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Optimizing Duration of Competitive Bidding: the Case of Ukrainian Public Procurement

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The paper considers the impact of the duration of competitive bidding on the organizer's payoff. Extending the duration of the bidding process has a two-fold effect on its results. On the one hand, it attracts a larger number of bidders, and competition between them leads to the better price for the auctioneer. On the other hand, such prolongation delays the receipt of money or necessary supplies, and time has value in itself. These two conflicting factors suggest that there must be an optimal duration of the bidding process. We propose a model that combines these considerations with the formal game-theoretic model of bidder's behavior. The procedure for determining the optimal duration of bidding has been developed using the institutional context of Ukrainian public procurement tenders. Derived optimality conditions have an intuitive economic interpretation and can be used either with an empirical distribution of bidders' reservation prices or with its analytical representation. Practical implementation of the proposed algorithm may improve the economic performance of the auctioneer, which may be particularly important for the public sector of the economy.

Keywords: *auction; procurement; bidding; duration; order statistic; Poisson process.*

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1. Introduction

The problem of effective use of public funds and assets has always attracted a lot of attention from economists, politicians, journalists, and the general public. One of the ways to improve the efficiency of the public sector is to employ competitive mechanisms of economic operations. Examples of such mechanisms are auctions and tenders, commonly known as competitive bidding.

Auctions are generally recognized as one of the most efficient ways of selling products in the imperfectly competitive environment. During the auction, buyers compete to obtain goods or services by offering increasingly higher prices. The popularity of auctions has significantly increased after the emergence of electronic trading platforms, which drastically reduced organizational costs. In the public sector, auctions are widely used for privatization, lease of public property, sale of seized assets, distribution of broadcast frequencies, etc.

Even more widespread is the use of reverse auctions, or tenders, for procurement of goods and services. In contrast to auctions, the sellers compete to obtain business from the buyer and prices will typically decrease as the sellers underbid each other. In most countries, this is the principal tool for the provision of goods, services, and public works in government contracting.

Two competitive bidding formats which are most commonly used in public procurement are first-price sealed-bids (FPSB) auctions and English auctions. In FPSB auctions, participants submit their bids to the auctioneer in the closed form (classically in sealed envelopes, henceforth the name). At a certain time which is specified in advance, envelopes are opened and the winner is selected based on the best price, which also becomes the contract price. This procedure is still widely employed in public procurement throughout the world.

In English auctions, the auctioneer sets the starting price. The bidders may offer a better price (typically by a specified margin). Bidding stops when only one bidder remains or when the deadline is reached. The winner is the one who, at the end of the auction, offered the best price, which she ends up paying.

In Ukraine, since 2016 all but the most mundane public purchases are carried out electronically using the Prozorro[1] on-line trading platform [1]. This platform has received wide critical acclaim and was selected by the European Bank for Reconstruction and Development as a recommended model for reforming electronic procurement in the European Union [2]. The procedures used for procurement are governed by the law "On public procurement" [3]. According to this legislation, the procurement process is split into two parts:

— the solicitation phase during which applications for participation in the tender are accepted, as well as initial bids;
— the auction phase which is run electronically among registered bidders as an English auction. Most often, the winner of competitive bidding is selected based on the price criterion, although it is possible to evaluate bids as a weighted average of different product characteristics (including price).

The most important distinction of the Prozorro format from FPSB auctions is that the exact number of bidders becomes common knowledge before the beginning of the second phase, which reduces uncertainty for participants.

Ukrainian legislation stipulates only the minimum length of the solicitation phase (15 days), while the actual duration of this phase is left open to the discretion of the tender organizer [3]. Extending the duration of the solicitation phase has a two-fold effect on the tender results. On the one hand, it may attract a larger number of bidders, and competition between them may lead to a better price for the auctioneer. On the other hand, such prolongation delays the delivery of necessary goods and services, and time has value in itself. These two conflicting factors suggest that there must be an optimal duration of the solicitation phase. The purpose of this paper is the development of the model that would formalize these considerations.

The rest of the paper is organized as follows. Section 1 reviews the related literature. Section 2 presents a procedure for estimating expected bidding results depending on the number of bidders based on the general auction theory. Section 3 proceeds to present a simple model for determining the optimal duration of the solicitation phase in the presence of time-dependent costs. The last section concludes and discusses possible directions for further research.

2. Literature overview

The behavior of bidders during the auction has been the subject of intense research in economic theory, operations research, and game theory over the past 60 years. We will provide a brief overview of the results obtained in auction theory, which are important for the further presentation of the material. It is useful to start with a brief overview of the most frequently used auction schemes.

In addition to the already mentioned FPSB and English auctions, there is a wide variety of other auction procedures – Japanese, Dutch, all-pay, Honolulu–Sydney and many others [4]. The choice of a particular bidding procedure and certain organizational settings (e.g. the starting price) indirectly affects the strategic behavior of bidders, which in turn changes the expected auction results. In the context of the public sector, the task of regulatory bodies is to choose such procedures and settings that would facilitate obtaining a socially desirable outcome.

Levers of influence on the behavior of the bidders may include:

i. setting the starting price of the auction;
ii. reward scheme (e.g. in most auctions the contract price is equal to the winner's bid, but in the Vickrey auction it is set at the level of the second-best offer);
iii. fees for participation in bidding (in a broad sense, they should also include costs of preparing tender documentation, technical support, etc.);
iv. magnitude of price change steps;
v. bidding deadlines, etc.

[1]This means "transparent" in Ukrainian.

Some of these tools have received considerably more attention in the literature than the others.

The impact of the procedural form of the auction on the auctioneer's results was a subject of intense research in economic theory since the seminal paper of Vickrey [5]. The most important result of the auction theory is the revenue equivalence theorem [6, 7], which establishes that (under certain technical assumptions) the expected outcome of all "reasonable" auction schemes will be the same. Moreover, the auctioneer cannot expect to obtain better results by employing any other conceivable auction procedure. A rigorous and thorough presentation of these results can be found in [8].

Optimal starting prices from the auctioneer's point of view were independently and almost simultaneously derived by Myerson [6] and Riley and Samuelson [7]. Bulow and Roberts provided a simple economic interpretation of these results by drawing parallels between the auctioneer and price-discriminating monopolist [9]. Levin and Smith [10], Li and Zheng [11] considered how bidders' strategies and the expected auction outcome will be affected by entry costs. Melnikov [12] derived conditions when holding auctions is economically preferable to buying products on the market in the presence of entry costs. McAfee and McMillan [13] study auctions where the number of bidders is not known in advance. David et al. [14] consider the optimal design of the English auction with discrete price steps.

The relationship between the duration of bidding procedures and the auction results was not comprehensively studied in the literature. Oliveira et al. [15] present empirical evidence that extending the duration of bidding leads to lower procurement prices, drawing from electronic auctions run by the Federal Government of Brazil. Zhang [16] studies how a deadline for selling a product may impact a monopolist's choice between posting a selling price and running a comparatively costly auction. Jusselin et al. [17] consider the problem of optimal auction duration in the institutional context of stock markets. Hafalir et al. [18] study the specifics of auctions for perishable products on the example of Sydney-Honolulu fish auctions.

The model presented below considers the issue of the optimal bidding duration in the institutional context of Prozorro procurement auctions in Ukraine, although it can easily be extended to other auction formats as well.

3. The model

Extending the duration of the bidding process has a dual effect on its outcome from the auctioneer's point of view.

The payoff from using competitive bidding is created by addressing a wider range of buyers or sellers. Competition between them provides the auctioneer with an opportunity to obtain a better price. From this perspective, lengthening auction duration should lead to a higher number of bidders, which should improve the expected outcome.

On the other hand, such prolongation delays the receipt of money (for auctions) or necessary products (for tenders), and time has value in itself. It also increases risks associated with quickly changing market conditions and makes planning more difficult.

The tradeoff between these conflicting considerations can be formalized by the model described below. Throughout the rest of the paper, we will talk about public procurement tenders, although the model can be applied to auctions as well with minor modifications.

3.1. Basic setting

Assume that the buyer holds a tender for procuring a certain product at the lowest price. Bids from potential sellers indexed by i are accepted during a time period $[0, t]$. Meanwhile, the buyer incurs waiting costs that are proportional to the duration of this period: $C(t) = ct$.

Each bidder has a certain reservation value v_i (i.e. the minimum price at which $s(he)$ is willing to supply the product), which is unknown to neither the buyer nor other bidders. We assume, however, that these reservation values are independent random variables drawn from a common distribution with support on $\left[\underline{V}, \overline{V}\right]$. This distribution is given by its cumulative distribution function (cdf) $F(x) = P\{v_i \leqslant x\}$, which is common knowledge among all auction participants. We will assume that the product can always be bought at the maximum price $p_{\max} = \overline{V}$.

The i-th bidder submits a single bid denoted by b_i. The single bid assumption is a procedural feature of sealed-bid auctions, but it can be applied without a loss of generality to other auction formats for reasons that will be discussed in the next section. Individual rationality implies that $b_i \geqslant v_i$.

Finally, we will assume that the arrival of bids to the buyer is governed by the Poisson process with intensity λ. Then the probability of receiving exactly n bids during the time t will be:

$$\pi(n,t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}. \tag{1}$$

The contract for supplying the product will be awarded to the lowest bidder. The price paid to the winner will be referred to as the contract price. Buyer's payoff is the difference between the maximum price and the contract price net of waiting costs.

3.2. Auction scheme and bidder's behavior

The relationship between sellers' reservation prices and their bids is rather complex and depends on the auction scheme. The key simplifying result is the revenue equivalence theorem [6–8]. The logic behind it is briefly outlined below.

Consider first the English auction scheme. In English auctions, bidding stops when only one bidder remains. To do this, the winner must submit an offer that is lower than the price of the penultimate bidder. It makes sense for participants to keep bidding as long as the current price exceeds their reservation value. Thus, ignoring the discreteness of price change steps, the winning bid should be equal to the second lowest among the reservation prices of auction participants.

In a seminal paper [5], Vickrey observed that the same result can be obtained with much less effort in a second-price sealed-bid (SPSB) auction, where the winner pays the price which is equal to the second best of submitted bids. He also proved that under such a bidding scheme, the dominant strategy for all the bidders would be truth-telling, that is, submitting an offer at the level of one's reservation price. This also relieves auction participants of the need to physically attend the event.

In FPSB auctions, the truth-telling strategy cannot be optimal because it leads to zero winnings for all the bidders. Therefore, participants' bids will be higher than their reservation values. However, it can be shown that in the Bayes-Nash equilibrium the winning bid would again match the second-best reservation value[2]. The same result holds for other auction formats that satisfy assumptions outlined in [19].

The revenue equivalence theorem allows abstracting away from the specifics of auction procedures actually used in a particular setting. Without loss of generality, we can assume that the auction is held using the SPSB scheme where it is optimal for the bidders to apply the truth-telling strategy. Hence, the contract price would be equal to the second-lowest reservation value of participating bidders.

3.3. Estimating the contract price

In probability theory, the k-th smallest value $x_{(k)}$ of the random vector $(x_1, x_2, \ldots, x_n)$ is known as the k-th order statistic. If components of this vector are independent and identically distributed (iid), then the probability density function (pdf) of the k-th order statistic is given by

$$f_{(k)}(x) = \frac{n!}{(k-1)!(n-k!)} f(x)[F(x)]^{k-1}[1-F(x)]^{n-k}, \tag{2}$$

where $F(x)$ and $f(x)$ are the cdf and the pdf of the components, respectively [20].

Considerations discussed in the previous section imply that the contract price p is a second order statistic $v_{(2)}$ of the bidders' reservation values $(v_1, v_2, \ldots, v_n)$. From equation (2), it follows that its

[2]This result relies on the assumption that the number of bidders is known in advance, which may be problematic for FPSB auctions used in public procurement. McAfee and McMillan [13] have shown that the "second best" result holds true for FPSB auctions with a stochastic number of buyers, but averaging is required not only over the distribution of bidders' reservation values, but also over the priors on the number of bidders. This potentially important issue is largely irrelevant for Prozorro auctions, since in them the bidding starts when the number of bidders is already known.

density is given by

$$f_p(x,n) = n(n-1)\,f(x)\,[F(x)]\,[1-F(x)]^{n-2}, \tag{3}$$

assuming $n \geqslant 2$ (otherwise $v_{(n-1)}$ is not defined).

If $n=0$, then there is not a single participating bidder and if $n=1$, there is no competition between bidders. In a public sector context, Ukrainian legislation stipulates that in both of these cases, tenders must be canceled and rescheduled. We will assume that in both of these cases the product will be procured at the maximum price.

Figure 1 shows the density of the contract price distribution for the standard uniform $U[0,1]$ (Figure 1a) and normal $N(1/2, 1/6)$ (Figure 1b) distributions of buyers' reservation prices.

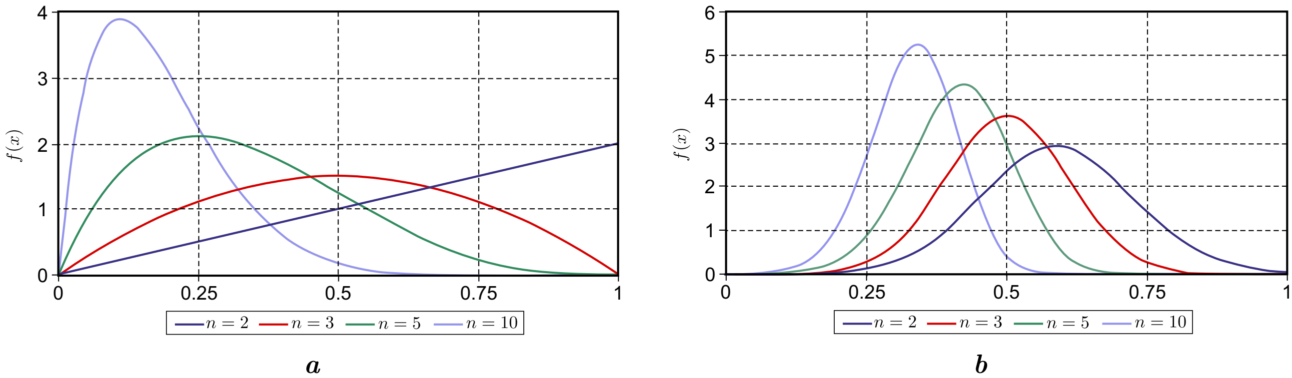

Fig. 1. Contract price distribution density for different distributions of bidders reservation prices: ***a*** – uniform $U[0,1]$, ***b*** – normal $N(1/2, 1/6)$.

Normal distribution parameters on Figure 1b are chosen so that by the three sigma rule the bulk of the distribution will be concentrated on the unit interval, which makes results easily comparable with the uniform distribution case. For the standard uniform distribution $F(x)=x$, $f(x)=1$ and direct substitution of these functions into formula (3) shows that the contract price will have a beta distribution with parameters $\alpha=2$, $\beta=n-1$. For the normal distribution, the pdf of the contract price cannot be expressed in elementary functions and was calculated numerically.

As can be seen from the graphs, the increase in the number of bidders shifts the contract price density to the left, i.e. to the area of lower prices.

Formula (3) allows computing the conditional expectation of the procurement price given the number of bidders as:

$$\overline{p}_n = \begin{cases} \mathbb{E}[p \mid n] = \displaystyle\int_{\underline{V}}^{\overline{V}} x\,f_p(x,n)\,dx, & n \geqslant 2, \\ \overline{V}, & n \in \{0,1\}. \end{cases} \tag{4}$$

For instance, in the standard uniform case

$$\overline{p}_n = \min\left\{\frac{2}{n+1}, 1\right\} \tag{5}$$

by the properties of beta distribution. Figure 2 shows the expected contract price as a function of the number of bidders and the distribution of their reservation prices.

The number of participating bidders, however, itself is a random variable with a distribution given by (1), which depends on the tender duration. Combining formulae (1) and (4), by the law of total probability, we arrive at the following expression for the expected contract price as a function of tender duration:

$$\overline{p}(t) = \sum_{n=0}^{\infty} \pi(n,t)\,\overline{p}_n = \sum_{n=2}^{\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \int_{\underline{V}}^{\overline{V}} x\,f_p(x,n)\,dx. \tag{6}$$

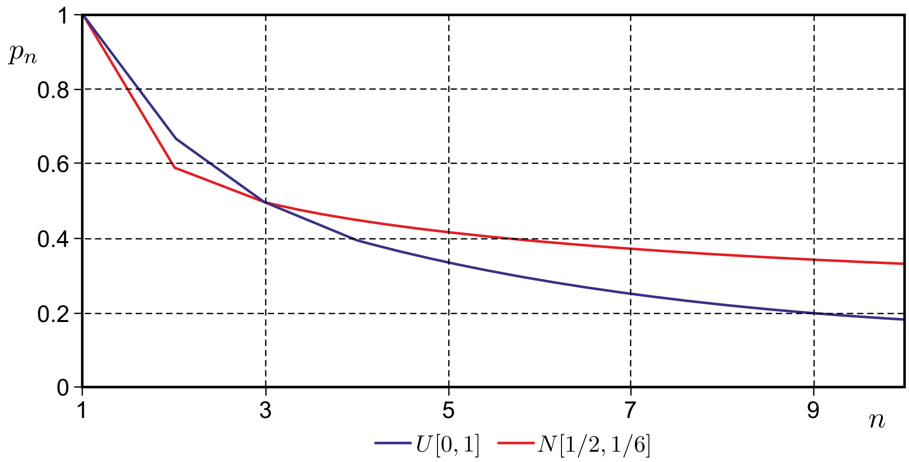

Fig. 2. Expected contract price as a function of the number of bidders.

3.4. Optimizing tender duration

Given variables defined above, we can set up the problem of optimizing tender duration as the following cost minimization problem:

$$V(t) = \overline{p}(t) + c\,t \underset{t}{\rightarrow} \min . \tag{7}$$

First term in the above expression is a decreasing function of time bounded below by $\underline{V}$. Second term increases linearly with t. Thus, there is a unique solution to the above optimization problem. Figure 3 shows the objective function for the standard uniform distribution of bidders' reservation values.

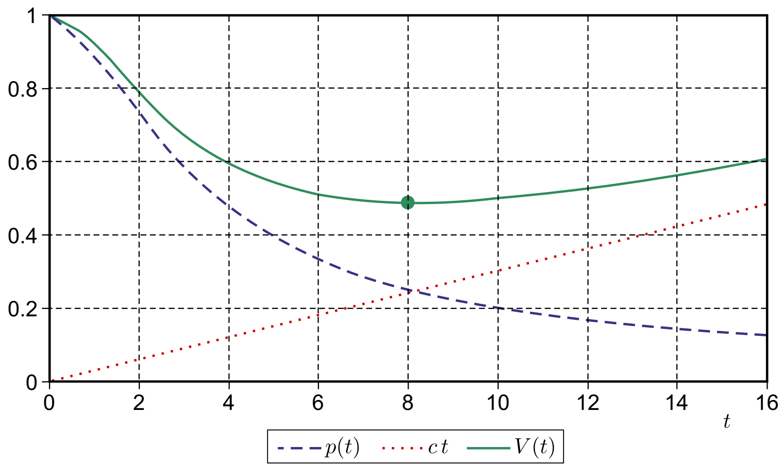

Fig. 3. Determining the optimal duration of the auction ($\lambda = 1$).

First-order conditions for the optimization problem (7) are:

$$\sum_{n=0}^{\infty} \frac{\partial \pi(n,t)}{\partial t} p_n + c = 0. \tag{8}$$

Above formula has a fairly standard economic interpretation: the expected marginal benefit from extending the auction by a unit of time should equal the marginal cost of this time.

Formula (8) can be transformed as follows. From formula (1) it follows that:

$$\frac{\partial \pi(n,t)}{\partial t} = \lambda \frac{n(\lambda t)^{n-1} e^{-\lambda t}}{n!} - \lambda \frac{(\lambda t)^n e^{-\lambda t}}{n!} = \lambda(\pi(n-1,t) - \pi(n,t)). \tag{9}$$

Plugging the above relationship into formula (8), we obtain:

$$\sum_{n=1}^{\infty}(\pi(n-1,t)-\pi(n,t))\,\overline{p}_n=\frac{c}{\lambda} \tag{10}$$

or

$$\sum_{n=0}^{\infty}\frac{(\lambda t)^n e^{-\lambda t}}{n!}(p_n-p_{n-1})=\frac{c}{\lambda}. \tag{11}$$

This allows for a more meaningful interpretation. The expression in the left-hand side of formula (11) is the expected reduction of the contract price from having an additional tender participant. For a Poisson distribution, the inverse of the intensity λ is the mean interval between events. Hence, the term c/λ on the right-hand side of formula (11) is the expected cost of waiting for an additional bidder. Thus, the optimal tender duration is such that the expected payoff from having an extra bidder equals the expected cost of waiting for this bidder.

Computing the objective function in (7) requires knowing the distribution of sellers' reservation values, which may be problematic to obtain. If there exists available information on posted sale prices, such as catalog listings, internet ads, etc., it may be easier to employ Monte Carlo methods to estimate p_n. Given a list of prices, one can draw a random sample of n elements, and take the second-highest price as the estimate of the result of the simulated auction. Repeating this procedure and averaging results will yield desired estimates.

4. Conclusion

The paper proposes a simple model that allows estimating the impact of the duration of competitive bidding on its expected results for the auctioneer. On this basis, a methodology for determining the optimal duration of bidding, applicable to a wide range of organizational mechanisms for conducting auctions, has been developed. The obtained formulas have meaningful and transparent economic interpretation. Under conditions of imperfect information about the market environment, forecasting bidding results and estimating the optimal duration of bidding can be carried out using Monte Carlo methods.

The practical implementation of the developed mechanism will help to streamline tender procedures and to improve their efficiency. In the commercial sector, the model can be relevant, in particular, for online auctions for advertising placement, which take place on a permanent basis in real time [21]. The adaptation of the model to this environment is an interesting topic for further research.

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Оптимізація тривалості конкурсних торгів на прикладі українських державних закупівель

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У статті розглянуто вплив тривалості конкурсних торгів на очікувану ціну продажу (для аукціонів) або закупівлі (для тендерів). Подовження тривалості торгів двояко впливає на їх результати. З одного боку, це приваблює більшу кількість учасників торгів, і конкуренція між ними призводить до кращої ціни для аукціоніста. З іншого боку, таке затягування затримує отримання грошей або необхідних речей, а час сам по собі має цінність. Ці два суперечливі фактори свідчать про те, що має бути оптимальна тривалість процесу торгів. У статті запропоновано модель, яка поєднує вказані міркування з теоретико-ігровою моделлю поведінки учасників торгів. Розроблено процедуру визначення оптимальної тривалості торгів в інституційному контексті української системи державних закупівель. Отримані умови оптимальності мають інтуїтивно зрозумілу економічну інтерпретацію і можуть бути застосовані як для емпіричного розподілу резервованих цін учасників торгів, так і для його аналітичної апроксимації. Практична реалізація запропонованого підходу сприятиме покращенню економічних показників діяльності організатора торгів, що є особливо важливим для державного сектора економіки.

Ключові слова: *аукціон; закупівлі; торги; тривалість; статистика замовлень; процес Пуассона.*