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**Vitaliy Krivda***, **Olha Sakno**, **Fedir Abramchuk**,
**Yevhen Haiek**, **Kostiantyn Kornilenko**,
**Anna Moskalenko**
Dnipro University of Technology
19, Dmytro Yavornytskyi Ave., Dnipro, 49005, Ukraine

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# AN ENHANCED TIRE ROLLING MODEL FOR IMPROVING STABILITY ON LOW-FRICTION SURFACES

***Summary.*** *Object of study: the mechanism for improving vehicle stability on low-friction surfaces within the active-safety control loop. Subject of study: the formation of lateral force in the tire–road interaction and its use for reproducing (accounting for) limit handling modes and adaptive stability control. Because many existing stability-assessment methods rely on simplified wheel/tire rolling models that do not capture the full set of factors governing vehicle dynamics, we develop a framework that explicitly represents the nonlinear nature of tire-road interaction and integrates tire-material properties, pavement microtexture, and dynamic loading into a single mathematical model. The practical significance is in the possibility of embedding the proposed model into ESP/ABS/TCS algorithms for online identification of road-friction parameters and adaptive tuning of intervention thresholds according to current thermoclimatic conditions. At the engineering-design level, the method enables the construction of stability "maps" for various operating scenarios and can be used for tire selection, test-maneuver planning, and calibration of yaw-stability functions. The proposed approach is novel in that it preserves universality while remaining extendable to account for tire-inflation pressure, tread wear, residual tread depth, and non-uniform axle load distribution. The results reveal a sigmoidal temperature sensitivity near 0 °C, a concave-down degradation of stability with increasing wetness, and a pronounced reduction in maximum lateral force capacity in icy scenarios. Comparison with quasi-linear estimates reveals that neglecting the post-peak decay of the lateral force leads to an overestimation of the tire's limit capability and an inflated stability margin during maneuvers. Overall, enhancing rolling models with explicit nonlinear effects is an effective tool for more accurate reproduction of limit modes and for reducing accident risk under reduced-friction conditions. Application domain: automotive production and automotive service operations.*

***Key words****: vehicle stability, tire rolling, effective coefficient of friction ($\mu_{eff}$), nonlinear model, slip angle, adhesion loss, pavement microtexture, tire temperature, wetting/icing, lateral force, ESP/ABS/TCS, stability maps.*

## 1. INTRODUCTION

Vehicle stability during motion is one of the fundamental operational properties that determine road safety, the reliability of the transport system, and the robustness of vehicle behaviour across a wide range of operating regimes. It governs the ability of a vehicle to maintain a prescribed trajectory, reject external disturbances, and remain controllable under diverse road conditions. In today's context of intensified road transport, growing traffic flows, and increasingly complex road-weather factors, ensuring stability becomes

* *Corresponding author. E-mail: vitaliy.krivda@gmail.com*

especially critical: even a slight reduction in the tire-road coefficient of friction can sharply degrade controllability, lengthen stopping distance, and trigger uncontrolled skids.

Operating on low-friction surfaces – wet, icy, or compacted snow pavements is particularly critical, as tire-road interaction becomes markedly more complex and nonlinear. Under these conditions, several processes substantially affect the generation of contact-patch forces: changes in the viscoelastic properties of rubber, a reduction in effective contact area, the formation of a thin water or ice film, pressure redistribution, and the localization of shear in the tread layer. These factors alter the balance between the adhesive and deformation components of friction, so that the tire-road system enters a mode of complex nonlinear interaction for which classical linear models lose adequacy.

Traditional theoretical approaches to stability assessment are based on linearized rolling models, in which the lateral force is assumed to be proportional to slip, and the friction coefficient is treated as a constant determined by the surface type. Such models are sufficiently accurate on dry asphalt but fail to capture the dynamics of the contact zone as friction decreases. They do not account for the deformational properties of the tire compound, the influence of pavement microtexture, thermoclimatic effects, or the stochastic variability of surface roughness. As a result, the computed relationships cannot adequately reproduce limit handling modes or determine the critical parameters that directly affect the operation of active safety systems, such as ABS, ESP, and TCS.

Contemporary automotive practice shows that improving road safety requires vehicle-dynamics models that are not only accurate but also adaptable in real-time. Stabilization, anti-lock braking, and traction-control systems rely on estimates of friction that can be significantly biased under rapid changes in temperature, moisture, or surface state. Imperfections in baseline models lead to two extreme scenarios: either delayed interventions, when the vehicle has already entered an unstable region, or overly conservative control that unnecessarily restricts vehicle performance. In both cases, the safety-controllability balance is disrupted, directly degrading operational effectiveness.

From the standpoint of tire rolling theory, a key unresolved issue is the description of the nonlinear dependence between lateral force and slip angle. Under real-world conditions, this relationship is S-shaped: an almost linear rise at small angles, followed by a saturation plateau and then a sharp drop caused by adhesion breakaway. The post-peak decay is crucial because it marks the onset of loss of directional stability. However, most simplified models represent only the pre-peak region, neglecting the breakaway phase and thus overestimating the vehicle’s limiting capability.

Accounting for thermoclimatic factors and surface roughness within a single unified model is challenging due to the differing physical nature of these processes. Temperature alters the elastic properties of rubber and shifts the balance between adhesion-driven and deformation-driven friction, whereas pavement microtexture forms mechanical interlocking with a saturating nature. In addition, the presence of water or ice films sharply reduces the adhesive component by introducing a hydrodynamic layer, further complicating the modelling. Hence, there is a need for an integrated model that couples a physically grounded dependence of the effective friction coefficient on temperature, wetting / icing, and microtexture with an analytical description of the nonlinear lateral force behaviour in the contact patch.

Given the above, the scientific problem is in the absence of rolling models capable of reproducing the combined nonlinear and stochastic processes of tire-road interaction under real operating conditions. Its solution requires an enhanced methodology that incorporates the physico-mechanical properties of tire materials, variable climatic parameters, and pavement characteristics. Implementing such an approach will yield more accurate analytical and experimental relationships, increase the fidelity of reproducing limit handling modes, enable effective adaptive control algorithms for active safety systems, and reduce the incidence of crashes caused by loss of stability on low-friction roads.

Therefore, improving rolling models by explicitly representing the nonlinear effects of tire-road interaction is not only a relevant scientific task but also a necessary prerequisite for advancing intelligent active-safety concepts. The development of such models will enhance vehicle control effectiveness, ensure predictable behaviour in critical situations, and support safe-driving strategies under variable friction conditions.

## 2. STATEMENT OF THE PROBLEM AND RELEVANCE OF THE STUDY

Ensuring vehicle stability on low-friction roads remains one of the most complex and most pressing issues in contemporary automotive engineering. Climate change, growing traffic volumes, and the increasing share of vehicles operated in urban environments lead to a substantial complication of road situations. Statistics indicate that a significant proportion of road accidents occur due to drivers losing control of their vehicles when driving on slippery or uneven surfaces. Under such conditions, even modern active-safety systems based on classical stability estimation algorithms do not always provide an adequate level of protection.

The complexity of the problem is in the fact that existing stability assessment methods rely on simplified rolling models that fail to account for the full set of factors influencing vehicle dynamics. In particular, linear approaches ignore changes in rubber properties at different temperatures, variations of the friction coefficient with pavement roughness and moisture, as well as the complex nonlinear relationships between friction force and slip angle. As a consequence, the obtained results often exhibit limited accuracy in practice, which precludes their effective use in vehicle control systems; that is, they produce systematic errors in peak and limit modes that directly affect control decisions and can degrade controllability precisely when it is most needed.

The relevance of this research is determined by the need to develop a new methodology that will improve existing rolling models and provide a more accurate reproduction (accounting) of vehicle behaviour under reduced-friction conditions. Such a methodology must reflect the nonlinear character of tire-road interaction processes and integrate factors related to tire materials, pavement microtexture, and dynamic loading. The expected outcome is improved accuracy in determining limit handling modes, leading to reduced accident rates, optimized ABS and ESP algorithms, and enhanced overall road safety.

The aim of the study is to develop and validate an enhanced nonlinear tire-rolling model, and to formulate criteria and an algorithm for adaptive improvement of vehicle stability on low-friction surfaces using stabilization and control systems (ESP/ABS/TCS).

To achieve this aim, the following tasks are addressed:

– to perform a critical analysis of existing tire-rolling models and identify their limitations under reduced friction;
– to develop an improved mathematical model that accounts for the nonlinear dependence of friction force on slip angle and the properties of tire materials;
– to investigate the influence of road and climatic factors on vehicle-stability parameters;
– to test the proposed model on typical road scenarios;
– to assess its effectiveness in predicting critical driving modes.

## 3. ANALYSIS OF THE RECENT RESEARCH AND PUBLICATIONS

The problem of ensuring vehicle stability and improving tire-rolling models has been actively studied by both domestic and international researchers. The textbook by V. Sakhno, V. Poliakov, V. Bilichenko at all systematizes basic approaches to determining vehicle operational properties, including dynamics, efficiency, stability, and controllability [1], emphasizing the importance of theoretical models for vehicle assessment. However, most presented relations are linear, which limits their applicability under complex driving conditions.

In the works of M. Podryhalo, V. Harmash at al., the need to move from simplified tire-contact models to more advanced approaches that capture nonlinear and dynamic characteristics is highlighted [2]. The proposed models enable a more in-depth analysis of tire rolling, particularly under reduced friction, although their practical application still requires further validation.

Klets, Pavlov, and Yarovyi examined the maneuverability of multi-axle vehicles with two steering platforms [3]. They proposed assessing stability and controllability by the criterion of maneuver execution time, which improves the fidelity of reproducing vehicle behavior under real operating conditions. This research is directly relevant to building adaptive motion models for slippery roads.

Recent studies on information technologies in vehicle maintenance emphasize the integration of telematics and condition monitoring systems into transport control processes [4]. Such approaches enable new algorithms for reproducing (accounting for) vehicle stability and improve the effectiveness of active safety systems.

International studies also confirm the relevance of the topic. Wolfgang Dick and Michael Holle consider combined steering control systems that enhance vehicle stability by enabling adaptive changes of driving modes [5]. The monograph by Charles L. Phillips and Royse D. Harbor analyzes modern automatic-control systems that utilize tire-rolling models as components of adaptive control [6]. Automatic trajectory tracking using the drive torques of the front and rear steering allows nonlinear tire effects to be incorporated, thereby improving vehicle stability under slippery conditions [7]. A Hardware-in-the-Loop (HiL) simulation environment enables the consideration of reaction forces for target loads during vehicle motion [8].

With the development of electronics and signal-processing technologies in the automotive industry, there is growing interest in enhancing vehicle stability via adaptive control of steer-by-wire systems [9, 10, 11]. However, studies have shown that suspension design and motion are dominant factors for stability. Research [12, 13], aimed at stability through observation-based and sliding-mode control with disturbance rejection, demonstrated significant deviations from expected outcomes due to consideration of a limited set of factors.

A kinematic model of a vehicle with four-wheel drive (4WD) and four-wheel steering (4WS), incorporating wheel slip on the surface, considers torque delivery to all four wheels and uses adaptive and predictive controllers to track the trajectory via front and rear steering angles [14]. The basic bicycle chassis model is augmented by a torque distribution mechanism among all four wheels and a refined description of slip in the contact patch, which enables a dynamic longitudinal-lateral model that accounts for 4WD / 4WS. According to stability analyses for platoons of autonomous vehicles equipped with V2X and for human-driven vehicles, an adaptive-control strategy is required [15]. Thus, an adaptive feedback-gain strategy can effectively improve the disturbance rejection of the traffic flow, reduce oscillations in speed and acceleration, and promote vehicle motion stability.

A significant portion of recent work uses nonlinear / empirical models, such as Pacejka, but their parameters are sensitive to temperature, moisture, and surface roughness. The proposed model makes these influences explicit and physically interpretable. A synthesis of the above studies indicates that, despite substantial progress in vehicle performance theory, there remain unresolved tasks in constructing tire-rolling models that adequately capture the nonlinear tire-road interaction. It defines the relevance of further research in this direction.

## 4. PRESENTATION OF THE MAIN MATERIAL

The problem of improving vehicle stability on slippery roads is directly linked to the adequacy of tire-rolling models used to describe tire-road interaction. Modern operating conditions are characterized by substantial variability of road-weather factors, giving rise to a wide spectrum of tire-road interaction modes. The accuracy with which this interaction is described determines the reliability of computed operational properties and, consequently, the ability of engineers to accurately predict stability and controllability in demanding situations.

In classical vehicle performance theory, tire rolling is traditionally described by a relationship between friction force and the magnitude of slip, assumed to be linear. This approach was historically based on experiments conducted on dry asphalt with relatively high friction. Under such conditions, the tire-road contact exhibits stable properties, and deviations from linearity remain within acceptable limits. Accordingly, linear models provide satisfactory accuracy and they are widely used in coursework, engineering practice, and in the development of baseline active safety algorithms.

However, when driving on slippery surfaces covered with water, ice, or compacted snow, the interaction conditions differ significantly. The friction coefficient decreases severalfold, the effective contact area shrinks due to a water film, and slip processes become nonlinear. Under these conditions, the friction force does not increase proportionally with slip; instead, it exhibits sharp, step-like changes. After a critical slip angle is reached, adhesion breakaway occurs, resulting in a sudden loss of stability. Linear models that do not account for this effect show significant errors and do not reflect the actual vehicle dynamics. Moreover, classical approaches ignore the deformational properties of rubber, pavement microprofile, and temperature effects. In practice, tire rubber changes its mechanical characteristics with deformation-cycle frequency and temperature: as temperature drops, elasticity decreases, negatively affecting friction. Surface moisture and roughness introduce additional stochastic factors that substantially alter the contact conditions. Neglecting these processes in calculations leads to systematic errors in determining limit modes.

Classical rolling models were established during the mid-twentieth-century, when the growth of automotive transport was underway, and the need to accurately represent dynamic and stochastic factors was less acute. They rely on simplifying assumptions that allow relatively quick calculations and results consistent with operation on dry, hard-surfaced roads. That is, we further employ the standard linearized small-slip assumption with cornering stiffness Cα under a constant classical, constant tire-road coefficient of friction, with the understanding that the effects of temperature, wetting, and roughness are not represented in this approximation and will be captured later via effective coefficient of friction explicitly parameterized by tire temperature ($T$), pavement microtexture ($k_r$), and wetting / icing level ($k_w$).

The basis of such models is the assumption of a linear relationship between wheel slip and the friction force generated in the contact patch. In a simplified form, this dependence can be expressed as:

$$F_y = \mu \cdot N \cdot \alpha, \tag{1}$$

where $F_y$ – is the lateral force acting on the wheel; $\mu$ is the coefficient of friction between the tire and the road; $N$ is the normal load on the wheel; and $\alpha$ is the slip angle.

This model allows one to determine the lateral resisting force at small slip angles ($\alpha<5^\circ$) and under high-friction conditions ($\mu\approx0.7–0.9$). Within this parameter range, the calculations provide satisfactory accuracy, which explains their wide use in engineering practice, textbooks, and methodological materials [1, 2]. However, at moderate and large slip angles ($\alpha>8–10^\circ$), the model's adequacy degrades sharply.

Within known approaches, linearized tire models remain a useful tool for initial estimates and for analyzing motion on dry pavements. Nevertheless, they have several limitations: a narrow range of applicability beyond small slip angles, an incomplete description of post-peak effects, and sensitivity to changes in thermoclimatic and surface conditions. In view of this, the present study employs an enhanced nonlinear description $F_y(\alpha)$with an explicit dependence $\mu_{eff}(T, k_w, k_r)$, which enables a more accurate reproduction of limit modes on low-friction surfaces and better agreement between computations and experimental observations.

In addition, classical models ignore a several factors that substantially affect wheel rolling. They do not account for the deformational properties of rubber, which vary with temperature and loading rate, as well as pavement microtexture and profile, which determine the effective contact area. Additionally, they overlook oscillatory processes in the tire that form a non-uniform pressure distribution in the contact patch, and dynamic changes in the friction coefficient under different climatic conditions.

Although classical rolling models formed the foundation of automotive theory and long served engineering needs, they cannot be regarded as adequate under modern operating conditions. Their limitations become especially pronounced on slippery roads, where the tire-road interaction exhibits a strongly nonlinear character. It necessitates the development of improved mathematical models that enable more accurate predictions of vehicle behavior in critical modes and, consequently, improved stability.

Experimental studies show that tire-road interaction is markedly nonlinear, particularly when the friction coefficient is reduced. At small slip angles, the growth of lateral force can indeed be approximated as proportional to $\alpha$. However, upon reaching a critical value $\alpha_{cr}$, the dependence approaches a saturation plateau, after which a decrease in force is observed due to partial or complete adhesion breakaway. This behaviour yields an S-shaped lateral force-slip angle curve.

Traditionally, these processes are described by the Pacejka model, which reproduces the characteristic S-shape. However, this model contains purely empirical coefficients *B, C, D, E* that lack a direct physical interpretation and require repeated experimental calibration for different conditions. To address this drawback, we propose an enhanced approach in which the effective friction coefficient is introduced as a function of operating factors, while the lateral-force dependence itself is described by an approximation that explicitly accounts for loss of stability. The proposed model is given by:

$$F_y(\alpha, T, k_r, k_w) = \mu_{eff}(T, k_r, k_w) \cdot N \cdot \frac{\alpha}{\alpha_{cr}} \cdot e^{-(\frac{\alpha}{\alpha_{cr}})^2}, \tag{2}$$

where $F_y$ is the lateral force acting on the wheel; $N$ is the normal load on the wheel; $\alpha$ is the slip angle; $\alpha_{\mathrm{cr}}$ is the critical slip angle at which the maximum lateral force is attained; and $\mu_{eff}(T, k_r, k_w)$ is the effective coefficient of friction, dependent on tire temperature $T$, pavement roughness $k_r$, and surface wetting $k_w$.

At small values of $\alpha$, the dependence approaches linearity, consistent with classical models. Near $\alpha \approx \alpha_{cr}$, the function attains a maximum determined by the effective friction coefficient and the load. When the slip angle exceeds the critical value, the lateral force decreases exponentially, corresponding to the adhesion breakaway. The function $\mu_{eff}(T, k_r, k_w)$ can be approximated by:

$$\mu_{eff}(T, k_r, k_w) = \mu_0 \cdot (1 - \beta_T(T_0 - T)) \cdot (1 - \beta_w k_w) \cdot (1 - \beta_r k_r), \tag{3}$$

where $\mu_0$ is the baseline friction coefficient for dry asphalt at temperature $T_0$; $\beta_T$ is the temperature-dependent reduction coefficient; $\beta_w$ is the moisture-influence coefficient; and $\beta_r$ is the surface roughness influence coefficient.

Dependencies (2) and (3) were calibrated using open experimental materials from proving ground and laboratory tire tests: lateral force-slip angle curves for different surfaces (dry / wet / ice) and consolidated intervals of the effective friction coefficient $\mu$ that account for micro-(macro)texture and temperature sensitivity [16–19]. The sources used provide examples of S-shaped $F_y(\alpha)$ curves with post-peak decay under low-friction conditions, as well as public ranges of $\mu$ for dry, wet, and icy surfaces, which enabled a physically meaningful and reproducible tuning of the model parameters.

Therefore, the model combines physically grounded tire and pavement parameters with a mathematical function that reproduces the nonlinear dynamics of adhesion breakaway. Its application makes it possible toenables the prediction of vehicle behaviour under slip more accurately and to assess critical driving modes.

The adequacy of stability prediction on a slippery road is determined not only by the choice of the lateral force-slip angle law, but also by properly accounting for environmental factors that change the "Lever Arm" of tire-surface interaction. The key among them are tire temperature, surface state (presence of a water / ice film), and pavement microtexture. Within the proposed approach, these factors are reduced to an effective friction coefficient that enters the nonlinear law of lateral force formation in the contact patch. This decomposition separates the "vertical" channel of environmental influences (via $\mu_{eff}$) from the "horizontal" channel of deformational kinematic effects (via $\alpha/\alpha_{\mathrm{cr}}$ and the exponential factor modelling adhesion breakaway).

To describe thermo-climatic sensitivity, a reduction / amplification multiplier of the baseline friction is used: $\mu_0$ is the baseline value for dry asphalt at temperature $T_0$; $k_w \in [0,1]$ is a dimensionless measure of wetting / icing (0 – dry; 5 – wet; 1 – ice); $k_r \in [0,1]$ is the normalized microtexture; $\beta_T$ is the temperature sensitivity (5–10 % per 10 °C within typical winter modes); $\beta_w$ is the intensity of friction degradation due to water / ice film; and $\beta_r$ is the roughness-influence coefficient. Such parameterization preserves the physical interpretability of the coefficients and allows their calibration in proving ground / on-road tests. The maximum lateral force in our model is attained near $\alpha \approx \alpha_{cr}$ and equals:

$$F_{y,max} = \mu_{eff} N \frac{\alpha}{\alpha_{cr}} e^{-1} = \frac{\mu_{eff} N}{e}, \quad (4)$$

that is, it decreases approximately in 2.718 times relative to the quasi-linear upper bound $\mu_{eff} N$ due to unavoidable breakaway losses in the contact patch. The transition from the single wheel model to the vehicle level is performed by summing the lateral reactions of all wheels, accounting for static and dynamic load transfer:

$$F_{y,\Sigma,max} = \sum_i F_{y,\max}^{(i)}. \quad (5)$$

In the numerical examples, a uniform load distribution is assumed, which yields conservative estimates of $K_s$; if needed, the lateral and longitudinal load transfer under braking / turning is considered. For assessing the stability of the entire vehicle, it is advisable to aggregate the tire's maximum lateral reaction over the four contact patches for a passenger car with uniform loading:

$$F_{y,\Sigma,max} = 4 \cdot F_{y,\max}. \quad (6)$$

Below is a numerical illustration for a typical passenger car ($m$ = 1500 kg, $l$ = 2.6 m, $h$ = 0.55 m; a single contact patch has $N$ = 3500N). We take $\mu_0$ = 0.85 at $T_0$ = 20 °C; $\beta_T$ = 0.07° °C (≈7 % per 10 °C), $\beta_w$ = 0.8 (strong degradation on ice), and $\beta_r$ = 0.2. The generalized stability criterion is computed as:

$$K_s = \frac{F_{y,\Sigma,\max}}{mgh/l}, \quad (7)$$

in the denominator is the estimate of the lateral load that must be resisted to maintain directional stability during a maneuver; for the given parameters, $mgh/l \approx 3113$N. Thus, it has been established (Table 1, Fig. 1):

*Table 1*

**Influence of temperature, moisture, and surface roughness on stability indicators**

| Surface scenario | $T$, °C | $k_w$ | $k_r$ | $\mu_{eff}$ | $F_{y,\max}$, N | $F_{y,\Sigma,\max}$, N | $K_s$ |
|---|---|---|---|---|---|---|---|
| Dry, rough asphalt | 20 | 0.0 | 0.6 | 0.952 | 1227 | 4908 | 1.58 |
| Wet asphalt | 5 | 0.5 | 0.6 | 0.511 | 659 | 2636 | 0.85 |
| Ice / compacted snow | −5 | 1.0 | 0.1 | 0.142 | 183 | 732 | 0.24 |

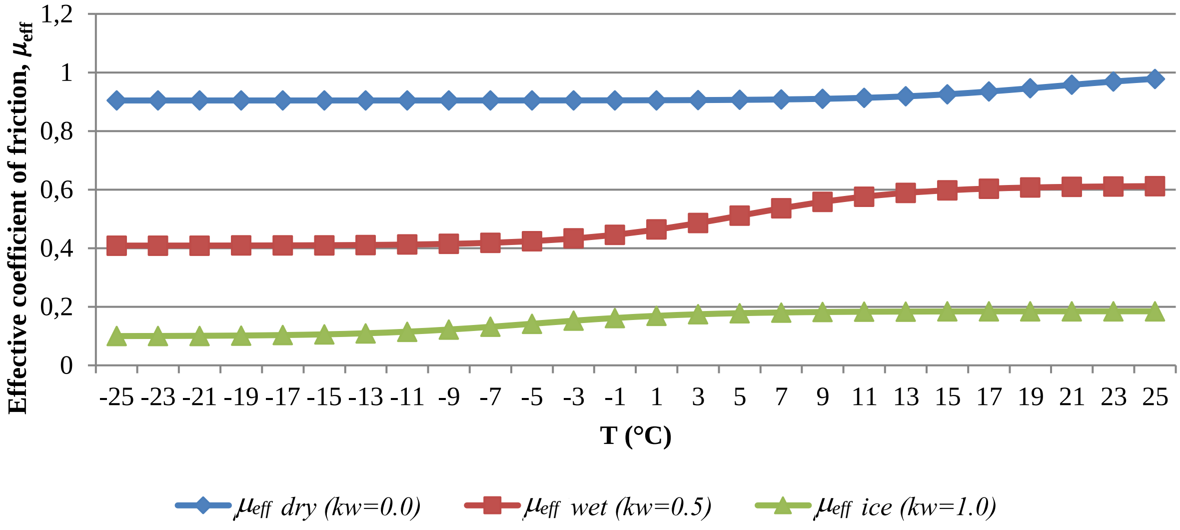


*Fig. 1. Dependence of $\mu_{eff}$ on temperature for different scenarios (dry, wet, ice) at fixed surface roughness*

The obtained $F_y(\alpha)$ curves for the "dry / wet / ice" scenarios reproduce the expected S-shape with a maximum near $\alpha_{cr}$ and a post-peak decay. According to the generalized criterion $K_s$, the transition from dry rough asphalt to wet pavement reduces stability by roughly a factor of two – from $K_s \gtrsim 1.5$ to $K_s \approx 0.8$–$0.9$ – whereas icing decreases the stability margin by a factor of 6–7 to $K_s \approx 0.2$–$0.25$. It is consistent with practical observations: on ice, even small slip angles quickly drive the system into the post-peak mode, where any further increase in $\alpha$ is accompanied by a drop in lateral force. The proposed exponential factor in the law $F_y(\alpha)$ yields conservative estimates of the maxima and explains why models without a post-peak term systematically overestimate tire limit capability. The application-level interpretation is as follows: during online estimation of $\mu_{eff}$ and $\alpha_{cr}$, the ESP module should reduce permissible lateral accelerations and steering angle gradients in proportion to the degradation of $\mu_{eff}$, preventing entry into the sharp decay region of $F_y$.

The resulting values show the expected transition from confident stability, $K_s$>1, on dry rough asphalt to borderline and unstable modes on wet and icy surfaces. Comparison with the "classical" $\mu N$ estimate underscores the practical importance of the breakaway exponential factor: it provides a conservative (and closer to real maneuver tests) upper bound for lateral force near $\alpha_{cr}$.

The parameter $\alpha_{cr}$ itself is a function of temperature and surface condition: as $\mu_{eff}$ decreases, the critical angle shifts to smaller values due to faster localization of shear in the rubber layer and the loss of the adhesive component. In practice, this means that ESP / ABS algorithms must adapt not only to the reduction in the "Height" of the $F_y(\alpha)$ curve but also to its compression along the $\alpha$ axis, lowering allowable steering angle gradients and target longitudinal decelerations.

Calibration of $\beta_T, \beta_w, \beta_r$ is performed using a rig or on-road tests with measurements of lateral / longitudinal reactions ($\mu$-split plates, circular track, S-turn). In particular, $\beta_T$ is identified via a series of runs in a thermal chamber or under natural cooling with tread temperature control; $\beta_w$ via gradations of water-film thickness/icing degree; and $\beta_r$ via profilometry/noise-based microtexture tomography with subsequent normalization to [0,1]. Because $\mu_{eff}$ in the model has a transparent physical interpretation, the calibration coefficients are robustly transferable across different tires of the same size after a one-time tuning. Thus, it has been established:

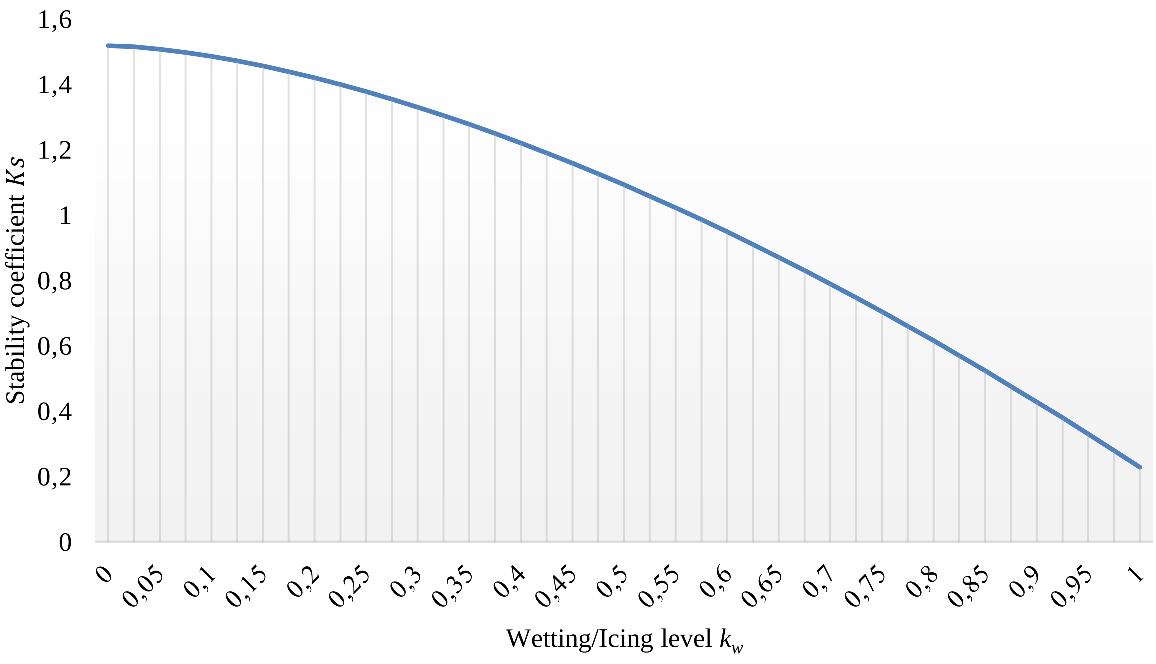


*Fig. 2. Plot of the nonlinear dependence of the stability criterion $K_s$ on the wetting level $k_w$*

It is important to emphasize that the proposed form $\mu_{eff}(T, k_r, k_w)$ is minimally sufficient for constructing adaptive control laws. If necessary, it can be enriched with additional determinants (tire inflation pressure, tread age / wear, normalized load $\lambda = N/N_{\text{ref}}$), as well as proxy signals from telematics (moisture / temperature estimates from a weather model, spectral features of micro-slip inferred from ABS module data). In this case, the transition from “offline calibration” to “online identification” of $\mu_{eff}$ occurs naturally and opens the way to implementing model predictive control (MPC) for yaw stabilization on slippery surfaces.

From an applied perspective, the numerical examples and the table show that even moderate cooling and wetting (the transition from 20 °C, dry, to 5 °C, wet) reduces the aggregate lateral load carrying capacity approximately in two times, whereas icing reduces it in 6–7 times. Accordingly, the target limits for anti-skid systems should be scaled at least in the same proportion, and trajectory planners should constrain lateral accelerations and corner-entry speeds in accordance with current estimates of $\mu_{eff}$.

In this study, it is assumed that tire-road interaction within the contact patch under reduced friction conditions is strongly nonlinear and cannot be adequately described by a linear proportionality between lateral force and slip angle. The effective coefficient of friction is treated as a function of environmental and surface factors, specifically, tire temperature, the level of wetting or icing, and pavement microtexture. The effects of these factors on friction have different physical origins: temperature sensitivity is associated with viscoelastic relaxation of the rubber and phase transitions in the water film; degradation under wetting / icing is caused by a reduction of the adhesive component and an increase in hydrodynamic effects; the beneficial effect of roughness arises from enhanced mechanical interlocking. To preserve the physical interpretability of parameters, the effective friction coefficient is modelled as a multiplicative combination of dimensionless factors, including temperature, moisture, and roughness, each with a nonlinear saturating form or a sharp transition near critical points.

The critical slip angle $\alpha_{cr}$ is considered a decreasing function of available friction: as $\mu_{eff}$ declines, the elastic deformation zone of the contact patch contracts and the transition to localized shear occurs earlier. In forming the lateral force, three characteristic regions are assumed: an initial near-linear rise, a saturation region near the critical angle, and a post-peak decay caused by adhesion breakaway. The proposed per-wheel lateral force formula combines these effects via a dimensionless relative slip parameter and an exponential damping factor, which is interpreted as an integral measure of losses in the contact patch during the development of plastic shear. Thus, it has been established:

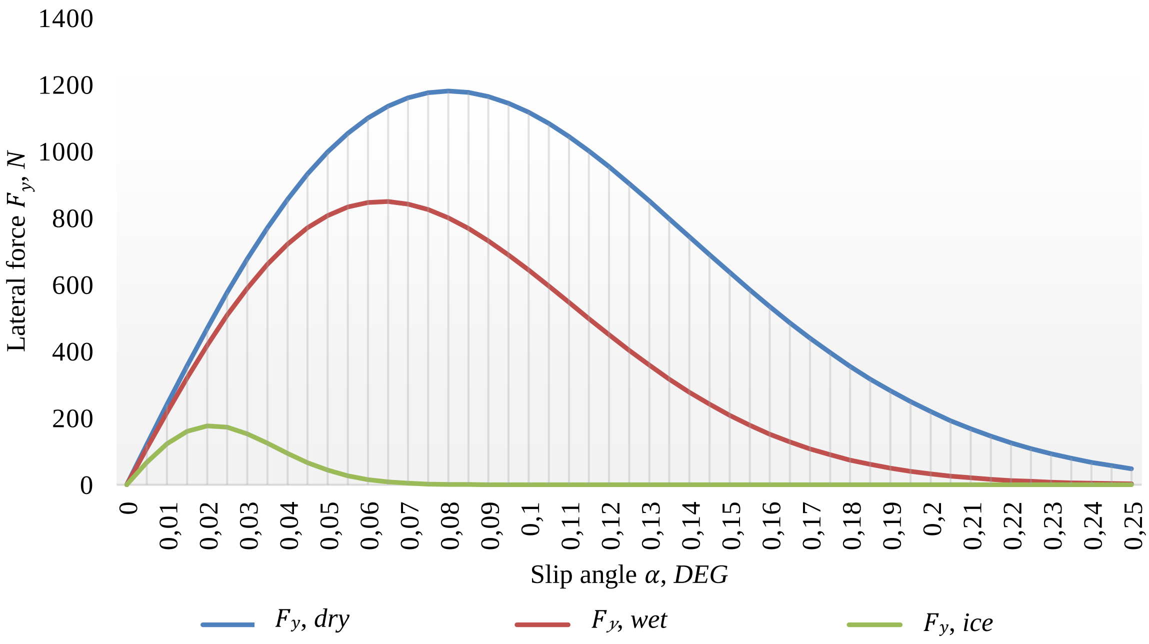


*Fig. 3. Nonlinear $F_y(\alpha)$ curves for different road surface conditions at $T = 0$ °C*

These assumptions are deemed to provide sufficient universality for adapting the model to different tires of the same stiffness class after a one-time coefficient calibration, and they enable lateral force-slip angle curves, and derived stability indicators, consistent with observations, suitable for integration into vehicle active safety algorithms.

The proposed methodology is suitable for integration into the active safety control loop of a vehicle and for use in engineering practice for stability assessment. At the vehicle systems level, it is advisable to implement it as a road friction estimation module that identifies $\mu_{eff}$ and $\alpha_{cr}$ in real-time using onboard sensor data. Data sources include wheel rotational speeds, steering angle, longitudinal and lateral accelerations, brake system pressure, ambient and tire temperatures, as well as telematics signals on precipitation and road surface state. The surface state estimate is obtained by minimizing the discrepancy between measured reactions and model predictions of lateral force in the parameter space $(T, k_w, k_r)$; the updated estimate $\mu_{eff}$ is then supplied to the ESP/ABS/TCS algorithms to adapt intervention thresholds and limit target lateral accelerations.

In the design-calculation cycle, the methodology is used to generate stability "maps" that specify allowable corner-entry speeds, trajectory radii, and steering gradients under different road-weather scenarios. Such maps can be built from the dependencies generated in the Excel workbook and employed to tune test maneuvers, select tires, and calibrate yaw stability functions. Additionally, the methodology is applicable to post-event analysis of crashes: by reconstructing weather conditions and surface state, one can obtain an estimate of the vehicle's limiting capability in a specific situation and compare it with the driver's actual actions.

## 5. CONCLUSIONS AND PERSPECTIVES FOR FURTHER RESEARCH

The proposed nonlinear model $F_y(\alpha)$ with an exponential post-peak decay reproduces the limit modes correctly: in winter scenarios, quasi-linear estimates overstate $F_{y,\max}$ by 25–35 %. When transitioning from "dry → wet", the stability margin decreases roughly in two times, and from "dry → ice" it decreases in 6–7 times. Introducing the effective coefficient $\mu_{eff}(T, k_w, k_r)$ enables real-time adaptation of ESP/ABS/TCS intervention thresholds, reducing delayed interventions and lowering the risk of entering the post-peak region of $F_y(\alpha)$. The methodology is suitable for building stability maps and calibrating winter settings. Further development should account for tire inflation pressure, tread wear, and non-uniform loading.

The methodology is consistent with current calibration practices for automotive systems. It can be implemented both as an engineering tool for constructing stability maps and as a software component of ESP/ABS/TCS with online parameter identification. The results lay the groundwork for extending the model to include tire pressure, tread wear, and uneven load distribution, as well as for integration with predictive MPC algorithms for motion stabilization on challenging surfaces.

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# УДОСКОНАЛЕНА МОДЕЛЬ КОЧЕННЯ КОЛЕСА ДЛЯ ПІДВИЩЕННЯ СТІЙКОСТІ НА СЛИЗЬКИХ ПОКРИТТЯХ

***Анотація.*** *Об'єкт дослідження – механізм підвищення стійкості автомобіля на слизьких покриттях у контурі активної безпеки. Предмет дослідження – процеси формування бічної сили у взаємодії "шина – дорога" та їх використання для відтворення граничних режимів і адаптивного керування стійкістю. Оскільки відомі методики оцінювання стійкості автомобіля ґрунтуються на спрощених моделях кочення колеса, які не враховують загальну сукупність факторів, що впливають на динаміку руху, розроблено механізм, який враховує нелінійність процесів взаємодії "шина – дорога", що дає змогу інтегрувати в математичну модель фактори, пов'язані із матеріалами шин, дорожнім мікропрофілем і динамічними навантаженнями. Практична значущість роботи полягає у можливості інтеграції запропонованої моделі в алгоритми ESP/ABS/TCS для онлайн-ідентифікації параметрів дорожнього зчеплення та адаптивного налаштування порогів втручання відповідно до актуальних термокліматичних умов. На рівні інженерного проєктування методика забезпечує побудову "карт" стійкості для різних сценаріїв експлуатації, які можна використовувати під час вибору шин, планування випробувальних маневрів і калібрування функцій курсової стабілізації. Запропонований підхід характеризується новизною, зберігаючи універсальність, і може бути розширений з урахуванням таких факторів, як тиск у шині, зношення протектора, залишкова висота рисунка протектора та нерівномірний розподіл навантаження між осями автомобіля. Отримані результати демонструють сигмоїдальну температурну чутливість у зоні близьких до нуля температур, деградацію (випукло вниз) стійкості у разі нарощування зволоження та істотне зменшення бічної вантажопідіймальності у крижаних сценаріях. Зіставлення із квазілінійними оцінками засвідчує, що ігнорування післяпікового спаду призводить до переоцінювання граничних можливостей шини та завищення запасу стійкості під час маневрування. Сукупність отриманих результатів підтверджує, що удосконалення моделей кочення колеса із явним урахуванням нелінійних ефектів є ефективним інструментом для підвищення точності відтворення (облікування) граничних режимів та зменшення аварійності в умовах зниженого зчеплення. Сфери застосування – виробнича сфера автомобільного транспорту, сфера сервісних послуг автомобільного транспорту.*

***Ключові слова:*** *стійкість автомобіля, кочення колеса, ефективний коефіцієнт зчеплення, нелінійна модель, кут ковзання, зрив зчеплення, мікрошорсткість, температура шини, зволоження / обмерзання, бічна сила, ESP/ABS/TCS, карти стійкості.*