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A watermarking scheme for color images that achieves optimality using the Transit Search Algorithm

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This paper describes an innovative watermarking method that combines discrete wavelet transform (DWT), Hessenberg decomposition (HD), and singular value decomposition (SVD). To do this, the main image and the watermark are divided into three channels (red, green and blue — RGB). Then, each part of the main image individually undergoes the steps of DWT, HD and SVD, while the watermark components are processed by SVD. Insertion of the watermark is carried out by adjusting the singular values of the watermark and the main image, using a watermark scaling factor (α). The optimal choice of α poses a challenge, so the transit search algorithm is employed to find a trade-off between visibility and robustness. To evaluate this method, comparisons are made with other studies using various optimization algorithms such as particle swarm optimization, artificial bee colony and fly optimization algorithm. The results of the experiments confirm the effectiveness of this technique.

Keywords: *watermark; transit search; color images; DWT; HD; SVD.*

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1. Inroduction

Watermarking, a means of safeguarding copyrights, involves implanting distinctive data into digital media like images, videos, or audio files [1–4]. Its primary goal is to facilitate the identification of the creator or proprietor while discouraging unauthorized usage or replication. The fundamental benchmarks for image watermarking encompass invisibility, resilience, efficiency, and cost, necessitating a continual quest for equilibrium among them. Addressing this challenge involves the utilization of diverse optimization algorithms like artificial bee colony (ABC) [5], particle swarm optimization (PSO) [6], fruit fly optimization algorithm (FOA) [7], and Transit Search (TS) [8–11]. This study concentrates on watermarking RGB color images utilizing three techniques: DWT, HD, and SVD. Initially, DWT and SVD [7] are applied to the R, G, and B constituents of both the original and watermarked images. Following this, the watermark data is integrated into these constituents, leveraging an optimal DWT band, watermark level, and scale factor selection to enhance similarity and perceptual resilience. During the watermark retrieval process, watermarks are extracted from the R, G, and B constituents of the watermarked image. Ultimately, a composite watermark is formulated by averaging these three constituents, yielding a robust watermark impervious to attacks like cropping, noise, and compression. The paper's structure unfolds as follows: Section 2 outlines the proposed methodology, while Section 3 delves into the experimental findings. Finally, Section 4 offers concluding remarks.

2. Preliminaries

2.1. Discrete wave transformation

The discrete wavelet transform (DWT) stands out as a key mathematical transformation widely used across science and engineering due to its versatility. It adeptly encapsulates image characteristics in terms of energy, proving highly effective in bolstering image processing against various watermarking attacks. Through DWT, the host image undergoes transformation into four distinctive sub-bands: LH, HL, HH, LL. Following a singular DWT level, a significant accumulation of host image details stands out notably within the LL sub-band. Leveraging wavelet theory permits additional decomposition until the sub-bands attain the preferred scale for embedding watermarks. Notably, the LL sub-band surpasses other sub-bands in its ability to withstand attacks induced by filters or compression. This outstanding attribute positions the LL sub-band as an exceedingly robust option for watermarking purposes.

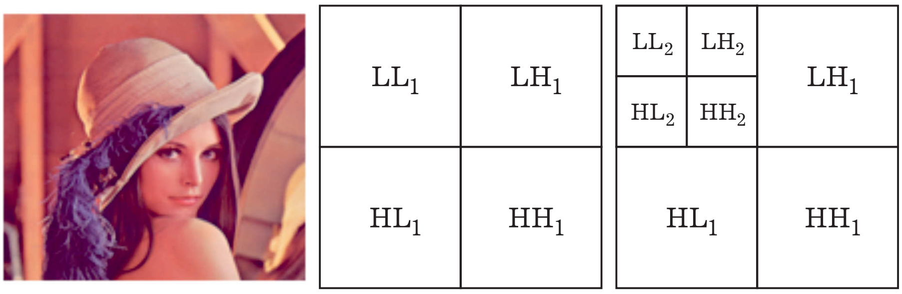


Fig. 1. Representation obtained by iterative decomposition of a signal using the discrete wavelet transform.

2.2. Hessenberg decomposition

Hessenberg decomposition involves converting a square matrix into a distinct higher Hessenberg form. An upper Hessenberg matrix holds all elements below the first subdiagonal (the diagonal just below the main diagonal) as zero. This transformation process can be defined by the following equation. Consider a square matrix A of size $n \times n$, and the aim is to derive its Hessenberg decomposition. The objective is to convert A into a higher Hessenberg matrix H, achievable through two matrices Q and its transpose, Q^T, meeting the condition:

$$A = Q\,H\,Q^T. \tag{1}$$

The process of obtaining the Hessenberg decomposition involves a series of orthogonal similarity transformations to zero out the subdiagonal elements below the first subdiagonal of matrix A, gradually transforming it into the upper Hessenberg form.

2.3. Singular value decomposition

SVD stands as a foundational method within linear algebra and signal processing, allowing the breakdown of a matrix into several essential components. Its application spans various fields, including data compression, image processing, data analysis, and solving linear systems. Breaking down a matrix A of size $N \times N$ into three primary components constitutes the essence of its singular value decomposition:

$$A = U\,S\,V^T, \tag{2}$$

where U is an orthogonal matrix of size $N \times N$, whose columns are the left singular vectors of A, S is a diagonal matrix of size $N \times N$, containing the singular values of A sorted in descending order: $S = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$. We can also write:

$$S = \begin{pmatrix} \lambda_1 & ... & 0 \\ ... & ... & ... \\ 0 & ... & \lambda_N \end{pmatrix} \tag{3}$$

with λ_i positive real values called singular values of A and satisfying $\lambda_1 > \lambda_2 > \ldots > \lambda_N$. V^T is the transpose of an orthogonal matrix V of size $N \times N$, whose columns are the right singular vectors of A, U and V are orthogonal matrices, $U^T\,U = V^T\,V = I_N$.

2.4. Transit search (TS)

Transit search is a method of detecting extrasolar planets that involves monitoring the brightness of a star to detect variations caused by a passing planet. The phases of transit search include:
Planning the observation: This involves determining which stars will be observed, with what instrument, and at what frequency.
Observation: This involves monitoring the brightness of the star to detect variations due to the transit of a planet in front of it. The observations can be made from the ground or from space.
Data processing: The raw data must be processed to remove noise and measurement errors, and to extract information about the transits of planets.
Transits detection: This involves finding signals in the data that indicate the presence of a transit of a planet in front of the star.
Confirmation: The transit candidates must be confirmed using other methods, such as the measurement of the planet mass by the radial velocity method or the detection of the light curve during the eclipse of the star.
Characterization: The properties of the planet must be determined from the data, such as size, mass, orbital period, distance from the star, etc.
Statistical analysis: The transit search data can be used to determine the frequency of planets of different sizes, at different distances from their star, etc.

3. Proposed watermarking scheme

Drawing from the DWT-HD-SVD method [7] and leveraging the TS algorithm, our novel approach aims to bolster watermarking robustness while enhancing quality and privacy. The specifics of the TS algorithm, are outlined below.

3.1. Integration algorithm

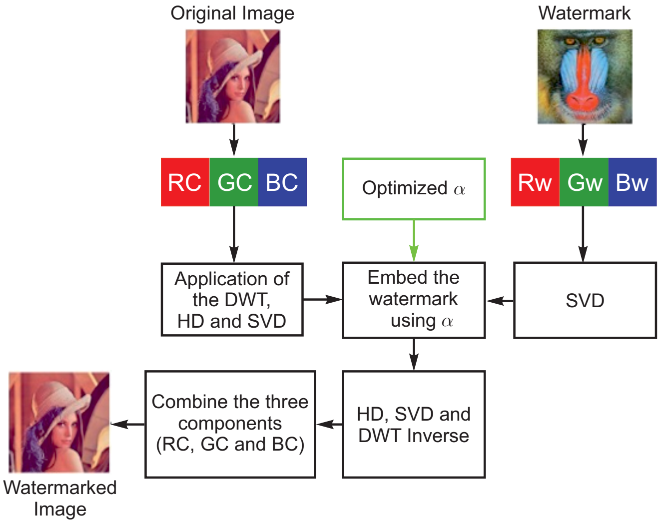


Fig. 2. Watermarking process of the proposed method.

Step 1 involves breaking down the original cover color image and the watermark color image into three components: RC, GC, BC, and Rw, Gw, Bw, respectively.
Step 2 the third levels of DWT are employed on components RC, GC, and BC.
Step 3 executes HD specifically on LL3.
Step 4 SVD is applied to the H matrix and the watermarked visual.
Step 5 adjusts the singular values of the host image in each subband using a scale factor α as

$$\begin{aligned} HSw\_hatR &= HSwR + \alpha_{opt} \times SwR, \\ HSw\_hatG &= HSwG + \alpha_{opt} \times SwG, \\ HSw\_hatB &= HSwB + \alpha_{opt} \times SwB. \end{aligned} \tag{4}$$

Step 6 Inverse SVD, inverse HD and inverse DWT, respectively.

Extraction algorithm

Step 1 The watermarked color image is first decomposed into three components (RC, GC and BC).
Step 2 Third DWT levels are applied to each component.
Step 3 HD is executed on LL3.

Step 4 Apply SVD to the H matrix.
Step 5 The extracted singular value Sw is calculated by

$$\begin{aligned} Sw\_hatR &= (HSw\_hatR - HSwR)/\alpha_{opt}, \\ Sw\_hatG &= (HSw\_hatG - HSwG)/\alpha_{opt}, \\ Sw\_hatB &= (HSw\_hatB - HSwRB)/\alpha_{opt}. \end{aligned} \tag{5}$$

Step 6 The extracted watermark W is reconstructed by inverse SVD and inverse DWT.

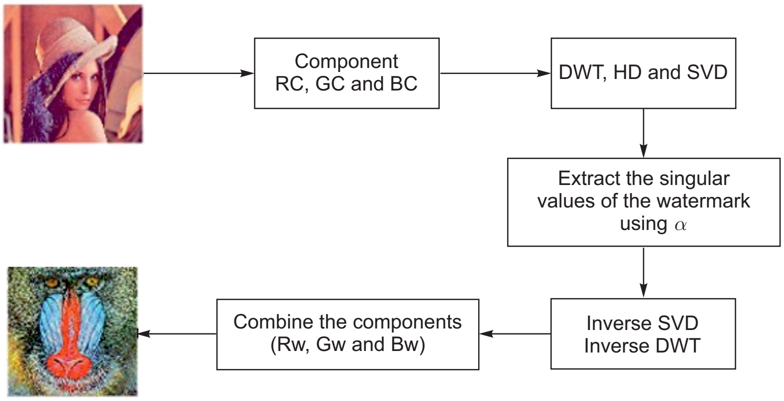


Fig. 3. Process of watermark extraction.

3.2. Optimization of the scaling factor using TS

Finding the exact force factor value that strikes a balance between the invisibility and robustness of integrated individual values is critical. To accomplish this, we employ the TS algorithm to autonomously seek the optimal α factor. To fulfill this objective, we rely on the objective function F, which amalgamates PSNR, SSIM, and NC criteria, formulated as

$$\text{Minimum}\, F(\alpha, n) = \frac{1}{n}\,\text{PSNR}(I, I^*) + \text{SSIM}(I, I^*) + \sum_{i=1}^{k} \text{NC}(W, W_i^*), \tag{6}$$

where n is the number of attacks.

The steps of the proposed algorithm

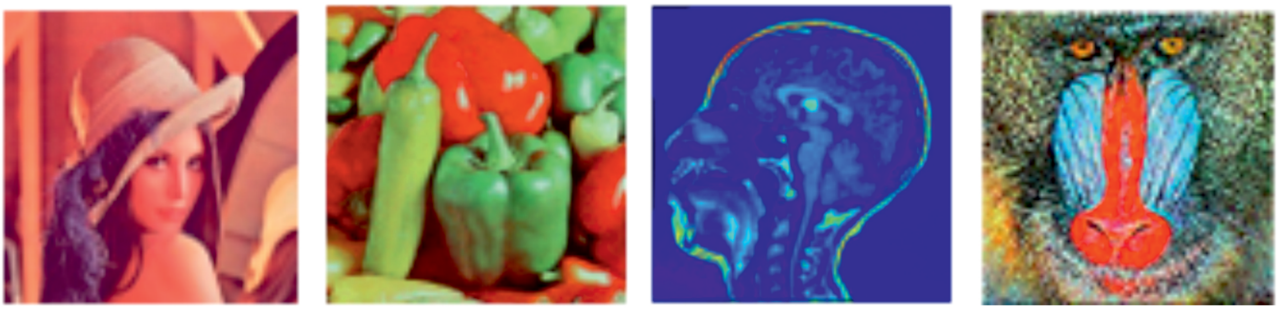
- Initialize the TS parameters.
- Generate the initial population randomly.
- Watermark embedding.
- Watermarked image.
- Applying different kinds off attacks.
- Watermark extraction.
- Calculate: PSNR, SSIM and NC.
- Fitness evaluation.
- If the fitness value is satisfactory, complete the optimization process and produce the best solution, then repeat back α.

4. Experimental results and analysis

Our watermarking method was implemented on the Lena image sized at 512 × 512, utilizing Matlab R2021a [12]. To assess the efficacy of our approach, various parameters were considered, including mean square error, PSNR, SSIM, NC [13].

4.1. Invisibility analysis

Invisibility denotes the inability of the human eye to discern the watermark concealed within the cover image. The α operator holds significant importance in assessing the watermarking system's performance. To appraise the visual fidelity of our proposed watermarking approach, we inserted the watermark from Figure 4d into the displayed cover images in Figures 4a–4c.



a (Lena) ***b*** (Peppers) ***c*** (Brain) ***d*** (Watermark image)

Fig. 4. Cover images.

In Figures $5a$–$5c$, the showcased images portray the watermark images generated using scaling factors α obtained from the TS algorithm, whereas Figures $5d$–$5f$ exhibit the extracted watermark images. This depiction emphasizes that there is no discernible deterioration in the watermarked image, underscoring the effectiveness of the proposed watermarking methodology.

a (Image of watermarked Lena) ***b*** (Image of watermarked peppers) ***c*** (Image of watermarked brain)

d (Watermark taken from (***a***)) ***e*** (Watermark taken from (***b***)) ***f*** (Watermark taken from (***c***))

Fig. 5.

Maintaining information security mandates that the watermarked image remains imperceptible to human eyes. Thus, assessing the watermark's invisibility stands as a crucial metric in gauging performance. The evaluator plays a pivotal role in appraising the watermarking scheme's efficacy. To gauge the visual quality of our proposed scheme, we embedded the 256-×-256 watermark image into Lena's cover image, sized at 512 × 512 [12].

As depicted in Figure 6, the experimental results showcase PSNR values nearing 40 dB and SSIM values approaching 1, indicating an almost indistinguishable similarity between the watermarked image and the original cover image. This close resemblance underscores the resilience of the suggested watermarking method in preserving visual quality for both the watermarked image and the extracted watermark. When comparing our approach to other image watermarking systems utilizing optimization algorithms like PSO, ABC, and FOA, our method based on the TS algorithm delivers a more imperceptible watermarked image compared to its counterparts.

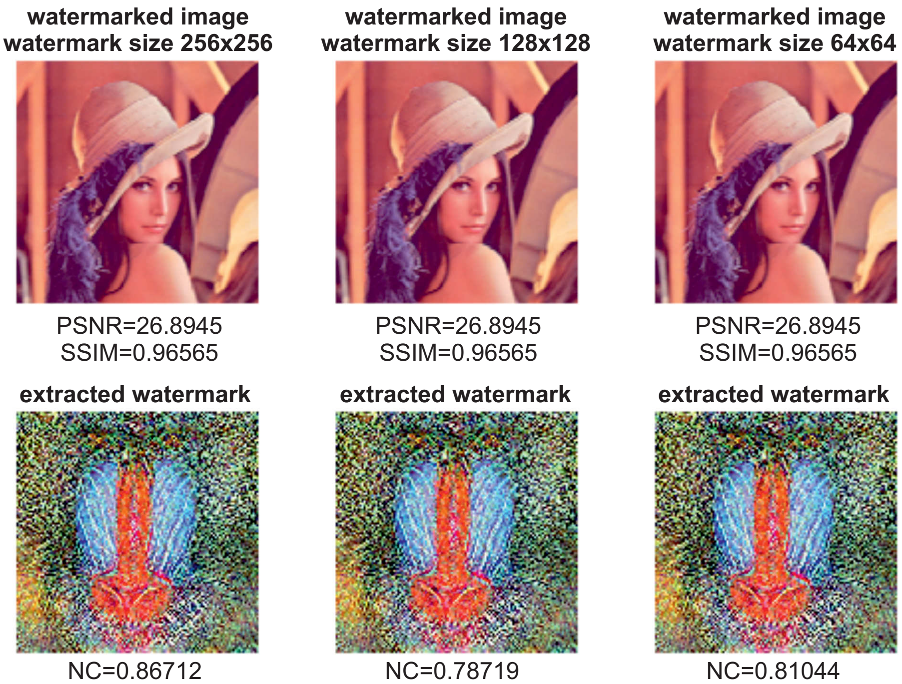


Fig. 6. Invisibility performance.

Table 1. Comparison between the PSNR and SSIM values of our proposed scheme and existing systems.

Optimization algorithm	PSNR	SSIM
Proposed method	56.43	0.9999
PSO	45.73	0.9984
ABC	37.82	0.9973
FOA	39.63	0.9995

4.2. Robustness analysis

In this section, we assess the durability of the proposed approach by subjecting the watermarked image to various assaults. This evaluation involves computing the NC (Normalized Correlation) between the extracted watermark and the original one. A watermark pattern exhibits higher resistance to attacks as the NC approaches 1, signifying heightened robustness. Conversely, a lower NC signifies diminished resilience. As illustrated in Figure 7, our proposed method exhibits substantial resilience against diverse attack types.

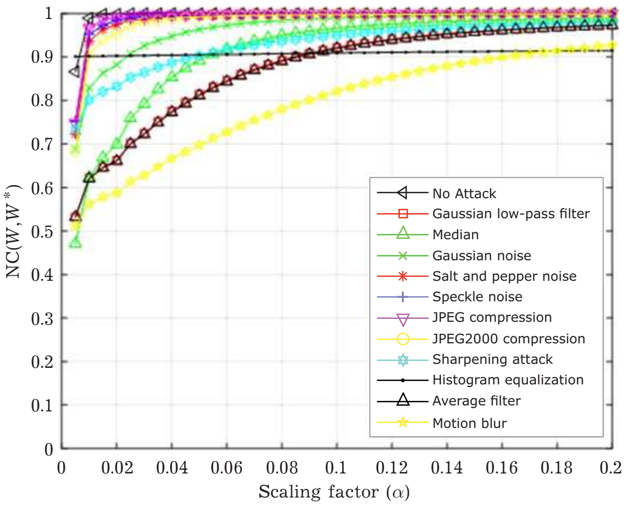


Fig. 7. NC of the extracted watermark under different attacks and scaling factors.

5. Conclusions

This paper introduces a novel watermarking technique utilizing DWT-HD-SVD, optimizing its scale factors through the TS algorithm. This approach enhances both the robustness and perceptual similarity of watermarking, demonstrating superior performance compared to alternative methods. As a prospect for further exploration, integrating Charlier moments with optimized parameters could potentially bolster the watermark's robustness in future iterations.

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Схема водяних знаків для кольорових зображень, яка досягає оптимальності за допомогою алгоритму транзитного пошуку

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У статті описано інноваційний метод водяних знаків, який поєднує дискретне вейвлет-перетворення (DWT), розкладання Гессенберга (HD) і розкладання за сингулярним значенням (SVD). Для цього основне зображення та водяний знак розділені на три канали (червоний, зелений і синій — RGB). Потім кожна частина основного зображення окремо проходить етапи DWT, HD і SVD, тоді як компоненти водяного знака обробляються SVD. Вставка водяного знака здійснюється шляхом налаштування одиничних значень водяного знака та основного зображення, використовуючи коефіцієнт масштабування водяного знака (α). Оптимальний вибір α викликає труднощі, тому алгоритм транзитного пошуку використовується для пошуку компромісу між видимістю та надійністю. Щоб оцінити цей метод, проводяться порівняння з іншими дослідженнями, які використовують різні алгоритми оптимізації, такі як оптимізація роя частинок, штучна бджолина колонія та алгоритм оптимізації мухи. Результати дослідів підтверджують ефективність запропонованого метода.

Ключові слова: *водяний знак; транзитний пошук; кольорові зображення; DWT; HD; SVD.*