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Use of Trapezoidal Weirs for Determining Seepage Discharge through Earth Dams

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Abstract

The paper presents a comprehensive analysis of the application of trapezoidal weirs coupled with automated measurement systems for the precise determination of seepage flow rates through earth dams at hydroelectric power plants. A key focus is placed on the hydraulic operating modes of these weirs, particularly the distinction between non-submerged and submerged conditions. The paper details the results of extensive laboratory studies conducted to quantify the impact of submergence on the discharge capacity of trapezoidal weirs. These experiments led to the derivation of a submergence coefficient, which is crucial for correcting standard discharge formulas. The findings demonstrate that neglecting this coefficient can lead to significant overestimation of actual flow rates, with errors exceeding 457%. The study proposes a refined formula and a corresponding graphical relationship to ensure accurate seepage monitoring, thereby enhancing the operational safety of hydraulic structures.

Keywords: trapezoidal weirs; laboratory studies; submergence coefficient; discharge capacity; measurement automation.

1. Use of weirs at hydropower plants in Ukraine

In the construction of hydroelectric power plants, earth dams are fundamental structures used to create a hydraulic head. A critical and inherent characteristic of these embankments is the presence of seepage through the dam's body and foundation. According to the U.S. Bureau of Reclamation's Water Measurement Manual [1], the diligent monitoring of this seepage is not merely a procedural task but a cornerstone of dam safety and operational integrity. Uncontrolled or excessive seepage can lead to internal erosion (piping), which compromises the structural stability of the dam and poses a significant risk. Consequently, continuous monitoring is conducted through a network of instruments and observations, tracking key parameters such as soil strength, the position of the phreatic surface (depression curve), seepage velocities, and hydraulic gradients.

To manage and quantify seepage, dams are equipped with special drainage systems designed to intercept and safely divert percolating water. These systems often include internal pipe drains, inspection wells, and collection channels. A crucial aspect of this monitoring program is the accurate measurement of the seepage flow rate at various points within the drainage network. For this purpose, various hydraulic measurement devices are employed, among which weirs are one of the most reliable and widely accepted solutions, as detailed by Ackers et al. in *Weirs and Flumes for Flow Measurement* [2].

For measuring flow in open, non-pressurized channels, thin-plate weirs offer a simple, cost-effective, and accurate method. These structures function by forcing the flow over a crest of a specific shape, creating a direct relationship between the upstream water depth (head) and the discharge. According to their geometry, weirs are classified as rectangular, triangular (V-notch), and trapezoidal, among others (Fig. 1) [3]. The trapezoidal weir, particularly the

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Cipolletti type with sides sloped at 1 horizontal to 4 vertical, is highlighted in the Water Measurement Manual as a standard design that automatically compensates for end contractions. Due to their reliability and robust performance, both trapezoidal and triangular weirs are the most commonly utilized types in field and laboratory settings for hydraulic measurements.

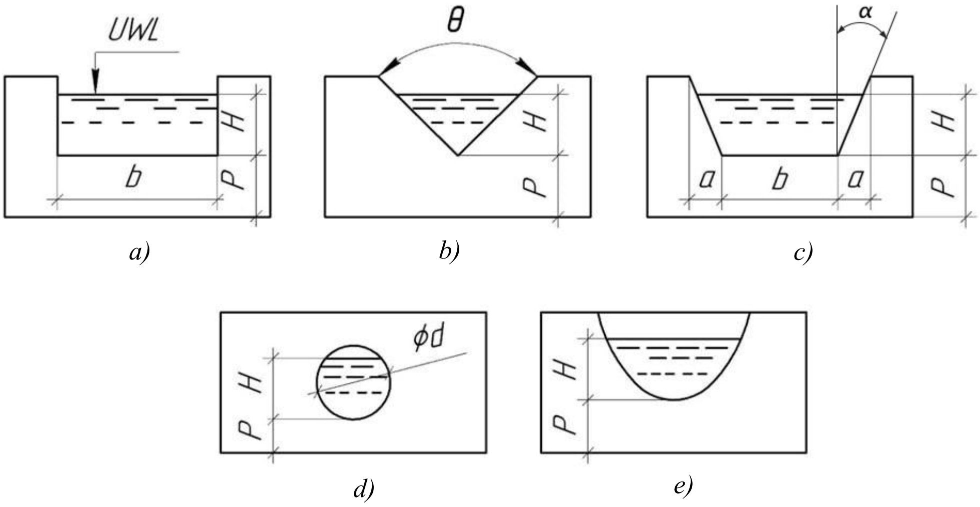

Fig. 1. Types of weirs for non-pressurized flows by cross-sectional shape: *a)* – rectangular, *b)* – triangular, *c)* – trapezoidal, *d)* – circular, *e)* – parabolic (where: UWL is upstream water level; *H* is the head on the weir crest; *P* is the weir crest height; *b* is the crest width; θ is the angle of the V-notch; α is the angle of the side slope; ø*d* is the diameter)

2. Types of hydraulic regimes for trapezoidal weirs

By the type of hydraulic regime, weirs are divided into submerged and non-submerged (free-flow) [4]. In the presence of a subcritical flow, a thin-plate weir is considered non-submerged if the downstream water level is below the weir crest elevation (Fig. 2). In laboratory settings, weirs are arranged to always operate in a non-submerged mode. In field conditions, efforts are also made to install weirs so that they are non-submerged. The water discharge through a non-submerged symmetric trapezoidal weir can be found using the following formula [5],[6]:

$$Q = m_d(b + 0.8 \tan \alpha \, H)H\sqrt{2gH}, \tag{1}$$

where H is the head on the weir crest; m_d is the weir discharge coefficient; b is the crest width; g is the acceleration due to gravity; α is the angle between the vertical and the side of the trapezoid. Here, H and b are expressed in meters, g in m/s², and the discharge Q in m³/s.

If the crest width b and head H are expressed in meters, and we assume $m_d = 0.42$, $m_n = 0.25$ for a trapezoidal thin-plate weir with an angle $\alpha = 14°$, and $g = 9.81$ m/s², the formula (1) takes the following form for discharge Q in m³/s:

$$Q = 1.86(b + 0.2H)H^{3/2}. \tag{2}$$

The calculated discharge values Q, according to formula (2), are presented in Table 1.

The head on the weir crest is measured using sensors, and the discharge is then determined from this head. It must be emphasized that formulas (1), (2), and consequently the results of automatic measurements, are valid only for a non-submerged weir, where the downstream water level (↓DWL) is below the crest elevation (Fig. 2,*a*), $\downarrow DWL < \downarrow weir\ crest$.

In the case of a submerged weir (Fig. 2,*b*), $\downarrow DWL > \downarrow weir\ crest$, the weir is submerged, and formulas (1), (2) contain a certain error due to not accounting for the effect of submergence. The magnitude of this error depends on the degree of submergence, and formulas (1) and (2) significantly overestimate the actual discharge value.

Table 1. Discharge values Q for a trapezoidal weir according to formula (2) with m_d = 0.42, m_n = 0.25, g = 9.81 m/s², b = 0.4 m.

H	*Q*	*H*	*Q*	*H*	*Q*	*H*	*Q*
m	m³/s	m	m³/s	m	m³/s	m	m³/s
0.05	0.009	0.14	0.042	0.26	0.111	0.44	0.265
0.06	0.011	0.15	0.046	0.28	0.126	0.46	0.286
0.07	0.014	0.16	0.051	0.30	0.141	0.48	0.307
0.08	0.018	0.17	0.057	0.32	0.156	0.50	0.329
0.09	0.021	0.18	0.062	0.34	0.173	0.60	0.450
0.10	0.025	0.19	0.067	0.36	0.190	0.70	0.588
0.11	0.029	0.20	0.073	0.38	0.207	0.80	0.745
0.12	0.033	0.22	0.085	0.40	0.226	0.90	0.921
0.13	0.037	0.24	0.098	0.42	0.245	1.00	1.116

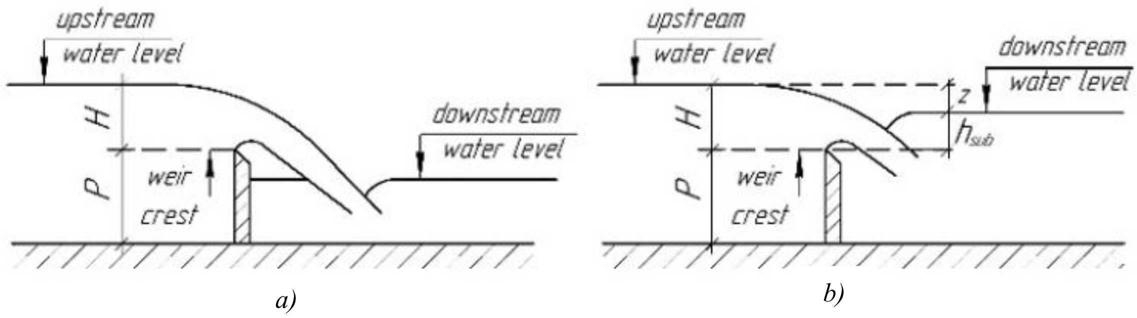

Fig. 2. Scheme of unsubmerged (*a*) and submerged (*b*) trapezoidal weir.

3. Laboratory investigation of the submergence effect

Formula (2), taking into account the effect of submergence, will take the below form:

$$Q = 1.86(b + 0.2H)\sigma_s H^{3/2}, \tag{3}$$

where σ_s is the submergence coefficient.

Using a symmetric trapezoidal weir with a crest width b = 0.4 m and an angle of inclination of the side relative to the vertical α = 14°, and an automated measurement system, a hydraulic modeling of the operation of trapezoidal weirs in a submerged mode was carried out in the laboratory conditions in the following way.

- The level of weir crest was measured as 20.09 mm.
- The water discharge Q_Δ was supplied to the flume and measured by a triangular weir located at the beginning of laboratory setup.
- The submerged regime was simulated in the downstream of the trapezoidal weir by means of different types of gates.
- The upstream water level (↓UWL) and downstream water level (↓DWL) were measured after the steady flow was formed in the flume.
- The depth downstream the weir was increased several times for the same water discharge in the flume Q_Δ. For each depth, the new values of ↓UWL and ↓DWL were measured.
- The water discharge Q_Δ was changed and all operations were repeated.
- Based on measured data the head H, submerged depth h_{sub} of trapezoidal weir and the flow water rate by formula (2) Q_{f2} were calculated.
- To find the influence from submerged depth on values of water flow rate the difference between the values Q_{f2} and Q_Δ was found.

The results of determining the submergence coefficient are presented in Table 2. Furthermore, there is a comparison of the actual discharge value with the theoretical one, which does not account for the influence of the submergence coefficient.

Table 2. Experimental values of σ_S for a trapezoidal weir.

Actual discharge (from triangular weir), Q_Δ, dm³/s	Head on trapezoidal weir, H, cm	Submergence depth, h_s, cm	h_s / H	Submergence coefficient, σ_S	Theoretical discharge (formula (2)) Q_{f2}, dm³/s	Deviation of actual from theoretical discharge, ΔQ, %
1	2	3	4	5	6	7
5.00	4.4	0.2	0.045	0.7125	7.02	40.36
13.67	20.5	20	0.976	0.1796	76.13	456.95
	14.8	14	0.946	0.3005	45.50	232.81
	11.6	10.5	0.905	0.4396	31.10	127.50
	7	1.9	0.271	0.9585	14.26	4.33
30.01	24.7	23.7	0.960	0.2925	102.61	241.92
	24.1	23.5	0.975	0.3043	98.63	228.66
	24	22.8	0.950	0.3063	97.97	226.47
	22	20.7	0.941	0.3522	85.22	183.96
	14.7	10.7	0.728	0.6667	45.01	50.00
	12.9	7	0.543	0.8178	36.69	22.27
	11.4	0.8	0.070	0.9914	30.27	0.86
40.75	26.6	25.7	0.966	0.3524	115.64	183.79
	26.4	25.1	0.951	0.3567	114.24	180.35
	24.9	23.1	0.928	0.3920	103.95	155.10
	19.7	15.5	0.787	0.5702	71.46	75.37
	16	9	0.563	0.7924	51.43	26.20
	15.2	6.5	0.428	0.8590	47.44	16.42
	14.5	2	0.138	0.9249	44.06	8.12
50.61	28.6	26.9	0.941	0.3891	130.07	157.00
	27.3	24.4	0.894	0.4196	120.61	138.31
	25.4	21.4	0.843	0.4715	107.34	112.09
	23	17.4	0.757	0.5531	91.50	80.80
	19.7	10	0.508	0.7082	71.46	41.20
	18.1	4.7	0.260	0.8101	62.48	23.45
	17.7	4	0.226	0.8392	60.31	19.16

Table 2 shows that if the influence of the submergence coefficient is not considered, the calculated discharge values using formula (2) will far exceed the actual values. The largest deviation is observed at low flow rates, reaching over 400%. Based on the data from Table 2, a graph of the relationship $\sigma_s = f(h_s/H)$ is plotted and compared with other experimental curves (Fig. 3).

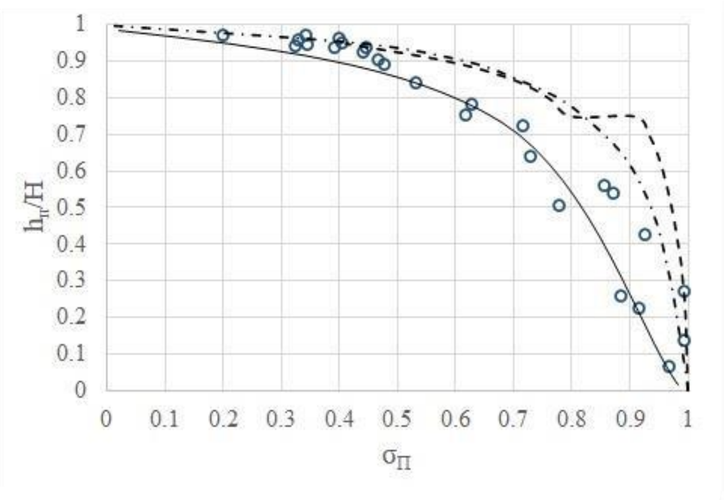

Fig. 3. Graph for determining the submergence coefficient $\sigma_s = f(h_s/H)$: ○ – experimentally obtained values of the submergence coefficient; - - - – submergence coefficient values according to Pavlovsky M.M. [8]; – · – – submergence coefficient values according to TUiN MES [8].

Thus, the general curve $\sigma_s = f(h_s/H)$ for trapezoidal weirs is shown in Fig. 4, and the values of the submergence coefficient are given in Table 3.

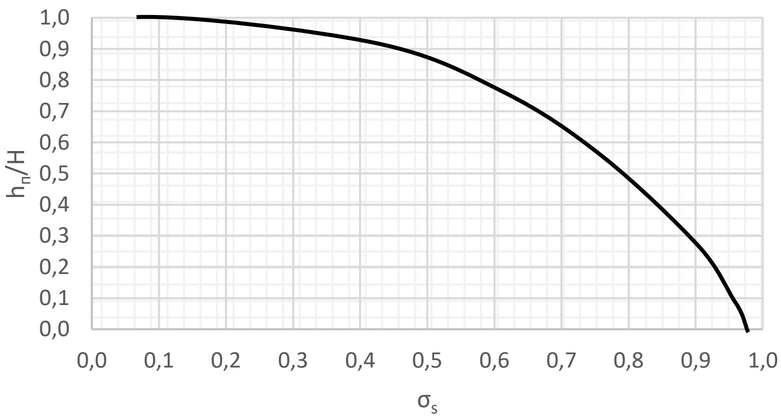

Fig. 4. Curve $\sigma_s = f(h_s/H)$ for trapezoidal weirs with a crest width b = 0.4 m.

Table 3. Submergence coefficient values for trapezoidal weirs with crest width b=0.4 m and side slope angle α=14°.

h_s/H	σ_s	h_s/H	σ_s
1.0	0.000	0.5	0.8
0.95	0.23	0.45	0.83
0.9	0.38	0.4	0.86
0.85	0.47	0.35	0.88
0.8	0.54	0.3	0.9
0.75	0.6	0.25	0.92
0.7	0.66	0.2	0.94
0.65	0.7	0.15	0.95
0.6	0.73	0.1	0.96
0.55	0.77	0.05	0.97
-	-	0	1.000

Taking into account the performed calculations and the obtained results, the final dependencies for determining the discharge capacity of trapezoidal weirs can be defined on the basis of formula (3).

4. Conclusion

The performed research demonstrates that operating trapezoidal weirs in a submerged mode without accounting for the submergence effect leads to significant errors in discharge measurement, reaching up to 457% in the considered range. To obtain reliable data, it is imperative to use the derived submergence coefficient σ_s. The study recommends determining the discharge on trapezoidal weirs with a crest width of b = 0.4 m using the corrected formula (3), where the submergence coefficient can be determined from the proposed experimental curve $\sigma_s = f(h_s/H)$. This methodology requires measuring water depths both upstream and downstream of the weirs, but provides a crucial improvement in the accuracy of seepage monitoring for earth dams.

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Використання трапецієвидних водомірів для визначення фільтраційних витрат земляних гребель

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Анотація

Забезпечення безпечної експлуатації гідроелектростанцій вимагає точного моніторингу фільтраційних процесів у земляних греблях. У статті представлено комплексний аналіз застосування трапецієвидних водомірів, оснащених автоматизованими системами вимірювання, як ефективного інструменту для визначення фільтраційних витрат. Детально розглянуто гідравлічні режими роботи водомірів, зокрема розкрито відмінності між незануреним та зануреним станом. Основну увагу приділено результатам лабораторних досліджень, проведених з метою кількісної оцінки впливу підтоплення на пропускну здатність водомірів. На основі експериментальних даних було виведено коефіцієнт підтоплення, що є ключовим для корекції стандартних розрахункових формул. Дослідження доводить, що ігнорування цього коефіцієнта призводить до значного завищення реальних значень витрати з похибкою, що може перевищувати 457%. У статті запропоновано удосконалену формулу та відповідну графічну залежність для забезпечення точного моніторингу, що суттєво підвищує надійність контролю за станом гідротехнічних споруд.

Ключові слова: трапецієвидні водоміри; лабораторні дослідження; коефіцієнт підтоплення; пропускна здатність; автоматизація вимірювань; фільтраційні втрати.