, it is necessary to determine its stress-strain state in more detail and study its dynamic characteristics. Due to the high costs of full-scale testing, the use of software products for numerical modeling is an urgent task.

Since one of the requirements for the elements of the mechanism used in elbow orthoses is weight reduction, the main task is to compare the natural frequencies, modes, and stress-strain state of the connection of gear elements made of steel 45 and PLA polymer.

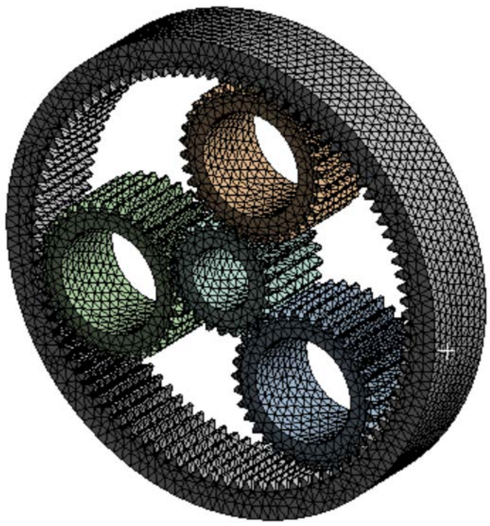

Fig. 1. Model of the planetary gearbox

Main Material Presentation

Creating the geometry is the first step of the analysis. The model of the planetary gear of the elbow orthosis gearbox was created in the ANSYS Workbench software package and is shown in Fig. 1. The planetary gear consists of a sun gear, three planet gears, and a fixed ring gear. The choice of meshing method and element size is contingent on the structure's geometry. In this investigation, a hexahedral finite element is employed due to its suitability and accuracy for the given shape.

The parameters of the planetary gear elements used in the design are given in Table 1. Mechanical characteristics of the materials from which the structural elements are made are given in Table 2.

Table 1

Planetary gear parameters

Parameter	Sun	Planet gear	Ring gear
Dividing diameter	9.6	13.6	38.4
Diameter of tooth tips	10.840	14.957	37.985
Initial diameter	10.097	14.303/13.381	37.871
Center distance	12.20	12.20	26
Module	0.4	0.4	0.4
Number of teeth	24	34	96
Width of the crown	8	8	8
Engaging angle	26.686	–	17.253
Tooth height	0.799	0.799	0.9
Tooth thickness	2.155	2.261	1.227
Average specific sliding	0.419	0.512	0.316
Face overlap	1.121	1.12	1.583
Safety factor of the tooth pedicle	5.333	2.548	3.415
Safety factor of the lateral surface of the tooth	1.541	1.609	2.231
Efficiency factor	0.978		

Table 2

Material properties

Parameter	Units of measure	Steel 45	PLA
Density	kg/m^3	7850	1250
Thermal expansion coefficient	C^{-1}	$1.2 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$
Young modulus	Pa	$2 \cdot 10^{11}$	$3.7 \cdot 10^{9}$
Poisson's ratio		0.3	0.3
Shear modulus	Pa	$7.69 \cdot 10^{10}$	$2.73 \cdot 10^{9}$

Determining the eigenfrequencies and performing a harmonic calculation is important because all gear elements operate at high speeds, i. e., vibrations occur, which can lead to an increase in the resonant frequency of the system.

At the first stage of the study, a modal analysis was performed to determine the vibration characteristics of the planetary gear. As a result of the calculation, the natural frequencies of the system made of steel 45 and PLA plastic were determined. The results of the comparative analysis are presented in the form of a diagram, which makes it possible to visually compare the obtained values (Fig. 2).

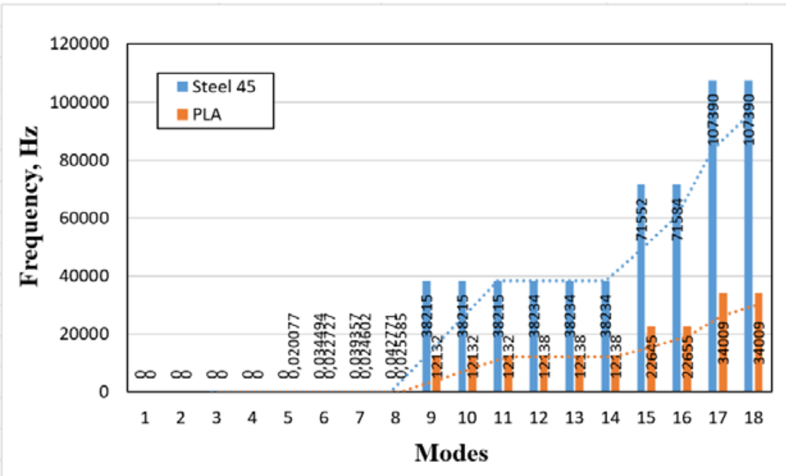

Fig. 2. Frequency (Hz) v/s modes

For each eigenfrequency obtained, the corresponding mode is plotted. The first 18 modes are shown in Fig. 3. It is important to note that the modes are not dependent on the material, so they are the same for steel 45 and PLA plastic.

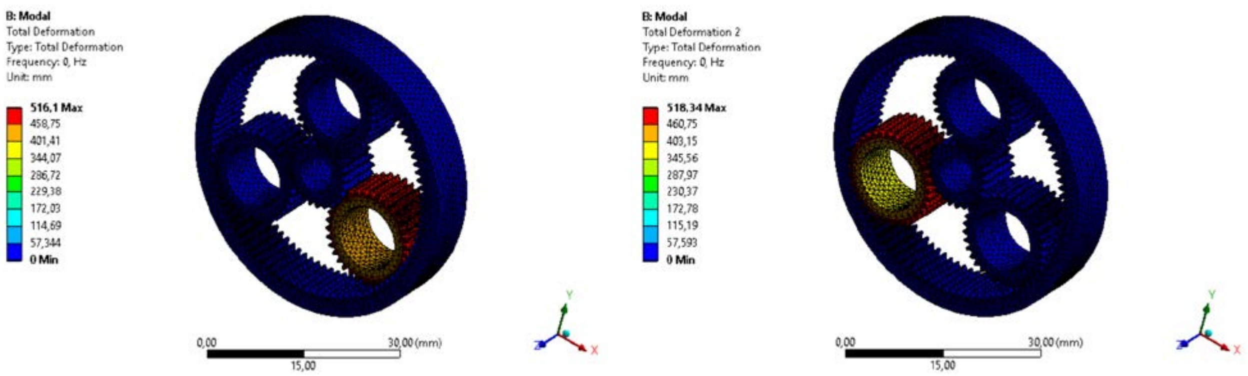

Fig. 3. Total deformation of the planetary gearbox from Mode 1–18 using ANSYS

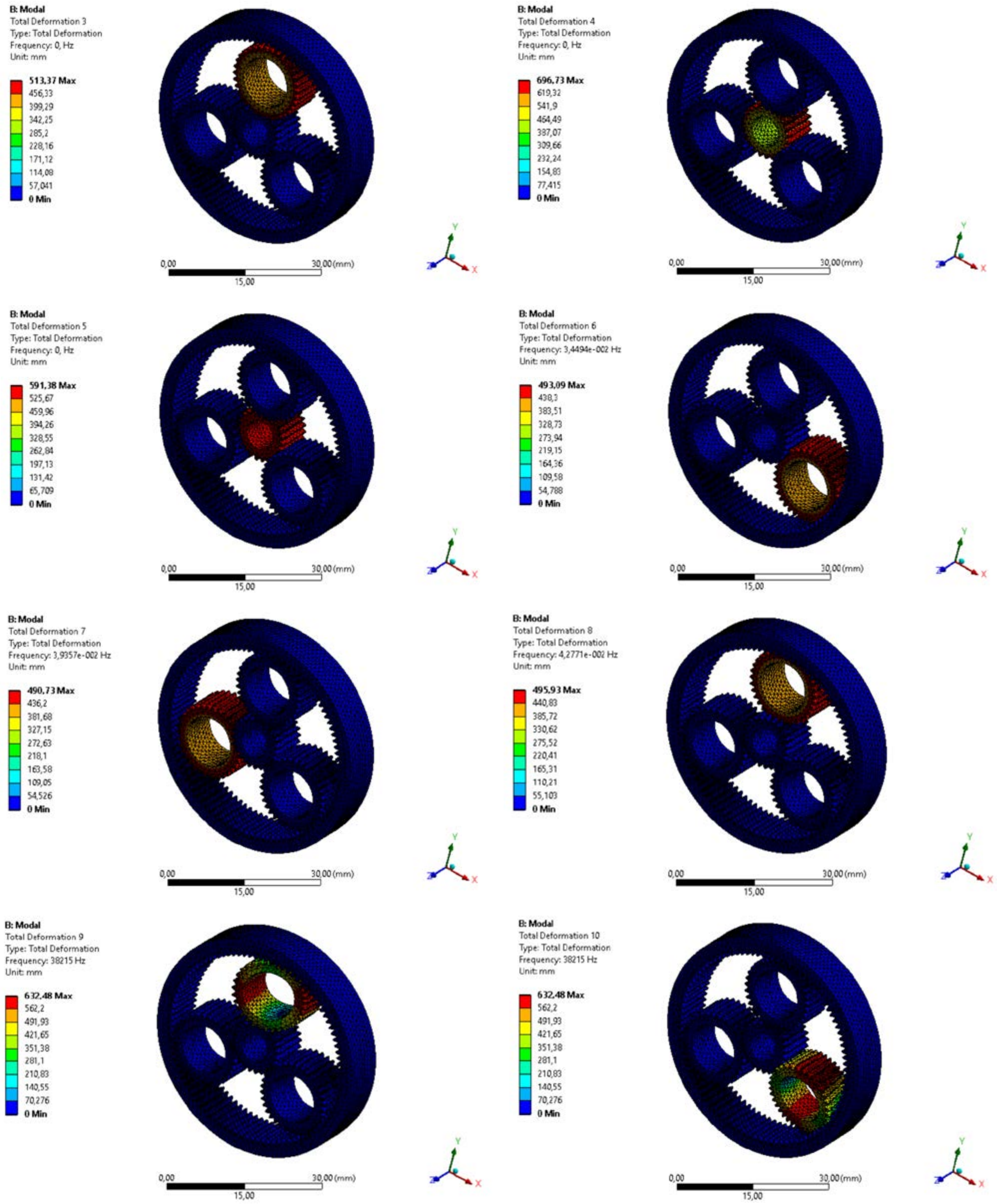

Fig. 3. (Continuation). Total deformation of the planetary gearbox from Mode 1–18 using ANSYS

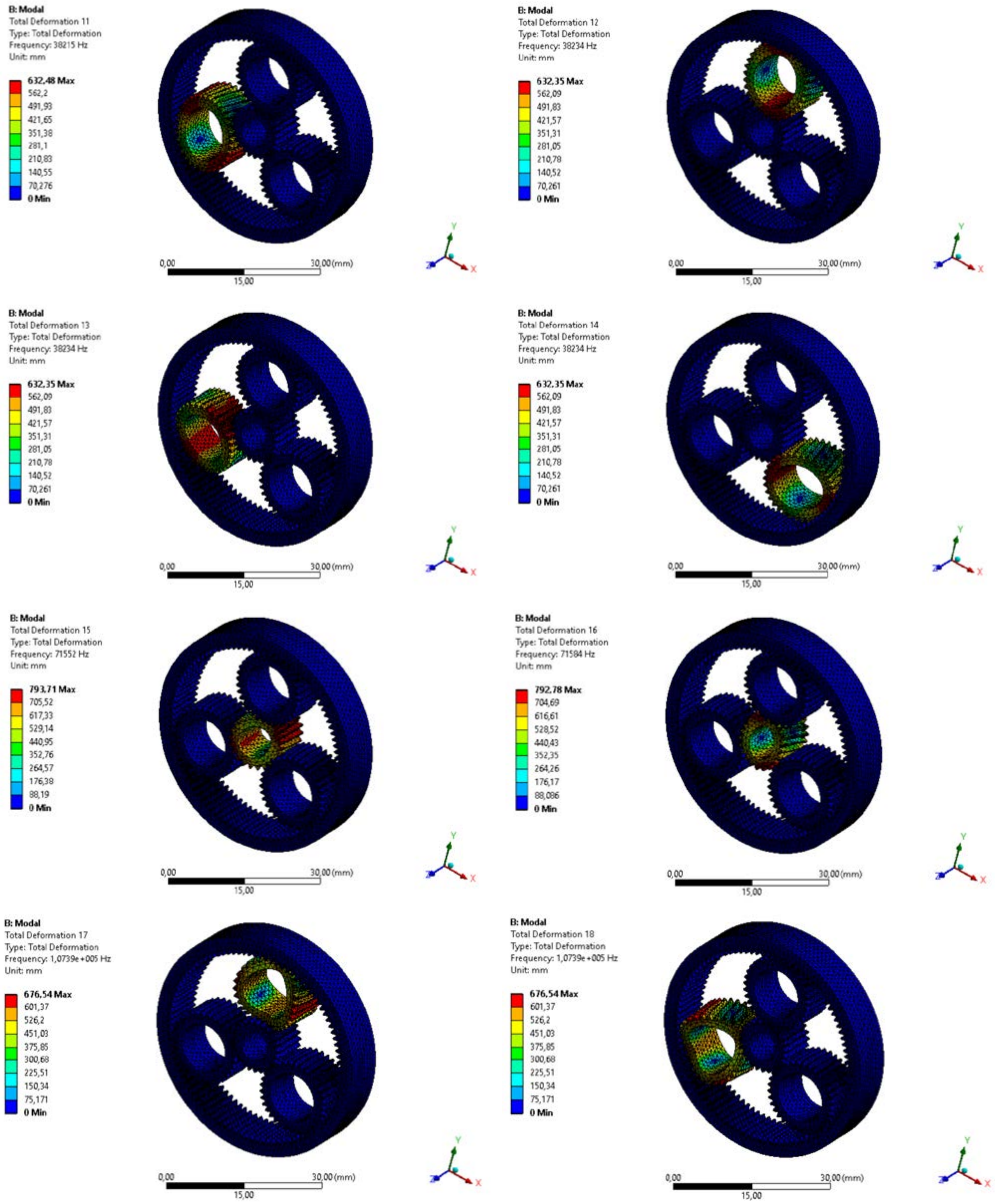

Fig. 3. (End). Total deformation of the planetary gearbox from Mode 1–18 using ANSYS

After obtaining the natural frequencies, a harmonic analysis is performed to study the behavior of the system when operating at certain frequencies close to the resonant ones. For this calculation, it is necessary to set appropriate boundary conditions, such as fixation and cyclic loading. The loading frequency was varied in the range of 0–110 kHz for 45 steel and 0–35 kHz for PLA plastic, respectively. These values were obtained from the modal analysis.

The boundary conditions of the system used for modeling are shown in Fig. 4. The outside of the ring gear is rigidly fixed and is assumed to be stationary. The internal elements of the system, the planet gears and the sun, are constrained as frictionless supports, i. e., the bodies can move and rotate freely in all directions except normal to the surface.

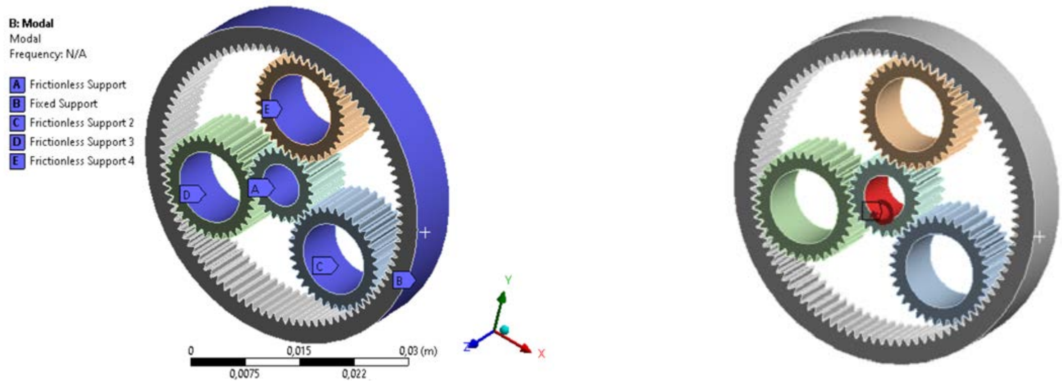

Fig. 4. System boundary and load conditions

During modeling, moments are applied to the inner surface of the sun: 1732 N·mm, 3464 N·mm, 5196 N·mm, corresponding to overloads of 100 %, 200 %, 300 %. This calculation is performed to prevent destruction of the planetary gearbox structural elements during operation.

Due to the technical characteristics and the corresponding load on the system, a harmonic analysis was performed for the planetary gears made of steel 45 and PLA plastic, applying each of the above torques. The result of the calculation is the graphs of displacements, stresses, and strains, respectively, along the 3 axes for a given range of cyclic loads (Fig. 6, *a*–*f*).

The next part of the study is to calculate the contact stresses between the sun and planet gear (Fig. 5). This study is carried out to estimate the values of the stresses arising in the engagement and to determine the location of their dislocation.

A finite element mesh was constructed, which is optimal for the corresponding calculation. For the surface, the size of the Tetrahedrons element was chosen to be 1 mm, and for the contact zone, in order to increase the accuracy of the calculation, the element size was 0.1 mm. Thus, 201074 nodes and 117829 elements were obtained (Fig. 5).

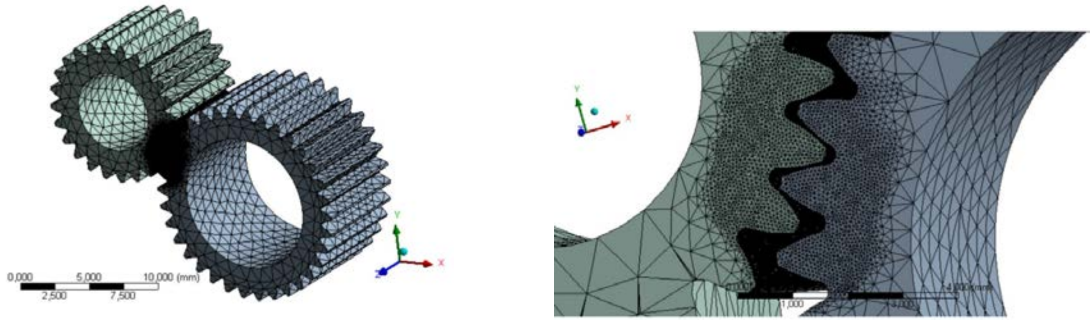

Fig. 5. Finite element mesh of the sun-planet gear contact

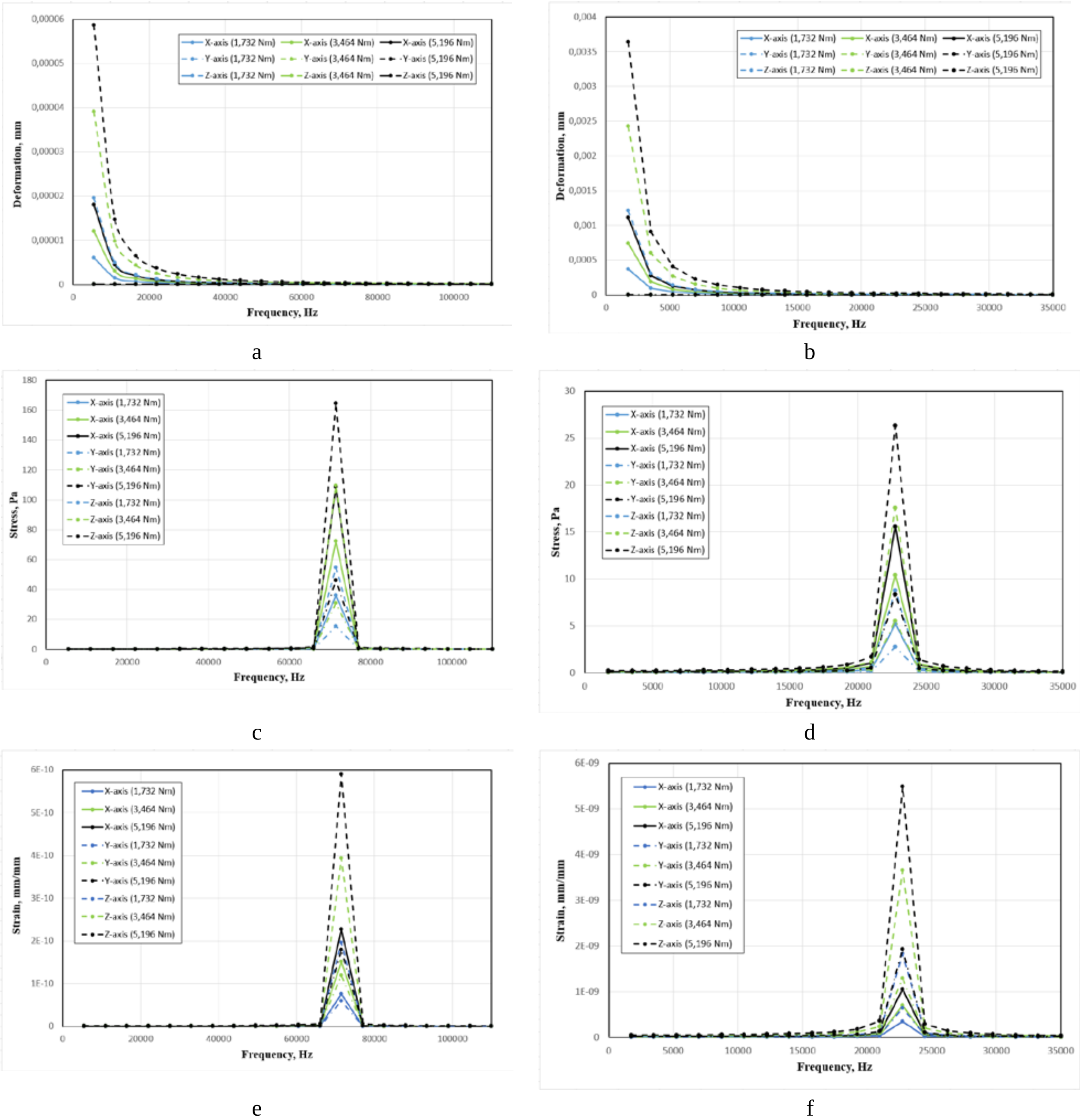

Fig. 6. Comparative analysis of results for Steel 45 and PLA gears: |
a, b – deformation; c, d – stresses; e, f – strains, respectively

The mesh was densified to obtain accurate values in the contact zone of the teeth since the contact surface is quite small.

The next step in the modeling is to set the boundary conditions for the contacting bodies of the system (Fig. 7, 8):

– 1732 N·mm, 3464 N·mm, 5196 N·mm torques are applied to the sun in turn, which corresponds to system overloads of 100 %, 200 %, 300 %, respectively;

– the method of fixing the sun is free displacement, for which all degrees of freedom are limited, except for rotation around the OZ axis;

– the method of fixing the satellite is frictionless support, which allows the movement of the element due to the action of another element or force without additional friction;

– the contact area of the teeth is set to frictional – interaction with friction. The friction coefficient for the calculation was 0.512.

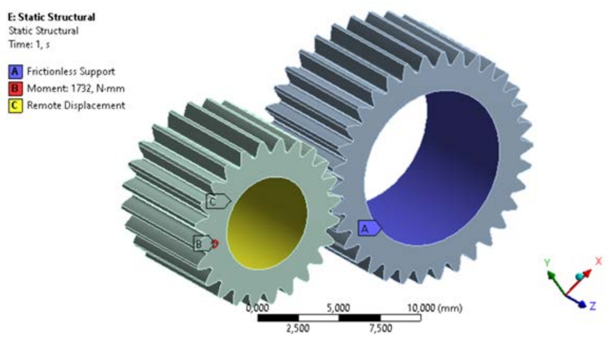

Fig. 7. Sun and planet gear boundary conditions

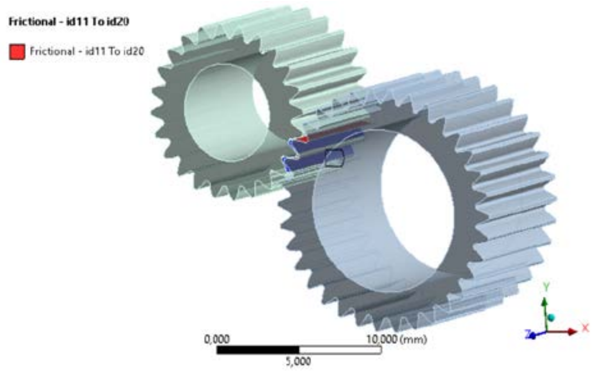

Fig. 8. Boundary conditions between the sun and planet gear teeth

The results of the static calculation of the contact problem to determine the equivalent stresses for each material and the corresponding load are shown in Fig. 9, *a–f*.

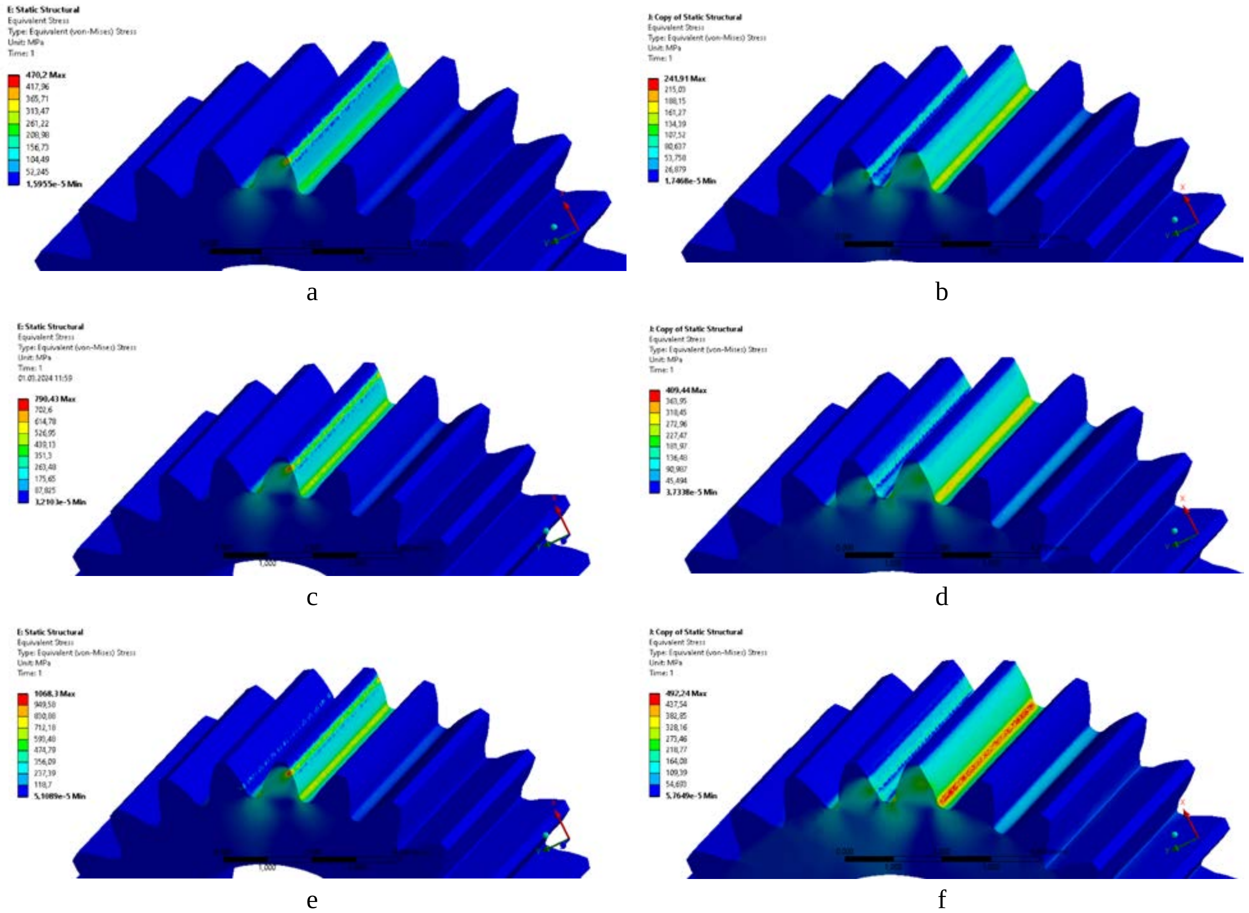

Fig. 9. Equivalent von mises stress for Steel 45 and PLA gears: a, b – 1,732 N·m; c, d n – 3,464 N·m; e, f – 5,196 N·m, respectively

To evaluate the results, we performed an analytical calculation. The input data and the design scheme for the calculation are shown in Table 1 and Fig. 10, respectively.

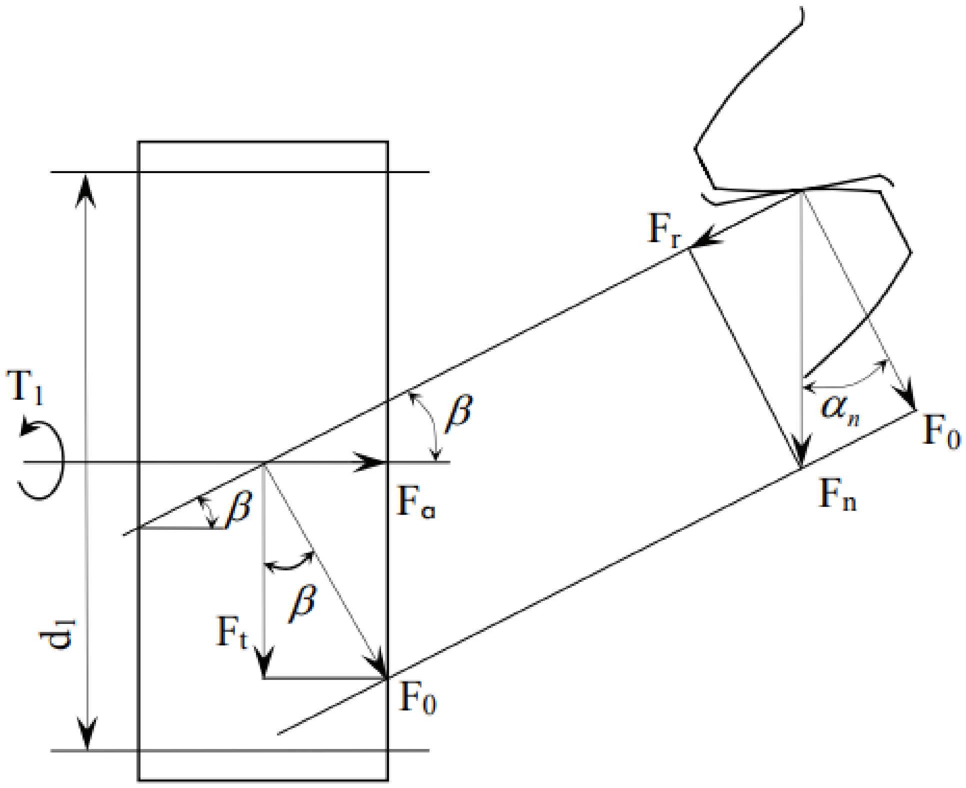

Fig. 10. The design scheme of the sun-planet gear engagement

Contact stresses are determined by the Hertz formula:

$$\sigma = Z_M \sqrt{\frac{q}{\rho_c}}. \tag{1}$$

A coefficient that takes into account the material properties of the sun and planet gear:

$$Z_M = \sqrt{\frac{2E_1E_2}{\pi\left(E_1\left(1-\mu_1^2\right)+E_2\left(1-\mu_2^2\right)\right)}}. \tag{2}$$

The particular circular force:

$$q = \frac{\omega_{ht}}{K_\varepsilon \varepsilon_\alpha \cos\alpha}, \tag{3}$$

where $K_\varepsilon -$ a coefficient that takes into account length fluctuations; $\alpha -$ gear engagement angle; $\varepsilon_\alpha = \left(1{,}88 - 3{,}2\left(\frac{1}{z_1}+\frac{1}{z_2}\right)\right)\cos\beta -$ coefficient of face overlap; $z_1, z_2 -$ number of teeth on the sun and planet gear, respectively; β – tooth angle; $E_1, E_2 -$ elasticity modulus of the sun and planet gear, respectively.

Particular calculated circular force:

$$\omega_{ht} = \frac{F_T}{b} K_{H\alpha} K_{H\beta} K_{Hv}, \tag{4}$$

where $K_{H\alpha}$ – load distribution coefficient; $K_{H\beta}$ – load concentration coefficient; K_{Hv} – dynamic coefficient; $F_T = \frac{2T}{d}$ – circular force; T – torque on the sun; b – width of the sun's crown; d – diameter of the dividing circle of the sun.

Total radius of curvature of tooth profiles:

$$\rho_c = \frac{\rho_1 \cdot \rho_2}{\rho_1 + \rho_2}, \tag{5}$$

where $\rho_1 = 0{,}015\ m$ – radius of curvature of the sun's tooth; $\rho_1 = 0{,}0135\ m$ – radius of curvature of the planet's tooth.

After substituting the appropriate values for the coefficients Table 1 and Table 2 into the Hertz formula Eq. (1), the corresponding contact stresses between the sun and planet teeth were obtained. The results of the comparative analysis of the obtained values of contact stresses by numerical and analytical methods are given in Table 3.

Table 3

Results of the comparative analysis

No.	Torque, N·m	Stress Steel 45 (ANSYS), MPa	Stress Steel 45 (Analytically), MPa	Stress PLA (ANSYS), MPa	Stress PLA (Analytically), MPa
1	1.732	470.2	469.28	241.91	240.62
2	3.464	790.43	772.07	409.44	343.87
3	5.196	1068.3	968.94	492.24	421.15

To illustrate the obtained values of contact stresses, the graphs in Fig. 11 are plotted. It can be seen that with an increase in torque, the error between the numerical and analytical calculations increases, but its value remains within acceptable limits.

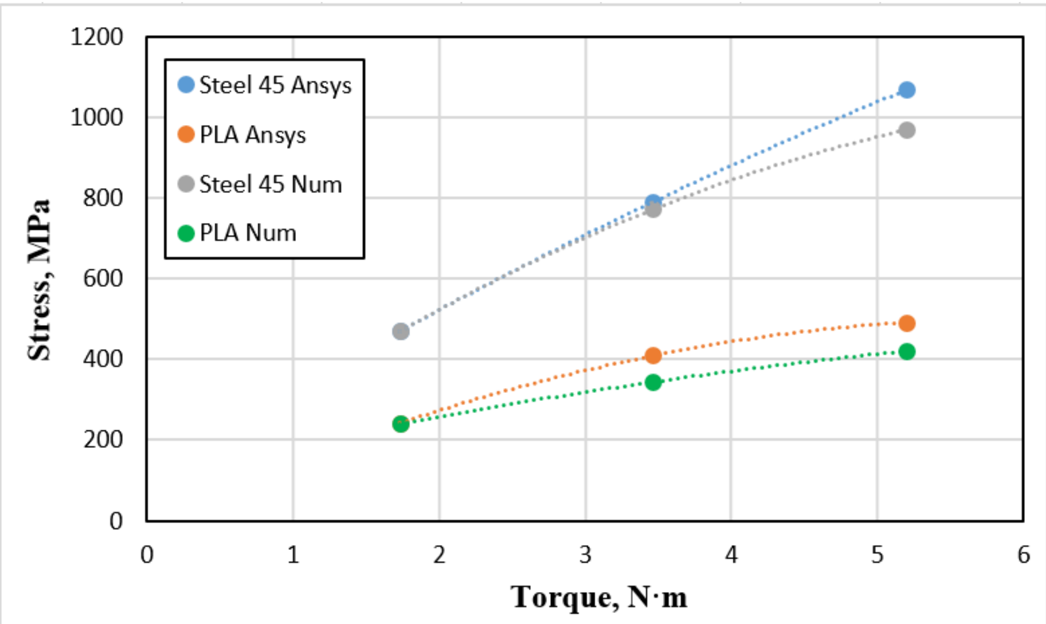

Fig. 11. Equivalent stresses calculated analytically and using ANSYS for different torques

Conclusions

After analyzing the obtained modes, the following conclusions can be made:

The first 8 frequencies are zero or close to 0. This is because each of the 4 bodies of the system (the sun and 3 planet gears) has 2 degrees of freedom, i.e., the stiffness in the respective directions is 0. Under this condition, ANSYS cannot obtain a solution to this problem and determines the natural frequency to be 0.

The next 6 frequencies have the same value. This frequency is the first bending frequency of the satellite in two mutually perpendicular planes. Since there are 3 planets in the system, we get 6 identical frequencies.

The next 2 frequencies have the same value. These frequencies, similar to the previous ones, are the bending frequencies of the sun in two mutually perpendicular planes.

The following frequencies, similar to the previous ones, will also be the same. These are the bending frequencies of planet gears in two planes simultaneously.

As a result of the harmonic analysis, a significant increase in stresses and strains was observed under cyclic loading at 71.5 kHz and 22.75 kHz for steel 45 and PLA, respectively. These values are close to the first two bending frequencies of the planetary gears. It can be concluded that this range is the most dangerous and can lead to resonance. Therefore, it is worth conducting a more detailed study of the system in this range of cyclic loads.

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