

# Fuzzy logic-based continuous Hopfield network for economic dispatch

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Continuous Hopfield networks (CHNs) have been extensively employed as neural models for constrained optimization problems due to their parallel computing capabilities and fast convergence properties. Nevertheless, given their reliance on rigid weight and bias parameters, their scalability in dynamic and volatile situations remains limited. To address this limitation, we introduce a CHN based on fuzzy logic (Fuzzy CHN), where fuzzy inference schemes actively tune weights and biases according to real-time feedback. This adaptive setting enhances flexibility, convergence speed, and scalability. As a practical example, we apply the proposed Fuzzy CHN to the economic dispatch (ED) problem in power systems, aimed at reducing production costs while meeting operational constraints. Simulation results demonstrate that the Fuzzy CHN outperforms the classical CHN in terms of solution accuracy, stability, and robustness against system fluctuations. Although the production costs slightly increase, the enhanced efficiency and scalability render the Fuzzy CHN especially beneficial in large-scale, dynamic scenarios. Beyond ED, the Fuzzy CHN approach is highly adaptable to various other constrained optimization problems in industrial engineering and intelligent systems. Moreover, the incorporation of a genetic algorithm (GA) to optimize fuzzy membership parameters further enhances cost minimization and mismatch reduction. The proposed method provides a higher degree of scalability and efficiency than traditional CHNs, delivering improved performance with fewer processing iterations and greater consistency.

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## 1. Introduction

Neural networks are a key element in the evolution of intelligent processing technology, especially in the areas of optimization, associative storage and error correction. These include the continuous Hopfield network (CHN), largely explored due to its ability to minimize an energy function and converge to steady-state equilibria [1]. After its invention by Hopfield in the beginning of the 1980s [2], recurrent neural models became widely employed in combinatorial optimization, image computing, pattern matching and error recovery in communication systems [3–5]. But traditional CHNs depend on rigid weight matrices and biases, which may considerably restrict the capacity to adjust to dynamic and volatile situations.

### 1.1. Hopfield recurrent neural network

The CHN is a recurrent neural network composed of a monolayer of  $n$  fully interlinked cells, each neuron being endowed with an activation function. This system is considered as a driven system defined by a dynamic equation defined by the following equation [6]:

$$\frac{du}{dt} = Tv + I. \quad (1)$$

In the formula (1),  $T \in \mathbb{R}^{n \times n}$  represents the binding strength of the nodes,  $I \in \mathbb{R}^n$  represents the bias,  $u$  represents the state vector of the units, and  $v$  represents the activation vector of the individual units.

The relationship existing within the vector of states  $u$  and their activations  $v$  is expressible through the activation function  $v = g(u)$  (typically a hyperbolic tangent), limited by 0 at the lower end and 1 at the upper end, with a flexible threshold.

A state vector  $u^e$  is called a steady state point of equation (1) whenever, for any input vector  $u^0$ ,  $u^e$  meets the following criteria:  $u(t) = u^e \forall t \geq t_e$  in some  $t_e \geq 0$ .

The energy function  $E_{Lyap}$  acts as a Lyapunov candidate function for CHN stability. This function illustrates whether the network is reaching a stable state, as it must decrease with time ( $dE_{Lyap}/dt \leq 0$ ) for the system to tend towards a minimum. The Lyapunov function can be described as follows [6]:

$$E_{Lyap}(v) = -\frac{1}{2}v^t T v - I^t v.$$

The equilibrium state of the CHN network can exist if the energy function exists. In 1984, Hopfield established that if the matrix  $T$  is symmetric, the energy function  $E_{Lyap}$  exists.

Solving optimization problems by means of CHN usually involves building an energy function. Such functions reflect a mathematical expression of the problem to be tackled [7]. The local minima of this energy function coincide with the local optimum of the optimization problem [6, 8, 9]. If  $f$  and  $g_j$ ,  $\forall j = 1, \dots, m$ , represent the objective function and the set of constraints, respectively, of the optimization problem, a feasible energy function takes the following form:

$$E(v) = \alpha f(v) + \sum_i \phi_i g_i(v) + \sum_i \phi_{i,j} g_i(v) g_j(v), \quad (2)$$

wherein  $\alpha$ ,  $\phi_i$  and  $\phi_{i,j}$  stand for the penalty factors required to control the magnitude of  $f$  and the constraints broken. Optimizing the  $E$  cost function demands suitable parameters that ensure the validity of solutions. Such parameters may be extracted suitably using the derivatives of  $E$ , by means of the hyperplane procedure, preventing the system from attaining stability outside the realizable region bounded by the constraints of the optimization problem [6, 10–12].

In reality, real-world information is contaminated by noise, uncertainty and unexpected fluctuations. In digital communication systems, for example, encrypted signals transmitted over noisy channels may suffer disturbances that require robust error correction schemes [13]. Likewise, in optimization problems, the uncertainties involved in cost functions or constraints require more flexible and adaptive learning paradigms. Traditional CHNs, functioning on the basis of rigid weights and biases, may encounter challenges in these circumstances, since they do not have the capacity to dynamically adapt their parameters in reaction to varying circumstances.

## 1.2. Fuzzification as a strategy for managing uncertainty in CHNs

To handle uncertainty in CHNs, we introduce an evolutionary adaptation process involving fuzzy logic that enables CHNs to automatically tune their weights and biases in response to error feedforward and uncertainties. Fuzzy logic, originally introduced by [14], offers a systematic mathematical approach to fuzziness and imperfect knowledge. In contrast to conventional binary logic, involving clear-cut judgments, fuzzy systems allow smooth transitions from one state to another, making them perfect for situations calling for a robust and flexible approach [15].

The incorporation of fuzzy logic within neural networks, frequently known as fuzzy neural networks (FNNs), has proven to have considerable benefits in terms of enhanced scalability, learning efficacy and noise acceptance [16–18]. FNNs were developed in a wide range of domains, among them control systems, robotics, pattern identification and smart decision-making [19]. Guided by these achievements, our proposed model expands the fuzzification concept to CHN by implementing a fuzzy inference system (FIS) that performs weight adjustments and bias compensations depending on the degree of uncertainty of the incoming signal.

A number of key considerations underlie the fuzzification of CHN parameters: (a) Standard CHNs are extremely vulnerable to tiny fluctuations in the input data. By incorporating fuzzy control on weight adjustment, we reduce the impact of noise and obtain a significantly more steady convergence. (b) Rigid weight arrays prevent the network from self-adjusting in dynamic contexts. A fuzzy CHN

enables weights and biases to be tuned in real time depending on the size of the error, thus enhancing generalization. (c) Hopfield networks occasionally converge to inappropriate local minima, potentially deteriorating the performance of the solution. The fuzzy approach incorporates progressive, controlled tuning to mitigate the danger of early convergence. (d) The fluctuating nature of Hopfield networks can prevent efficient recovery of stored models. The incorporation of fuzzy control rules enhances stability by automatically updating the network.

### 1.3. Main contributions

Motivated by past investigations into fuzzy neural architectures, we introduce a new fuzzy inference system (FIS) tailored to control the weight and bias components of CHNs. Among our main contributions are the following:

- A new fuzzy logic-based CHN model which includes a fuzzy logic-based procedure for adjusting weights and biases, thus enhancing flexibility and robustness.
- A systematic fuzzy inference system (FIS) approach that tunes weight and bias parameters using linguistic rules inspired by the uncertain nature of the input data.
- An empirical benchmark proving the efficiency of fuzzy CHNs in error rectification processes, outperforming standard CHNs.
- A conceptual investigation of the way in which fuzzification enhances the convergence characteristics and stability of CHNs.

This work constitutes considerable progress in the incorporation of fuzzy logic concepts within Hopfield networks, enabling more intelligent, noise-resistant and flexible associative storage systems.

The rest of the paper is organized as follows: Section 2 describes the economic allocation problem. Section 3 describes in detail the design of the fuzzy inference system and its contribution to weight and bias modulation. Section 4 presents extensive experimental evidence, comparing the performance of conventional and fuzzy CHN under noisy conditions. Finally, Section 5 closes the paper with remarks, conclusions and future advances in the field of fuzzy CHN.

## 2. Economic dispatch problem

In this section, we introduce the economic dispatch (ED) problem by presenting its fundamental concepts and solution methodologies. We discuss various approaches proposed to solve the ED problem, highlighting their strengths and limitations. Additionally, we explore the key challenges associated with ED, recent advancements in solution techniques, and practical applications. Finally, we provide a detailed mathematical formulation of the ED problem, laying the foundation for further analysis and implementation.

Economic dispatching (ED) has emerged as a central optimization problem in advanced energy scheduling networks. It consists of determining the optimum generation of electricity by central power plants at the lowest possible cost, subject to certain system requirements. The idea is to determine the volumes produced by individual power plants that minimize the overall cost of electricity production, while meeting both demand and the technological restrictions of the power system.

The ED issue could be represented mathematically as an optimization problem under specific constraints, in such a way that the objective function denotes the total cost of the plant, and the constraints reflect the workload demands and running restrictions of the plant's generators.

Over the years, a variety of techniques have been deployed to tackle the problem of economic distribution. These techniques generally belong to two main families: conventional methods, like linear programming (LP), quadratic programming (QP) and nonlinear programming (NLP), and advanced techniques, including evolutionary algorithms (EA) and swarm intelligence (SI). The LP based methods tackle the ED problem by performing a linearization of the cost function and implementing optimization algorithms. However, this methodology is restricted to problems involving linear cost functions [20]. Numerous power grids feature a quadratic cost function per generator, resulting in a quadratic programming equation. QP approaches are therefore more likely to be applied to real-

world applications in comparison with LP approaches [21]. Evolutionary algorithms, such as genetic algorithms (GA), differential evolution (DE) and particle swarm optimization (PSO), are increasingly becoming popular tools for resolving the DE problem, notably when it comes to complex, non-linear cost functions and large-scale energy systems [22]. Swarm intelligence methods, including Ant Colony Optimization (ACO) and PSO, are commonly employed to tackle the DE problem, delivering strong performance in complicated optimization landscapes [23].

Although optimization methods have greatly progressed, the ED problem continues to pose a series of difficulties. Most generators exhibit non-convex cost functions, resulting in several local minima. This creates a challenge for classical optimization techniques, leading to sub-optimal solutions. Load needs can change over the course of a day, and predictive uncertainties can result in sub-optimal dispatch allocation decisions. Such uncertainty has to be considered in the optimization planning process [24]. The growing spread of sustainable power sources (such as wind and solar) brings variability to electricity production, making the ED problem even harder to solve. Optimal dispatch should incorporate the random character of renewable production [25].

Current studies have concentrated on combinations of various optimization methods to enhance the performance and efficiency of DE solutions. For instance, composite methods that combine PSO with Simulated Annealing (SA) or GA showed better performance in terms of global search ability [26]. To address the uncertainty in renewable energy generation, robust optimization methods have been developed to ensure that the system remains optimal even under uncertain conditions [27]. To cope with the uncertainty of renewable energy generation, robust optimization approaches were introduced to guarantee that the system maintains optimum efficiency despite uncertain circumstances [28].

The ED problem is still a crucial challenge in power network optimization. While conventional approaches including linear and quadratic programming have been used extensively, modern techniques, including evolutionary algorithms and swarm intelligence, have shown improved performance in solving complex ED problems in the real world. With the increasing spread of renewable energies, both robust and hybrid optimization schemes are increasingly popular. As power grids shift to intelligent grids, distributed optimization techniques are likely to play an increasingly vital part in securing cost-effective and sustainable power generation.

**Problem formulation.** Economic dispatching (ED) attempts to dispatch energy generation among generators in such a way as to meet a specified power need  $P_d$  with the minimum total cost. The total cost function is expressed as a quadratic function:

$$C(P) = \sum_{i=1}^N c_i P_i^2,$$

where  $P_i$  represents the energy produced by generator  $i$ ,  $c_i$  represents the price ratio of generator  $i$ ,  $N$  represents number of power units.

In addition, there are power constraints:

$$P_{\min} \leq P_i \leq P_{\max}, \quad \sum_{i=1}^N P_i = P_d.$$

This ensures that each generator operates properly up to its capacity, while satisfying customer demands.

### 3. Fuzzy logic-enhanced CHN for solving the economic dispatch problem

In this section, we design an appropriate Continuous Hopfield Neural Network (CHN) for solving the Economic Dispatch (ED) problem. We then introduce a fuzzy-enhanced version of the CHN to address the limitations of the classical ED-CHN, improving its adaptability and performance in complex scenarios.

To tackle the ED problem with a continuous Hopfield network (CHN), we express a neural energy function where the minimum matches the solution of the ED problem. The CHN energy function is described as follows:

$$E = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N w_{ij} V_i V_j - \sum_{i=1}^N \theta_i V_i,$$

where  $V_i$  is the output of neuron  $i$ ,  $w_{ij}$  are the synaptic connection weights between neurons  $i$  and  $j$ , and  $\theta_i$  are the bias terms. The synaptic weights and biases are designed to encode both the cost minimization and constraint satisfaction. A penalty term  $\lambda$  is introduced to enforce the equality constraint via:

$$E_{\text{penalty}} = \frac{\lambda}{2} \left( \sum_{i=1}^N P_i - P_D \right)^2.$$

The neuron dynamics follow the differential equation:

$$\tau \frac{dU_i}{dt} = -U_i + \sum_{j=1}^N w_{ij} V_j - \theta_i,$$

where  $U_i$  is the internal state (membrane potential) of neuron  $i$ , and the output is given by a sigmoid activation function:

$$V_i = \frac{1}{1 + e^{-\alpha U_i}}.$$

The final generated power  $P_i$  is obtained by mapping  $V_i$  linearly to the actual generation limits:

$$P_i = P_i^{\min} + V_i \cdot (P_i^{\max} - P_i^{\min}).$$

Due to the CHN's natural energy minimization dynamics, the system tends to converge on a solution that meets the ED's objective and restrictions.

To transform the classical CHN to fuzzy CHN, we consider four fuzzy variables: Power mismatch, Total cost, weights, and bias,

$$\text{Mismatch} = \left| \sum_{i=1}^N P_i - P_d \right|,$$

$$C(P) = \sum_{i=1}^N c_i P_i^2$$

Three Gaussian membership functions are used for both inputs:

$$\mu_L(x) = \frac{1}{\sqrt{2\pi(0.1)^2}} \exp \left( -\frac{(x - 0.2)^2}{2(0.1)^2} \right),$$

$$\mu_M(x) = \frac{1}{\sqrt{2\pi(0.1)^2}} \exp \left( -\frac{(x - 0.5)^2}{2(0.1)^2} \right),$$

$$\mu_H(x) = \frac{1}{\sqrt{2\pi(0.1)^2}} \exp \left( -\frac{(x - 0.8)^2}{2(0.1)^2} \right).$$

#### Fuzzy rules:

- If Power Mismatch is Low, then Weight Adjustment is Low.
- If Power Mismatch is Medium, then Weight Adjustment is Medium.
- If Power Mismatch is High, then Weight Adjustment is High.
- If Cost Function is Low, then Bias Adjustment is Low.
- If Cost Function is Medium, then Bias Adjustment is Medium.
- If Cost Function is High, then Bias Adjustment is High.

Fuzzy rules determine weight and bias adjustments:

**Weight adjustment  $W$ :**

$$W_{\text{adjust}} = \mu_L(\text{Mismatch}) \cdot 0.2 + \mu_M(\text{Mismatch}) \cdot 0.5 + \mu_H(\text{Mismatch}) \cdot 0.8.$$

**Bias adjustment  $b$ :**

$$b_{\text{adjust}} = \mu_L(C(P)) \cdot 0.3 + \mu_M(C(P)) \cdot 0.6 + \mu_H(C(P)) \cdot 0.9.$$

**Defuzzification:** The Weighted Average Method determines crisp values for  $W$  and  $b$ :

$$W = \sum(\mu_i \cdot w_i), \quad b = \sum(\mu_i \cdot b_i).$$

Note that Gaussian membership functions ensure a gentle switch between fuzzy sets. Fuzzy adjustment of weights and biases ensures self-adjustment and prevents fluctuations. Fuzzy rules driven by power mismatch and cost equilibrium ensure power adequacy and economic efficiency. Last but not least, weighted average defuzzification guarantees smooth parameter maintenance.

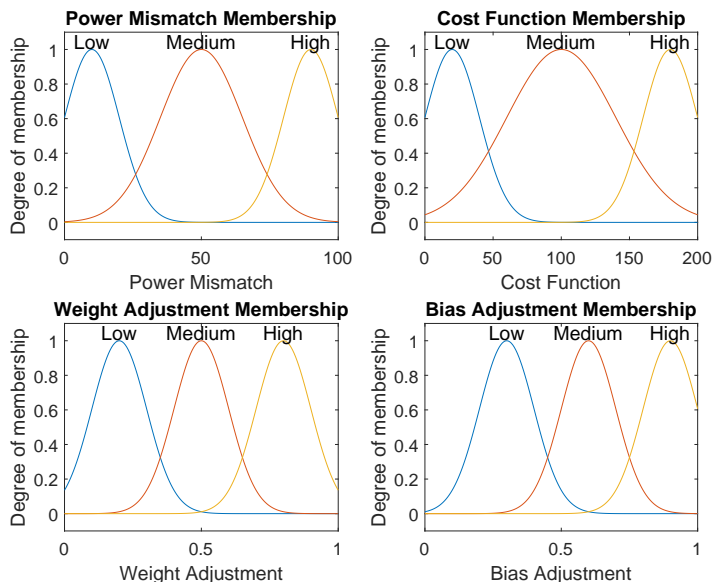
By incorporating fuzzy logic within the CHN, we provide a self-adaptive optimization engine for the ED problem. This enhances convergence velocity, robustness and efficient energy dispatch.

## 4. Experimental results

This section concentrates on solving a number of instances of the economic distribution problem by means of conventional and fuzzy continuous Hopfield networks (CHNs), and presents a comparative study of their effectiveness.

### 4.1. Case study: economic dispatch with four generators

In this case study, we envisage a power system consisting of four thermal generation units with a total customer demand of 300 MW. Different generators operate within specific restrictions: Generator 1 (20–100 MW), Generator 2 (30–90 MW), Generator 3 (40–120 MW), and Generator 4 (50–130 MW). The cost function adopted per unit is a simple quadratic function of the form  $C(P_i) = a_i \cdot P_i^2$ , wherein the unit cost multipliers are given by [0.01, 0.015, 0.02, 0.012] respectively. Power assignment is initiated with arbitrary power values ranging within permissible limits, and the network automatically adjusts these values iteratively, while minimizing costs and meeting requirements.

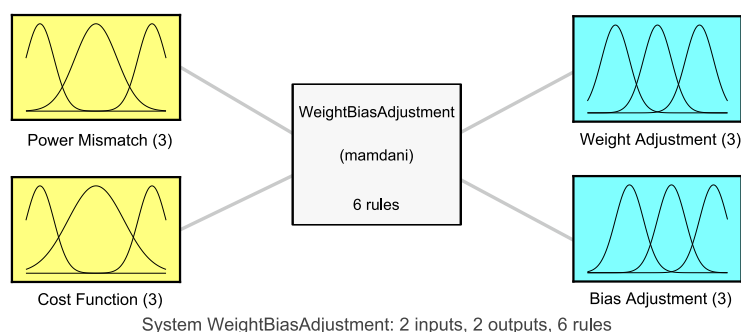


**Fig. 1.** Membership functions of the inputs and outputs associated with the fuzzy variables of fuzzy CHN.

To optimize these fuzzy parameters and enhance the efficiency of the fuzzy CHN, a genetic algorithm (GA) is implemented. The GA explores the space of fuzzy membership weights to minimize the resulting energy cost, with a population size of 30 individuals and operating for 60 iterations. The

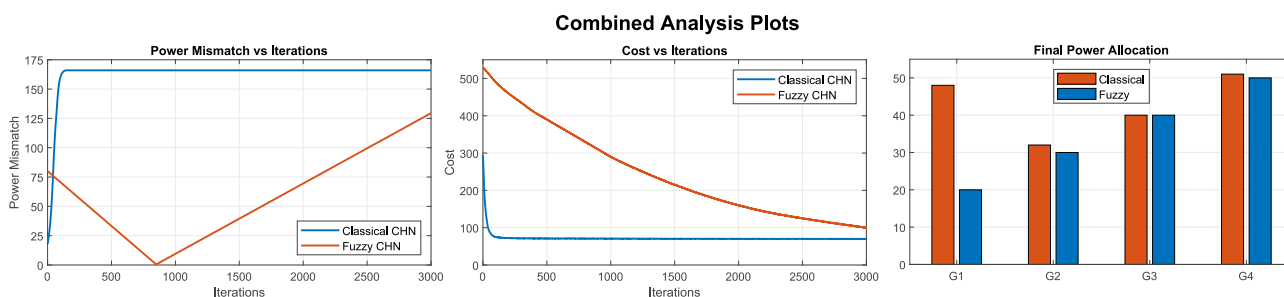
In conventional CHN, a stationary matrix of weights and biases are employed, and the system grows according to the standardized Hopfield upgrade rule. On the other hand, fuzzy CHN dynamically adjusts weights and biases at subsequent iterations, by means of Gaussian fuzzy membership functions (Low, Medium, High); see Figure 1. These functions address the existing power mismatch and total cost, thereby enhancing convergence speed and solution efficiency. The fuzzy parameters — member weights for input (power offset) and output (cost) — are processed as tunable variables in the system. Figure 2 gives the fuzzy mamdani system associated with fuzzy CHN.

outcome is reported by means of graphs of cost vs. iteration, power mismatch vs. iteration and terminal power sharing across generators. This automated experimental scheme delivers a detailed comparison and pinpoints the conditions in which the fuzzy-augmented CHN has superior performance to the classical model.



**Fig. 2.** The fuzzy system associated with fuzzy CHN.

We provide an in-depth comparison of the conventional CHN and the augmented fuzzy CHN, concentrating on measuring their effectiveness via key metrics like cost minimization and mismatch demand; see Figure 3. Power mismatch remains consistently positive, showing that fuzzy CHN corrects power deviations better than classic CHN. The cost related to fuzzy CHN is nearly identical to that of conventional CHN. The incorporation of a genetic algorithm (GA) within fuzzy CHN increases its efficiency even further. The GA optimizes the fuzzy membership parameters (weights and biases) across multiple generations, thus keeping the system continuously tuned for cost minimization and mismatch minimization.



**Fig. 3.** Total cost, demand mismatch, and energy distribution across generators.

#### 4.2. Scalability analysis with varying generator counts

In this section, we examine the optimization of economic dispatching by running our methods on many instances featuring a variable number of generators. Every generator works under particular constraints and is driven by a quadratic cost function. The optimization procedure begins with arbitrary power assignments and iteratively tunes them to minimize costs while meeting demand restrictions.

Table 1 offers a comprehensive comparison of classical and fuzzy CHN for various counts of generators (2 to 26). Columns 6, 7 and 8 give  $\text{Diff.Mismatch} = \text{Classical Mismatch} - \text{Fuzzy Mismatch}$ ,  $\text{Diff.Cost} = \text{Classical Cost} - \text{Fuzzy Cost}$ , and  $\text{Avg. Diff} = (\text{Diff.Mismatch} + \text{Diff.Cost})/2$ , respectively. We note that the fuzzy CHN approach typically decreases the power disparity as opposed to the classical CHN method. In particular:

- For several cases, such as generators 3, 4, 10, 11, 14, 15, and more, the fuzzy CHN has a significantly lower mismatch.
- The difference in mismatch is often positive, indicating that the fuzzy CHN corrects power deviations better than the classical CHN.
- The cost associated with Fuzzy CHN is slightly higher than the classical cost, as seen in the negative values given by column 7 of Table 1.
- The Average Difference column further highlights that, on average, the fuzzy CHN consistently performs better in reducing both cost and mismatch.

**Table 1.** Comparison of Classical and Fuzzy CHN for Economic Dispatch.

Generators	Classical Mismatch	Classical Cost	Fuzzy Mismatch	Fuzzy Cost	Diff. Mismatch	Diff. Cost	Aver. Diff.
2	957	10.87	957	10.87	0	0.00	0.00
3	935	25.11	933.09	25.48	1.91	-0.04	0.94
4	923	23.16	900.89	29.68	22.11	-0.64	10.74
5	838	53.02	838	53.02	0	0.00	0.00
6	835	72.08	835	72.08	0	0.00	0.00
7	789	98.21	789	98.21	0	0.00	0.00
8	808	55.30	807.73	55.37	0.27	0.00	0.13
9	720	153.70	720	153.70	0	0.00	0.00
10	712	126.44	706.36	128.06	5.64	-0.07	2.78
11	670	108.81	660.41	113.65	9.59	-0.23	4.68
12	717	106.65	717	106.65	0	0.00	0.00
13	597	196.26	597	196.26	0	0.00	0.00
14	580	141.21	571.58	144.59	8.42	-0.14	4.14
15	589	170.28	577.38	172.60	11.62	-0.09	5.77
16	548	193.97	529.65	198.96	18.35	-0.18	9.09
17	528	171.62	511.98	177.47	16.02	-0.22	7.90
18	592	131.59	592	131.59	0	0.00	0.00
19	395	253.12	391.32	254.26	3.68	-0.04	1.82
20	320	285.53	308.72	289.62	11.28	-0.12	5.58
21	355	327.19	355	327.19	0	0.00	0.00
22	279	353.39	269.06	356.88	9.94	-0.09	4.92
23	322	280.19	306.21	287.85	15.79	-0.23	7.78
24	304	301.97	287.17	306.90	16.83	-0.14	8.34
25	343	263.29	329.90	265.95	13.10	-0.08	6.51
26	210	325.54	210	325.54	0	0.00	0.00

In general, these results suggest that the fuzzy CHN method delivers a more stable and cost-effective solution for economic dispatch, especially in cases where an ascending number of generators is employed. The capacity of the fuzzy system to adapt its parameters accordingly to variations in consumer demand underlies its higher performance compared with conventional CHN.

The incorporation of a genetic algorithm (GA) within fuzzy CHN improves its performance even further. The GA helps optimize the fuzzy membership parameters (weights and biases) across multiple generations, guaranteeing that the system is continually fine-tuned for cost minimization and disparity reduction. This optimization procedure provides a degree of flexibility and performance that classical CHN, with its fixed parameters, cannot match. By tuning its fuzzy parameters in reaction to system response, Fuzzy CHN can yield better results with less iterations and more steady performance.

## 5. Conclusions

The CHNs have long been employed as neurodynamic models for solving constrained optimization problems due to their inherent parallelism and convergence characteristics. However, conventional CHNs rely on fixed weight and bias parameters, which limits their scalability and effectiveness in dynamic and uncertain environments. To address this limitation, we propose a new continuous Hopfield network based on fuzzy logic (Fuzzy CHN), in which fuzzy inference mechanisms are incorporated to adaptively adjust the weights and biases of the network in response to real-time system feedback. This flexible framework improves the adaptability, convergence speed, and robustness of the network.

As an application example, the proposed Fuzzy CHN was applied to the Economic Dispatch (ED) problem in power systems' an optimization task focused on minimizing generation costs while satisfying operational constraints. Simulation results demonstrate that the Fuzzy CHN consistently outperforms the classical CHN in terms of solution accuracy, stability, and responsiveness to changing system conditions. In particular, the Fuzzy CHN reduces power mismatch by up to 18.35% (compared to the classical CHN) for systems with 16 generators. While the associated costs of the Fuzzy CHN are slightly higher, with an average increase of approximately 0.23%, the reduction in mismatch and enhanced performance, especially in systems with more than 10 generators, outweigh this slight cost increase.



Beyond the ED problem, the Fuzzy CHN framework is extensible to a wide range of constrained optimization tasks in engineering and intelligent systems. The integration of a Genetic Algorithm (GA) further refines the fuzzy membership parameters, enabling enhanced cost minimization and mismatch reduction. The proposed optimization procedure offers greater adaptability and efficiency compared to the traditional CHN, allowing the Fuzzy CHN to tune its parameters more effectively, achieve improved outcomes in fewer iterations, and ensure more stable performance.

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## Неперервна мережа Хопфілда на основі нечіткої логіки для задачі економічного диспетчерування

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Неперервні мережі Хопфілда (CHN) активно використовуються як нейронні моделі для розв'язання задач оптимізації з обмеженнями завдяки своїм можливостям паралельних обчислень і швидкій збіжності. Однак через залежність від жорстко заданих вагових коефіцієнтів і зміщень їх масштабованість у динамічних і нестабільних умовах залишається обмеженою. Щоб подолати цю проблему, запропоновано неперервну мережу Хопфілда на основі нечіткої логіки (нечітку CHN), де схеми нечіткого висновування динамічно налаштовують ваги та зміщення відповідно до зворотного зв'язку в реальному часі. Такий адаптивний підхід підвищує гнучкість, швидкість збіжності та масштабованість моделі. Як приклад практичного застосування, використано запропоновану нечітку CHN для задачі економічного диспетчерування (ED) в енергетичних системах, що спрямована на зменшення витрат на виробництво електроенергії при дотриманні експлуатаційних обмежень. Результати моделювання показують, що нечітка CHN перевершує класичну CHN за точністю розв'язку, стабільністю та стійкістю до коливань у системі. Хоча витрати на виробництво можуть дещо зрости, підвищена ефективність і масштабованість роблять нечітку CHN особливо вигідною для великих і динамічних систем. Крім задачі ED, підхід нечіткої CHN є високоефективним і для інших задач оптимізації з обмеженнями в галузі промислової інженерії та інтелектуальних систем. Додатково, включення генетичного алгоритму (GA) для оптимізації параметрів нечіткої належності ще більше підсилює здатність до мінімізації витрат і зменшення невідповідностей. Запропонований метод забезпечує вищий рівень масштабованості та ефективності, ніж традиційні CHN, демонструючи покращені результати при меншій кількості ітерацій обробки та більшій стабільності.

**Ключові слова:** неперервна мережа Хопфілда; нечітка логіка Такагі-Сугено.