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# Circulation Flow of Polyacrylamide Aqueous Solution between Coaxial Cylinders with Smooth Surfaces

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**Abstract**

A mixer with coaxial arrangement of inner and outer circular cylinders with smooth surfaces is considered. During the mixer operation, a circulating flow is formed between the cylinders with the same conditions of motion along the entire length of the annular gap. The smooth surfaces of the inner and outer vertical cylinders reduce the possibility of the polymer solution destruction as an agent for reducing hydraulic resistance due to the high speed of rotation of the inner cylinder and/or the outer cylinder. The hydrodynamics of the mixing process of an aqueous solution of polyacrylamide (PAA) with a mass concentration of 100 ppm in the gap between the cylinders, the inner one of which was rotating, were estimated by the dependence of the power criterion $N_P$ on the modified Reynolds criterion $Re_m$. The mixing of the aqueous solution of PAA occurred in a laminar mode with a decrease in $N_P$ at a certain fixed value of $Re_m$ compared to water.

**Keywords:** coaxial cylinders; rotor with smooth surface; power number; Reynolds number.

## 1. Definition of the problem to be solved

In mixers intended for the preparation of emulsions and related heterogeneous systems, the efficiency of the mixing process is significantly increased by broadening the spectrum of operating fluid velocities within the recirculating flow. Under a coaxial configuration of the stator and rotor, an annular gap is formed between the elements, ensuring uniform hydrodynamic conditions along its entire length (Fig. 1) [1]. A principal advantage of the proposed design, in comparison with conventional mixers, lies in the substantially enlarged active area of the lateral surface of the working element.

The stator diameter of the installation was $D = 0.142$ m, while the rotor diameter was $d = 0.113$ m, with a height of $H = 0.133$ m (Fig. 2). The aspect ratio of the rotor diameter was $\Gamma_d = 0.796$, and the width of the annular gap was $b = 0.0145$ m. These values were determined according to the following relationships:

$$\Gamma_d = \frac{d}{D}, \tag{1}$$

$$b = \frac{D-d}{2} = \frac{D}{2} \cdot \left(1 - \Gamma_d\right). \tag{2}$$

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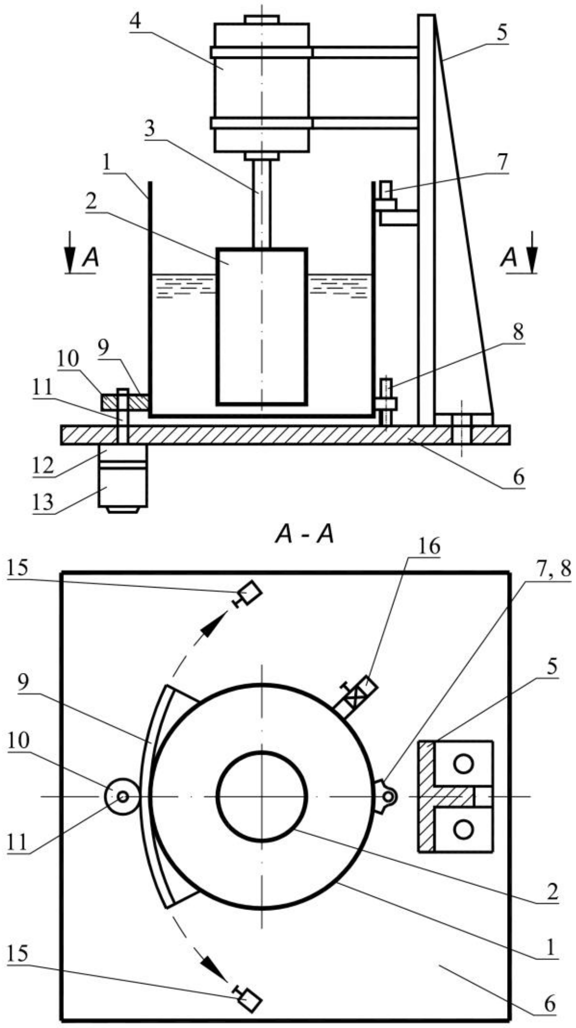


Fig. 1. Scheme of the mixer with coaxial cylinders:
1 – outer cylinder-stator; 2 – inner cylinder-rotor; 3 – drive shaft; 4 – electric motor; 5 – riser; 6 – plate;
7, 8 – suspension; 9 – part of the gear sector; 10 – gear wheel; 11 – shaft; 12 – reducer; 13 – reverse electric motor;
14, 15 – winding pole limit witches; 16 – outlet pipeline.

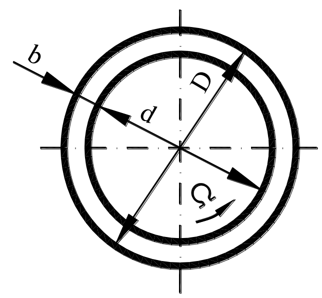


Fig. 2. Scheme of coaxial arrangement of cylinders.

## 2. Analysis of the recent publications and research works on the problem

Flow between rotating cylinders depends, in particular, on surface conditions [2]. The rotor, serving as the working body of the mixer, had a smooth surface.

The mixing process can be intensified by increasing the active area of the rotor's lateral cylindrical surface, for example, by introducing surface roughness. Thus, when the outer cylinder with a diameter of $D = 0.140$ m rotated relative to the stationary inner cylinder with a diameter of $d = 0.120$ m and a height of $H = 0.150$ m, a polyethylene oxide solution (Polyox WSR-301) at a concentration of 20 ppm rapidly lost its rheological properties. Both cylinders were roughened by machining longitudinal and transverse grooves on their surfaces, forming closely spaced tetrahedral pyramids with an apex angle of approximately 60° and a height of 0.00125 m (Fig. 3). In this case, the geometric similarity ratio of the rotor diameter was $\Gamma_d = 0.857$, and the width of the annular gap was $b = 0.010$ m [3].

The static pressure generated by a screw pump, positioned vertically in an open tank contained an aqueous solution of polyethylene oxide (Polyox FRA-1368-A-01) at concentrations ranging from 12.5 ppm to 100 ppm, initially increased to a maximum value and then gradually decreased to a minimum. The screw rotated relative to a stationary smooth cylinder. The screw protrusions had a pitch of 0.0016 m and a height of 0.0075 m (Fig. 4). In this configuration, the width of the annular gap was $b$=0.0005 m [4].

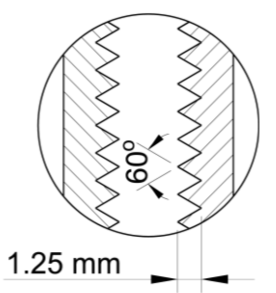


Fig. 3. Scheme of formation of rough surface of coaxial cylinders.

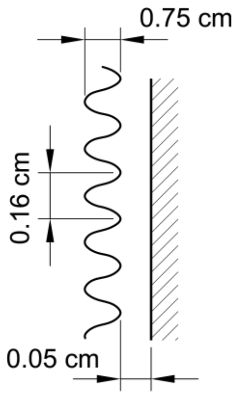


Fig. 4. Scheme of the formation of helical protrusions on the screw surface.

Although the mixing process is less intensive when the rotor and cylinder have smooth surfaces, the flow of a polymer solution through the annular gap between roughened cylinders may lead to its degradation under the action of shear stresses. Moreover, this degradation is irreversible [5]. But even with the flow in the annular gap with a width of $b = D/28$ between smooth cylinders, the polymer solution can be destroyed due to the high rotation speed of the outer cylinder [6].

The goal of this paper is to investigate the circulation flow of liquid between coaxial cylinders with a smooth rotor.

### 3. Presentation and discussion of the research results

An aqueous solution of polyacrylamide (PAA) with a concentration of $C = 100$ ppm by mass was examined. For its preparation, an 8% technical gel (Technical Specifications 6-01-1049-92) with a molecular weight of $1.8 \cdot 10^6$ was used. The concentration was determined based on the content of anhydrous PAA in the solution, calculated as $C = 0.0001$ kg PAA/kg. This fluid exhibits shear-thinning behaviour, which is described by Oswald's law

$$\tau = m \cdot \dot{\gamma}^n \tag{3}$$

with flow consistency index $m = 0.0027$ Pa·s$^n$ and flow behaviour index $n = 0.83$. To determine the flow consistency index and flow behavior index, a rotational viscometer VSN-3 was used.

The molecular weight was calculated from the Mark-Houwink equation [7]:

$$[\eta] = 3.73.10^{-4} \cdot M^{0.66} \tag{4}$$

by the value of the intrinsic viscosity [η] in a 0.1N $NaNO_3$ solution at a temperature of 30 °C. The intrinsic viscosity was [η] = 5.0 dl/g, it was determined by the standard method using a Bischof viscometer with a capillary diameter of 0.0006 m.

The hydrodynamic equation for mixing processes [8] for our case has the following form

$$N_P = f(\mathrm{Re}_m, \Gamma_d, \Gamma_H), \tag{5}$$

where $N_P$ is the power number defined as follows[8], [9]

$$N_P = \frac{P}{\rho \cdot N^3 \cdot d^5}, \tag{6}$$

where $P$ is the power (W); ρ is the fluid density (kg/m$^3$); $N$ is the number of rotor revolutions (s$^{-1}$); $\mathrm{Re}_m$ is the Reynolds number for mixing [8], [9]

$$\mathrm{Re}_m = \frac{\rho \cdot N \cdot d^2}{\mu}, \tag{7}$$

where μ is the dynamic viscosity (Pa·s); $\Gamma_H$ is the aspect ratio of the stator height

$$\Gamma_H = \frac{H}{D}. \tag{8}$$

In the studies, the temperature of water and PAA aqueous solutions was 26 °C. The fluid density for PAA aqueous solution was determined using a pycnometer and for water it was determined depending on the temperature. The dynamic viscosity for water was determined depending on the temperature.

Since the stator diameter in the experimental setup is constant, and the rotor in this experiment has the same height and diameter, the aspect ratio of the stator height $\Gamma_H$ = 1.077 = const and the aspect ratio of the rotor diameter $\Gamma_d$ = 0.796 = const. Therefore, the hydrodynamic equation for mixing processes can be rewritten as [8], [9]

$$N_P = a \cdot \mathrm{Re}_m^k, \tag{9}$$

where $a$ is the coefficient; $k$ is the exponent.

Mixing occurs under laminar flow region with a slope of −1.0 (Fig. 5). For PAA aqueous solution, in comparison with water, a reduction in the $N_P$ value is observed at a given fixed value of $\mathrm{Re}_m$. For example, at $\mathrm{Re}_m \approx$ 57,000, the relative change is $\Delta N_P/N_P$ = 77.1%. The relative change was determined as:

$$\frac{\Delta N_P}{N_P} = \left[1 - \frac{(N_P)_s}{(N_P)_w}\right] \cdot 100, \tag{10}$$

where $\frac{(N_P)_s}{(N_P)_w}$ is a criterion characterizing the hydrodynamic efficiency of PAA aqueous solutions of a given concentration, and the indices $s$ and $w$ correspond to PAA aqueous solutions and water, respectively.

In this case, the values of the $N_P$ decrease with increasing $\mathrm{Re}_m$ for both water and PAA aqueous solutions. Furthermore, for identical $\mathrm{Re}_m$ values, lower $N_P$ values are consistently observed for PAA aqueous solutions compared to water. This behavior corresponds to the dependence $C_F = f_1(\mathrm{Re}_b)$ [10], where $C_F$ is the friction coefficient in the annular gap between the cylinders [11], defined as

$$C_F = \frac{G}{\mathrm{Re}_b^2}, \tag{11}$$

where $G$ is the relative torque on the rotor; $\mathrm{Re}_b$ is the Reynolds number based on the gap width $b$ (m).

The relative torque on the rotor is defined as follows

$$G = \frac{\rho \cdot G}{\mu^2 \cdot H}; \tag{12}$$

The Reynolds number based on the gap width is calculated according to the following formula

$$\mathrm{Re}_b = \frac{\rho \cdot \left( \Omega \cdot \frac{d}{2} \right) \cdot b}{\mu}, \tag{13}$$

where Ω is the rotational speed of rotor (rms), defined as

$$\Omega = 2\pi \cdot \mathrm{N}. \tag{14}$$

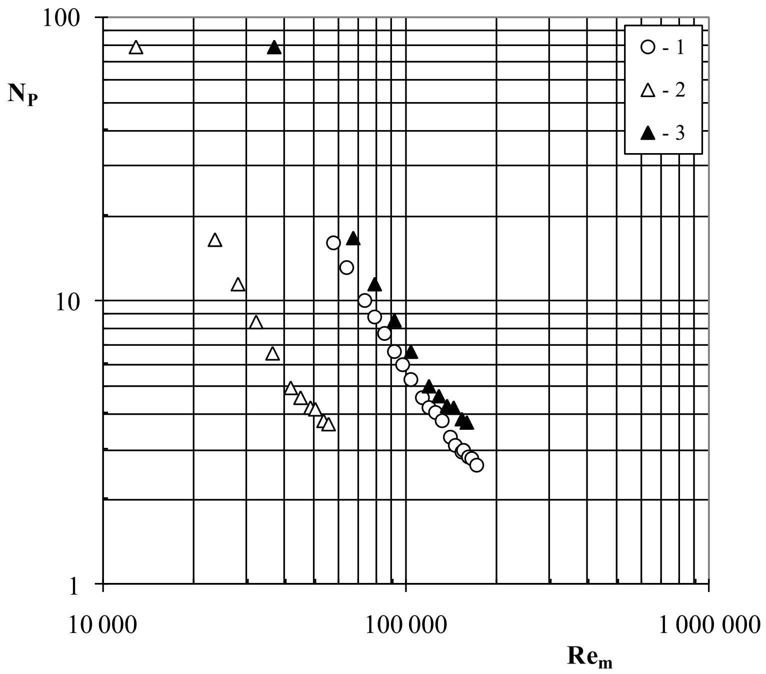


Fig. 5. Dependence of the power number on the Reynolds number for mixing for water (1) and for PAA aqueous solutions (2, 3), with calculations performed for the PAA aqueous solution (2) and for water (3).

Mechanical blade mixers operating in laminar regimes mix only those liquid layers directly adjacent to the blades [8]. But a mixing using a three-blade marine impeller operating at rotational speed of 75 rpm was achieved of minimal degradation of Magnafloc 5250 aqueous solution with a concentration of 100 ppm [12]. At low Reynolds numbers the PAA at a concentration of 80 ppm by weight can be mixed in a simple flow in a curved channel. As a solvent for the polymer was used a solution of 65% saccharose and 1% NaCl in water [13].

PAA aqueous solutions with concentrations up to 100 ppm may still be regarded as Newtonian fluids, albeit with increased viscosity [14]. Therefore, calculating the $N_P$ and $\mathrm{Re}_m$ numbers using specific gravity and kinematic viscosity, as for water, makes it possible to determine the actual (technical) values of these criteria in comparison with water. Thus, for PAA aqueous solution at identical values of $\mathrm{Re}_m$, the $N_P$ value is slightly larger (approximately 1.26 times) than that for water. In this case, the relative change is $\Delta N_P/N_P \approx -26\%$. This behaviour resembles the dependence $C_F = f_2(\mathrm{Re}_b)$ [15].

## 4. Conclusion

For a mixer with coaxial arrangement of the inner and outer cylinders, the roughness of their surfaces is justified. Their smooth surface reduces the possibility of degradation of drag reducing polymers due to the high speed of rotation of the inner cylinder and/or the outer cylinder.

Mixing of an aqueous solution of polyacrylamide with a mass concentration of 100 ppm in the gap between the circular cylinders, the inner one of which was rotating, occurred in a laminar mode with a decrease in the power number at a certain fixed value of the Reynolds number for mixing compared to water.

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# Циркуляційна течія водного розчину поліакриламіду між коаксіальним циліндрами з гладкими поверхнями

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### Анотація

Розглянуто змішувач з коаксіальним розташуванням внутрішнього та зовнішнього круглих циліндрів з гладкими поверхнями. При роботі змішувача між циліндрами утворюється циркуляційна течія з однаковими умовами руху по усій довжині кільцевого проміжку. Гладкі поверхні внутрішнього та зовнішнього вертикальними циліндрів зменшують можливість деструкції полімерного розчину як агента зменшення гідравлічного опору через велику швидкість обертання внутрішнього циліндра або/та зовнішнього циліндра. Гідродинаміку процесу перемішування водного розчину поліакриламіду (ПАА) з масовою концентрацією 100 ppm в проміжку між циліндрами, внутрішній з яких обертався, оцінювали залежністю критерію потужності $N_P$ від модифікованого критерію Рейнольдса $Re_m$. Перемішування водного розчину ПАА відбувалося за ламінарного режиму зі зменшенням $N_P$ за певного фіксованого значення $Re_m$ порівняно з водою.


**Ключові слова:** коаксіальні циліндри; гладкий ротор; критерій потужності; критерій Рейнольдса.