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Solution of the dynamic problem of thermoelasticity in stresses for a strip

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The method for solving a two-dimensional dynamic thermoelasticity problem in stresses for a strip with a rectangular cross-section is proposed. The initial system of equations in stresses, which describes the plane-stress state of the strip, is selected. A solution method is developed based on approximating the distributions of all components of the dynamic stress tensor using cubic polynomials with respect to the thickness coordinate of the strip. This reduces the original system of two-dimensional unsteady equations to a system of one-dimensional unsteady equations involving integral characteristics along the thickness. To solve the resulting system of equations, a finite integral transform is applied with respect to the transverse coordinate of the strip, and the Laplace transform is used for the time variable. General solutions to the considered dynamic thermoelasticity problem under unsteady thermal and force actions on the strip are presented. A criterion for assessing the bearing capacity of the strip is proposed. The analysis reveals the existence of four types of resonant frequencies under the specified unsteady conditions.

Keywords: *strip; non-stationary thermal and force actions; plane-deformed state; components of the stress tensor; stress intensities; resonant frequencies.*

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1. Introduction

During their operation, plate elements are subjected to intense unsteady temperature and force effects. To assess their bearing capacity, it is necessary to solve dynamic thermoelasticity problems formulated in terms of stresses. This makes it possible to determine the stress intensity σ_i. According to the Huber-Mises criterion, $\sigma_i \leqslant \sigma_d$ [1], where σ_d is the dynamic limit of elastic deformation of the element material. The formulation of the problem of thermoelasticity under stresses was considered, in particular, in [2–5]. They mainly provide solutions to static problems of thermoelasticity in stresses and a few solutions to some one-dimensional dynamic problems. For a more complete study of the thermoelastic state of structural elements, it is necessary to solve two-dimensional problems. The initial systems of equations for two-dimensional dynamic problems for plates and cylindrical bodies were written in [6,7]. The aim of this paper is to construct a solution to the two-dimensional dynamic problem of thermoelasticity for a strip in stress.

2. Relationship of the dynamic problem of thermoelasticity for the strip

In the Cartesian coordinate system $Ox_1x_2x_3$, we consider a strip of rectangular cross-section with thickness $2h$ and width $2d$. The material of the strip is homogeneous and isotropic. The strip is subjected to unsteady volume force $\boldsymbol{F} = \{F_1; 0; F_3\}$ and the temperature field T. Their expressions do not depend on the coordinate x_2, and the surfaces of the strip $x_3 = \pm h$ and $x_1 = \pm d$ are free from surface force load. Under these conditions, we have a plane-deformed state of the strip. As a starting point, we choose the system of equations of the plane dynamic problem of thermoelasticity in stresses [7, 8]

$$\Delta_1 \Psi - \frac{1}{C_1^2}\frac{\partial^2 \Psi}{\partial t^2} = 2\rho\,\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2 T}{\partial t^2} - \frac{\alpha E}{1-\nu}\Delta_1 T - \frac{1}{1-\nu}\left(\frac{\partial F_3}{\partial x_3} + \frac{\partial F_1}{\partial x_1}\right), \tag{1}$$

$$\Delta_1\sigma_{13}-\frac{1}{C_2^2}\frac{\partial^2\sigma_{13}}{\partial t^2}=-\frac{\partial^2\Psi}{\partial x^1\partial x_3}-\frac{\partial F_1}{\partial x_1}-\frac{\partial F_3}{\partial x_3}, \tag{2}$$

$$\frac{\partial^2\sigma_{11}}{\partial x_1^2}-\frac{1}{C_3^2}\frac{\partial^2\sigma_{11}}{\partial t^2}=-\frac{\partial^2\sigma_{13}}{\partial x_1\partial x_3}-\frac{\partial F_1}{\partial x_1}+\frac{\partial^2}{\partial t^2}\left[\rho\,\alpha(1+\nu)T-\rho\,\frac{\nu(1+\nu)}{E}\Psi\right], \tag{3}$$

$$\sigma_{33}=\Psi-\sigma_{11},\quad \sigma_{22}=\nu\Psi-\alpha ET. \tag{4}$$

Here, $\Psi=\sigma_{11}+\sigma_{33}$, $C_1^2=(1-\nu)(1+\nu)^{-1}(1-2\nu)^{-1}\rho^{-1}E$, $C_2^2=E/\left[2\left(\rho(1-\nu)\right)\right]$, C_1 and C_2 are the propagation velocities of elastic waves of expansion and deformation; $\Delta_1=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$; $C_3^2=2C_2^2$; α, ν are the coefficients of linear thermal expansion and Poisson's ratio; E is the Young's modulus; ρ is the density; t is the time. The initial conditions for the defining functions at $t=0$ are as follows

$$\sigma_{11}=\sigma_{13}=\Psi=\sigma_{22}=\sigma_{33}=0,\quad \dot\sigma_{11}=-\frac{\alpha E\dot T}{(1-2\nu)},$$
$$\dot\sigma_{13}=0,\quad \dot\sigma_{22}=-\frac{\alpha E\dot T}{(1-2\nu)},\quad \dot\sigma_{33}=-\frac{\alpha E\dot T}{(1-2\nu)},\quad \dot\Psi=-\frac{2\alpha E\dot T}{(1-2\nu)}. \tag{5}$$

Here, $\dot T$, $\dot\Psi$, $\dot\sigma_{11}$, $\dot\sigma_{13}$, $\dot\sigma_{22}$, $\dot\sigma_{33}$ are time derivatives t of the determining functions.

When the surfaces of the strip are free from force loading, the system of boundary conditions on the surfaces $x_3=\pm h$ has the form:

$$\frac{\partial^2\Psi^\pm}{\partial x_1^2}-\frac{1}{C_4^2}\frac{\partial^2\Psi^\pm}{\partial t^2}=\rho\,\alpha(1+\nu)\frac{\partial^2\Psi^\pm}{\partial t^2}-\frac{\partial F^\pm}{\partial x_1}-\frac{\partial}{\partial x_1}\left(\frac{\partial\sigma_{13}}{\partial x_3}\right)^\pm,\quad \sigma_{13}^\pm=0,\quad \sigma_{11}^\pm=\Psi^\pm. \tag{6}$$

Accordingly, on the surfaces $x_1=\pm d$ it is written

$$\frac{\partial^2\Psi_*^\pm}{\partial x_3^2}-\frac{1}{C_4^2}\frac{\partial^2\Psi_*^\pm}{\partial t^2}=\rho\,\alpha(1+\nu)\frac{\partial^2T_*^\pm}{\partial t^2}-\frac{\partial F_{*3}^\pm}{\partial x_3}-\frac{\partial}{\partial x_3}\left(\frac{\partial\sigma_{*13}}{\partial x_1}\right)^\pm,\quad \sigma_{*13}^\pm=0,\quad \sigma_{*33}^\pm=\Psi_*^\pm. \tag{7}$$

Here, $C_4^2=2(1-\nu)^{-1}C_2^2$, $T^\pm=T(x_1,\pm h,t)$, $F_i^\pm=F_i(x_1,\pm h,t)$;

$$\overset{\pm}{\sigma}_{*ik}=\sigma_{ik}\left(\pm d;x_3;t\right),\quad T_*^\pm=T\left(\pm T;x_3;t\right),\quad F_{*i}^\pm=F_i\left(\pm T;x_3;t\right).$$

The solution of the initial problem (1)–(4) is reduced to the joint solving two interconnected wave equations (1) and (2) and the subsequent solving of equation (3) under initial (5) and boundary (6)–(7) conditions.

3. Methodology for constructing the solution

To solve the system of equations (1)–(4), the functions ψ and σ_{13} are approximated by the thickness coordinate x_3 by cubic polynomials

$$\psi=\sum_{j=1}^{4}\psi_{j-1}(x_1,t)\,x_3^{j-1},\quad \sigma_{13}=\sum_{j=1}^{4}\alpha_{13(j-1)}(x_1,t)\,x_3^{j-1}. \tag{8}$$

The coefficients ψ_{j-1}, $\alpha_{13(j-1)}$ of the approximation polynomials (8) are given by the integral characteristics of the functions ψ and σ_{13}

$$N=\frac{1}{2h}\int_{-h}^{h}\psi\,dx_3,\ M=\frac{3}{2h^2}\int_{-h}^{h}\psi\,x_3\,dx_3,\ N_{13}=\frac{1}{2h}\int_{-h}^{h}\sigma_{13}\,dx_3,\ M_{13}=\frac{3}{2h^2}\int_{-h}^{h}\sigma_{13}\,x_3\,dx_3 \tag{9}$$

and the given boundary values of these functions. We obtain the expressions

$$\psi_{(0)}=\frac{3}{2}N-\frac{1}{4}\psi_*,\quad \psi_{(1)}=\frac{5}{2h}M-\frac{3}{4h}\psi_{**},\quad \psi_{(2)}=\frac{3}{4h^2}\psi_*-\frac{3}{2h^2}N,\quad \psi_{(3)}=\frac{5}{4h^3}\psi_{**}-\frac{5M}{2h^2},$$
$$\alpha_{13(0)}=\frac{3}{2}N_{13},\quad \alpha_{13(1)}=\frac{5}{2h}M_{13},\quad \alpha_{13(2)}=-\frac{3}{2h^2}N_{13},\quad \alpha_{13(3)}=-\frac{5}{2h^3}M_{13}.$$

Here, $\psi_*=\psi^++\psi^-$, $\psi_{**}=\psi^+-\psi^-$. The system of equations for determining the integral characteristics of N, M, N_{13}, M_{13} is obtained by integrating equations (1), (2) according to formulas (9) using expressions (8) is as follows:

$$\left(\frac{\partial^2}{\partial x_1^2}-\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}-\frac{3}{h^2}\right)N=\Phi_1-\frac{3}{2h^2}(\psi^++\psi^-),\quad \left(\frac{\partial^2}{\partial x_1^2}-\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}-\frac{15}{h^2}\right)M=\Phi_2-\frac{15}{2h^2}(\psi^+-\psi^-),$$

$$\left(\frac{\partial^2}{\partial x_1^2}-\frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}-\frac{3}{h^2}\right)N_{13}=\frac{1}{2h}\left[\frac{\partial}{\partial x_1}(\psi^+-\psi^-)+(F_1^+-F_1^-)\right]+\frac{\partial F_3^{(1)}}{\partial x_1},$$
$$\left(\frac{\partial^2}{\partial x_1^2}-\frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}-\frac{15}{h^2}\right)M_{13}=\frac{3}{2h}\left[\frac{\partial}{\partial x_1}(\psi^++\psi^-)+(F^++F^-)-\frac{3}{h}\left(\frac{\partial N}{\partial x_1}+F_1^{(1)}\right)\right]+\frac{\partial F_3^{(2)}}{\partial x_1}. \quad (10)$$

The initial conditions will be as follows

$$N(x_1,0)=0,\quad M(x_1,0)=0,\quad N_{13}(x_1,0)=0,\quad M_{13}(x_1,0)=0,\quad \dot N(x_1,0)=-\frac{2\alpha E}{1-2\nu}\dot T_1(x_1,0),$$
$$\dot M(x_1,0)=-\frac{2\alpha E}{1-2\nu}\dot T_2(x_1,0),\quad \dot N_{13}(x_1,0)=0,\quad \dot M_{13}(x_1,0)=0.$$

The functions Φ_1 and Φ_2 in the equations (10) are as follows

$$\Phi_1=2\rho\,\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2 T_1}{\partial t^2}-\frac{\alpha E}{1-\nu}\left\{\frac{\partial^2 T_1}{\partial x_1^2}+\frac{1}{2h}\left[\left(\frac{\partial T}{\partial x_3}\right)^+-\left(\frac{\partial T}{\partial x_3}\right)^-\right]\right\}-\frac{1}{1-\nu}\left[\frac{1}{2h}(F_3^+-F_3^-)+\frac{\partial F_1^{(1)}}{\partial x_1}\right],$$
$$\Phi_2=2\rho\,\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2 T_2}{\partial t^2}-\frac{\alpha E}{1-\nu}\left\{\frac{\partial^2 T_2}{\partial x_1^2}+\frac{3}{2h}\left[\left(\frac{\partial T}{\partial x_3}\right)^+-\left(\frac{\partial T}{\partial x_3}\right)^-\right]-\frac{3}{2h^2}(T^+-T^-)\right\}$$
$$-\frac{1}{1-\nu}\left[\frac{3}{2h}(F_3^++F_3^-)-F_3^{(1)}+\frac{\partial F_1^{(2)}}{\partial x_1}\right].$$

For the boundary values of $\psi^\pm$ of the function ψ on the surfaces of $x_1=\pm d$ of the band, we obtain the equation

$$\frac{\partial^2\psi^\pm}{\partial x_1^2}-\frac{1}{c_4^2}\frac{\partial^2\psi^\pm}{\partial t^2}=\rho\,\alpha(1+\nu)\frac{\partial^2 T^\pm}{\partial t^2}-\frac{\partial F_1^\pm}{\partial x_1}+\frac{1}{h}\frac{\partial}{\partial x_1}\left(5M_{13}\pm 3N_{13}^{(d)}\right)$$

and initial conditions

$$\psi^\pm(x_1,0)=\psi(x_1,\pm h,0)=0,\quad \dot\psi^\pm(x_1,0)=\dot\psi(x_1,\pm h,0)=-\frac{2\alpha E}{1-2\nu}\dot T^\pm(x_1,\pm h,0).$$

Considering the conditions for conjugation of the values of the functions ψ and σ_{13} at the vertices of the rectangle of the strip cross-section [9] on the functions N, M, N_{13}, M_{13}, $\Psi^\pm$ along the coordinate x_1, we obtain the following boundary conditions

$$\frac{d^2N(\pm d,t)}{dt^2}+\frac{3c_4^2}{h^2}N(\pm d,t)=\frac{c_4^2}{2h}\left[F_3^+(\pm d,t)-F_3^-(\pm d,t)\right]-c_4^2\rho\,\alpha(1+\nu)\frac{d^2T_1(\pm d,t)}{dt^2}, \quad (11)$$
$$\frac{d^2M(\pm d,t)}{dt^2}+\frac{15c_4^2}{h^2}M(\pm d,t)=3c_4^2\left\{\frac{1}{2}\left[F_3^+(\pm d,t)-F_3^-(\pm d,t)\right]+F_3^{(1)}(\pm d,t)\right\}$$
$$+\frac{3c_4^2}{2h}\left(\frac{dN_{13}}{dx_1}\right)_{x_1=\pm d}-c_4^2\rho\,\alpha(1+\nu)\frac{d^2T_2(\pm d,t)}{dt^2}, \quad (12)$$
$$\psi^\pm(\pm d,t)=0,\quad N_{13}(\pm d,t)=0,\quad M_{13}(\pm d,t)=0.$$

In accordance with the inhomogeneous boundary conditions (11)–(12) on the functions N and M, we present the solutions of the first two equations of the system (10) in the form

$$N(x_1,t)=\frac{1}{2}\left\{[N_*(d,t)+N_{**}(-d,t)]+\frac{x_1}{d}\left[N_*(d,t)-N_{**}(-d,t)\right]\right\}+N_0(x_1,t),$$
$$M(x_1,t)=\frac{1}{2}\left\{[M_*(d,t)+M_{**}(-d,t)]+\frac{x_1}{d}\left[M_*(d,t)-M_{**}(-d,t)\right]\right\}+M_0(x_1,t).$$

Then the functions $N_0(x_1,t)$ and $M_0(x_1,t)$ are defined from the following equations

$$\left(\frac{d^2}{dx_1^2}-\frac{1}{c_1^2}\frac{d^2}{dt^2}-\frac{3}{h^2}\right)N_0=\Phi_1-\frac{3}{2h^2}(\psi^++\psi^-)+\left(\frac{1}{c_1^2}\frac{d^2}{dt^2}+\frac{3}{h^2}\right)\frac{1}{2}\left[(N_*+N_{**})+\frac{x_1}{d}(N_*-N_{**})\right],$$
$$\left(\frac{d^2}{dx_1^2}-\frac{1}{c_1^2}\frac{d^2}{dt^2}-\frac{15}{h^2}\right)M_0=\Phi_2-\frac{15}{2h^2}(\psi^+-\psi^-)+\left(\frac{1}{c_1^2}\frac{d^2}{dt^2}+\frac{15}{h^2}\right)\frac{1}{2}\left[(M_*+M_{**})+\frac{x_1}{d}(M_*+M_{**})\right]$$

under uniform boundary conditions on the surfaces $x_1=\pm d$. Here, $N_*=N(d,t)$, $N_{**}=N(-d,t)$, $M_*=M(d,t)$, $M_{**}=M(-d,t)$.

The system of interdependent equations (10) is solved by applying the finite integral transform [10] with kernel $K(\alpha_k, x_1) = \frac{\sin\alpha_k(x_1+d)}{\sqrt{d}}$, where $\alpha_k = \frac{\pi k}{2d}$ and the Laplace transform in time t. After the transformations, we obtain the expressions of the functions $\psi^{\pm}$, N, M, N_{13}, M_{13} in the form

$$\psi^{\pm}(x_1,t) = \sum_{k=1}^{\infty} \left\langle \int_0^t \left\{ \frac{c_4^3}{\alpha_k} \sin\frac{\alpha_k(t-t_0)}{c_4} \int_{-d}^{d} \frac{dF_1^{\pm}(x_1,t_0)}{dx_1} K(\alpha_k,x_1)\,dx_1 \right.\right.$$
$$\left.\left. +\rho\,\alpha(1+\nu)c_4\alpha_k \sin\frac{\alpha_k t_0}{c_4}\,\tilde{T}^{\pm}(\alpha_k, t-t_0)\right\} dt_0 - \rho\,\alpha(1+\nu)c_4^2\,\tilde{T}^{\pm}(\alpha_k,0) \right\rangle K(\alpha_k,x_1), \quad (13)$$

$$N(x_1,t) = \frac{1}{2}\left[(N_* + N_{**}) + \frac{x_1}{d}(N_* - N_{**})\right] + \sum_{k=1}^{\infty} \left\langle \int_0^t \left\{ \frac{3c_1^2}{2h^2}\left[\tilde{\psi}^{+}(\alpha_0,t_0) + \tilde{\psi}^{-}(\alpha_k,t_0)\right] - \Phi_1(\alpha_k,t_0)\right\}\right.$$
$$\times \frac{1}{c_1\sqrt{\alpha_k^2+\frac{3}{h^2}}} \sin\left[c_1\sqrt{\alpha_k^2+\frac{3}{h^2}}(t-t_0)\right] dt_0 + \frac{1}{\alpha_0}\left[(-1)^k N_*(0) - N_{**}(0)\right]$$
$$\left. -\,\alpha_k \frac{c_1}{\sqrt{\alpha_k^2+\frac{3}{h^2}}} \int_0^t \left[N_*(t_0)(-1)^k - N_{**}(t_0)\right] \sin\left[c_1\sqrt{\alpha_k^2+\frac{3}{h^2}}(t-t_0)\right] dt_0 \right\rangle K(\alpha_k,x_1), \quad (14)$$

$$M(x_1,t) = \frac{1}{2}\left[(M_* + M_{**}) + \frac{x_1}{d}(M_* - M_{**})\right] + \sum_{k=1}^{\infty} \left\langle \int_0^t \left\{ \frac{15c_1^2}{2h^2}\left[\tilde{\psi}^{+}(\alpha_k,t_0) - \tilde{\psi}^{-}(\alpha_k t_0)\right] - \tilde{\Phi}_2(\alpha_k,t_0)\right\}\right.$$
$$\times \frac{1}{c_1\sqrt{\alpha_k^2+\frac{15}{h^2}}} \sin\left[c_1\sqrt{\alpha_k^2+\frac{15}{h^2}}(t-t_0)\right] dt_0 + \frac{1}{\alpha_k}\left[(-1)^k M_*(0) - M_{**}(0)\right]$$
$$\left. -\,\alpha_k \frac{c_1}{\sqrt{\alpha_k^2+\frac{15}{h^2}}} \int_0^t \left[M_*(t_0)(-1)^k - M_{**}(t_0)\right] \sin\left[c_1\sqrt{\alpha_k^2+\frac{15}{h^2}}(t-t_0)\right] dt_0 \right\rangle K(\alpha_k,x_1), \quad (15)$$

$$N_{13}(x_1,t) = -\sum_{k=1}^{\infty} \int_0^t \left\{ \frac{1}{2h}\left[\tilde{F}_1^{+}(\alpha_k,t_0) + \tilde{F}_1^{-}(\alpha_k,t_0)\right] + \int_{-d}^{d} \frac{dF_3^{(1)}(x_1,t_0)}{dx_1} K(\alpha_k,x_1)\,dx_1 \right\}$$
$$\times \frac{c_2}{\sqrt{\alpha_k^2+\frac{3}{h^2}}} \sin\left[c_2\sqrt{\alpha_k^2+\frac{3}{h^2}}(t-t_0)\right] dt_0\, K(\alpha_k,x_1), \quad (16)$$

$$M_{13}(x_1,t) = \sum_{k=1}^{\infty} \int_0^t \left\langle \frac{3}{h}\tilde{F}_1^{(1)}(\alpha_k,t_0) - \frac{3}{2h}\left[\tilde{F}_1^{+}(\alpha_k,t_0) + \tilde{F}_1^{-}(\alpha_k,t_0)\right] + \frac{3}{2hd}\left[N_*(t_0) - N_{**}(t_0)\right]\right.$$
$$\left. \times \frac{1-(-1)^k}{\alpha_k\sqrt{d}} - \int_{-d}^{d} \frac{dF_3^{(2)}(x_1,t)}{dx_1} K(\alpha_k,x_1)\,dx_1 \right\rangle \frac{c_2}{\sqrt{\alpha_k^2+\frac{15}{h^2}}}$$
$$\times \sin\left[c_2\sqrt{\alpha_k^2+\frac{15}{h^2}}(t-t_0)\right] dt_0\, K(\alpha_k,x_1). \quad (17)$$

Here, the boundary values of the functions N and M at the ends of $x_1 = \pm d$ of the strip are as follows

$$N(\pm d,t) = \int_0^t \left\{ \frac{c_4^2}{2h}\left[F_3^{+}(\pm d, t-t_0) - F_3^{-}(\pm d, t-t_0)\right] \frac{h}{3c_4} \sin\frac{3c_4 t_0}{h} \right.$$
$$\left. - \frac{3c_4^3\rho\,\alpha(1+\nu)}{h} T_1(\pm d, t-t_0) \sin\frac{3c_4 t_0}{h} \right\} dt_0 - c_4^2\rho\,\alpha(1+\nu)\,T_1(\pm d, 0),$$

$$M(\pm d,t) = \int_0^t \left\langle \left\{ \frac{3c_4^2}{2}\left[F_3^{+}(\pm d, t-t_0) - F_3^{-}(\pm d, t-t_0)\right] + 3c_4^2 F_3^{(1)}(\pm d, t-t_0)\right.\right.$$

$$+ \frac{3c_4^2}{2h}\left(\frac{dN_{13}(x_1,t-t_0)}{dx_1}\right)_{x_1=\pm d}\Bigg\}\frac{h}{15c_4}\sin\frac{15c_4t_0}{h} - \frac{15c_4^3}{h}\rho\,\alpha(1+\nu)\,T_2(\pm d,t-t_0) \times \sin\frac{15c_4t_0}{h}\Bigg\rangle dt_0 - c_4^2\rho\,\alpha(1+\nu)\,T_2(\pm d,0).$$

According to the expressions (13)–(17) of the functions $\psi^{\pm}$, N, M, N_{13}, M_{13} functions, ψ and σ_{13} are written in the form. Here, the boundary values of the functions N and M at the ends of $x_1 = \pm d$ of the strip are equal

$$\psi(x_1,x_3,t) = \frac{3}{2}\left(1-\frac{x_3^2}{h^2}\right)N(x_1,t) + \frac{5}{2}\left(\frac{x_3}{h}-\frac{x_3^3}{h^3}\right)M(x_1,t) - \frac{1}{4}\left(1-3\frac{x_3^2}{h^2}\right)\psi_*(x_1,t) - \frac{1}{4}\left(3\frac{x_3}{h}-5\frac{x_3^3}{h^3}\right)\psi_{**}(x_1,t),$$

$$\sigma_{13}(x_1,x_3,t) = \frac{3}{2}\left(1-\frac{x_3^2}{h^2}\right)N_{13}(x_1,t) + \frac{5}{2}\left(\frac{x_3}{h}-\frac{x_3^3}{h^3}\right)M_{13}(x_1,t).$$

Here, $\Psi_*(x_1,t) = \Psi^+ + \Psi^-$, $\Psi_{**}(x_1,t) = \Psi^+ - \Psi^-$.

Accordingly, for the component σ_{11} of the stress tensor, we obtain the expression

$$\sigma_{11}(x_1,t) = c_3^2\sum_{k=1}^{\infty}\int_0^t\Bigg\langle\rho(1+\nu)\Bigg\{\frac{\nu}{E}\left[\tilde{\psi}(\alpha_k,0)-\tilde{\psi}(\alpha_k,t-t_0)\,\alpha_k c_3\sin(\alpha_k c_3 t_0)\right] - \alpha\left[\tilde{T}(\alpha_k,0)-\tilde{T}(\alpha_k,t-t_0)\,\alpha_k c_3\sin(\alpha_k c_3 t_0)\right]\Bigg\} + \frac{\sin(\alpha_k c_3 t_0)}{\alpha_k c_3}\int_{-d}^{d}\frac{dF_1(x_1,t-t_0)}{dx_1}K(\alpha_k,x_1)\,dx_1\Bigg\rangle dt_0\,K(\alpha_k,x_1).$$

The stress components σ_{22} and σ_{33} are found using the algebraic relations (4). Note that the solutions to the quasi-static problem of thermoelasticity in stresses for the strip are given in [9].

To predict the strip carrying capacity from the known components $\sigma_{11}(x_1,x_3,t)$, $\sigma_{13}(x_1,x_3,t)$, $\sigma_{22}(x_1,x_3,t)$, $\sigma_{33}(x_1,x_3,t)$ of the stress tensor and temperature $T(x_1,x_3,t)$ are determined by the formula [2,4,6]

$$\sigma_i = \sqrt{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6\sigma_{13}^2}\Big/\sqrt{2}$$

the stress intensity σ_i and compare it, according to the Huber–Mises criterion, with the elastic strain limit σ_d of the strip material.

4. Conclusion

The obtained general solutions of the dynamic problem of thermoelasticity for a strip make it possible to predict the bearing capacity of a plate element in the form of a strip under nonstationary temperature and force actions. This technique allows us to estimate the resonant effects of the thermoelastic behavior of the strip at four types of natural frequencies of its mechanical vibrations

$$\omega_1 = c_1\sqrt{\alpha_k^2+\frac{3}{h^2}},\quad \omega_2 = c_1\sqrt{\alpha_k^2+\frac{15}{h^2}},\quad \omega_3 = c_2\sqrt{\alpha_k^2+\frac{3}{h^2}},\quad \omega_2 = c_2\sqrt{\alpha_k^2+\frac{15}{h^2}},$$

and is the scientific basis for the selection of safe temperature and power loading conditions of plate elements in the form of a strip.

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Розв'язок динамічної задачі термопружності у напруженнях для смуги

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Запропоновано методику побудови розв'язку двовимірної динамічної задачі термопружності у напруженнях для смуги прямокутного перерізу. За вихідну вибрана система рівнянь у напруженнях, що описує плоско-деформований стан смуги. Розвинуто методику побудови розв'язку даної задачі яка ґрунтується на апроксимації розподілів всіх компонент тензора динамічних напружень кубічними поліномами за товщинною координатою смуги. У результаті система вихідних двовимірних нестаціонарних рівнянь на ці компоненти зведена до системи одновимірних нестаціонарних рівнянь на інтегральні характеристики за товщинною координатою даних компонент. Для розв'язування отриманої системи рівнянь використано скінчене інтегральне перетворення за поперечною координатою смуги та інтегральне перетворення Лапласа за часовою змінною. Записано загальні розв'язки розглядуваної динамічної задачі термопружності за нестаціонарних температурних і силових дій на смугу. Запропоновано критерій для оцінки несучої здатності смуги. Виявлено резонансні частоти чотирьох типів для смуги за розглядуваних нестаціонарних дій.

Ключові слова: *смуга; нестаціонарні теплові і силові дії; плоско-деформований стан; компоненти тензора напружень; інтенсивності напружень; резонансні частоти.*