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# Synthesis of Automatic Control System for Low-Temperature Separator

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**Abstract**

The gas produced from the well under high pressure is fed to a complex preparation unit, where solid impurities and water are removed. Purified natural gas contains valuable components such as condensate, as well as heavy hydrocarbons, butane and propane. To extract associated components from gas (condensate and heavy hydrocarbons) low-temperature separation is used. The temperature regime in the separator is maintained by the energy of the compressed gas. When the gas passes through the throttle, due to the Joule-Thomson effect, the pressure and temperature decrease. The technological regime in the separator is provided by the single-loop automatic control systems for pressure and condensate level control. As shown by the studies carried out by the authors of the paper, the low-temperature separation as a control object is characterized by internal cross-links. Their presence significantly reduces the efficiency of single-loop control systems. To improve the quality of the control process, an autonomous control system for the low-temperature separation process was synthesized. A cross-coupling compensator was included in the control circuit of such a system, resulting in two independent single-loop automatic control systems. Based on the developed mathematical model, the transfer function of the compensator is synthesized and a method for determining the parameters of PI controllers is developed. The essence of the method is that on the complex plane of the roots of the characteristic equation, the positions of the roots are determined, which should ensure the desired quality of the control process. The placement of the roots is selected from the condition of the minimum of the generalized quadratic criterion of the quality of the control process.

**Keywords:** low-temperature separation; mathematical model; autonomous system; compensator; PI controller; quadratic criterion.

## 1. Introduction

In the case when the gas extracted from the well under high pressure enters the pre-treatment unit, then low-temperature separation (LTS) is used to separate the condensate from the gas. The decrease in gas temperature, which is below the dew point, occurs due to adiabatic expansion of the gas (Thomson-Joule effect). The effectiveness of the LTS process depends on the degree of compliance with the technological regulations, which are implemented by means of automation. The main technological parameters that affect the technological process are the temperature in the separator, the condensate level and the gas pressure in the low-temperature separator.

The analysis of the dynamic properties of the LTS process showed that there are cross-links between the input and output values. The presence of cross-links significantly complicates the automatic control process, since changing the task on the first control channel will cause an undesirable change in the output value on the second channel. A similar negative event will occur when changing the input value on the second input of the separator control system.

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The quality of the low-temperature separator control process can be significantly improved by compensating for cross-coupling in some way. One possible way to eliminate the negative impact of cross-coupling is to use the autonomous control method.

## 2. Analysis of literature sources

The current state of automation of technological processes is characterized by the widespread introduction of microprocessor technology for the implementation of complex control algorithms that were inaccessible to traditional automation tools, with the help of which single-loop automatic control systems were implemented. In this regard, the interest of both scientists and practitioners in methods of automatic control of multidimensional objects, which are quite common in oil and gas production, petrochemical, metallurgical, chemical and other industries, has increased.

The use of single-loop systems for automatic control of complex multidimensional objects did not provide the desired quality indicators of the control process [1]. With the advent of microprocessor control tools that can implement complex control algorithms, new methods for controlling multidimensional objects were developed. Such methods should be divided into two groups. The first of them is the synthesis of control systems in the state space; the second is the synthesis in the frequency domain.

The first group consists of methods of analytical design of regulators (modal control [2], Riccati method [2], [4] – [6], $H^{\infty}$ optimization [2]). The second group includes analytical synthesis of automatic control systems and separation of control loops [9] (autonomous control). Systems synthesized using methods of analytical controller design, $H^{\infty}$optimization and modal control are static systems in which control errors are nonzero.

The method of analytical synthesis of automatic control systems assumes that the transfer function of the control device is the ratio of two polynomials. The synthesis of such a system is carried out under the condition that the specified location of the zeros and poles of the system is achieved and the necessary indicators of the quality of the control process are provided. The control device synthesized in this way can have a high order of the transfer function. In [4] an example of the synthesis of a control device for an object with a third-order transfer function is given. As a result, a control device is obtained, the transfer function of which is of the sixth order, which significantly complicates the structure of a controller suitable for implementing such a transfer function.

Almost all methods of synthesis of control systems for multidimensional objects solve the problem of finding the so-called consistency matrix, the purpose of which is to compensate for cross-connections, which makes it possible to improve the qualitative and quantitative indicators of the control process.

The aim of the work is to synthesize an autonomous control system for the LTS process to increase the efficiency of condensate separation from produced gas from the well. To achieve the goal, it was necessary to solve the following tasks:

- synthesize a cross-coupling compensator and determine the parameters of its matrix transfer function.
- for the two obtained independent control loops, find the PI controller tuning parameters and evaluate the quality of the low-temperature separator control process.

## 3. Synthesis of cross-link compensator

The synthesis of an autonomous control system involves the inclusion of a cross-coupling compensator with the transmission circuit in the control loop $W_{eq}(p)$ (Fig. 1). The matrix transfer function of the controller $W_{cl}(p)$ is a diagonal matrix whose elements are the transfer functions of the controllers. The matrix transfer function of the system, which is depicted in Fig. 1, will be as follows:

$$W_{yu_d}(p) = \left(I + W(p)\right)^{-1} W(p), \tag{1}$$

where $W(p) = W_{yu}(p)W_{eq}(p)W_{cl}(p)$ is the matrix transfer function of the open system; $W_{yu}(p)$is the matrix transfer function of the low-temperature separator.

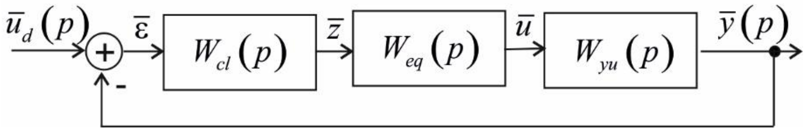


Fig. 1. Block diagram of an autonomous control system.

In [1] it is shown that the matrix transfer function for the low-temperature separator problem is as follows:

$$W_{yu} = (Ip - A)^{-1}B,$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$; $B = \begin{bmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

Since $(Ip - A)^{-1} = \frac{1}{\Delta(p)}\begin{bmatrix} p - a_{22} & -a_{12} \\ -a_{21} & p - a_{11} \end{bmatrix}$, then

$$W_{yu}(p) = \frac{1}{\Delta(p)}\begin{bmatrix} -a_{12}b_{21} & (p - a_{22})b_{12} - a_{12}b_{22} \\ (p - a_{11})b_{21} & (p - a_{11})b_{22} - a_{21}b_{12} \end{bmatrix}. \quad (2)$$

The elements of the matrix transfer function (2) determine the dynamic properties of the transmission channels of the effects of input quantities on the outputs of the object. Therefore,

$$W_{yu}(p) = \begin{bmatrix} w_{11}(p) & w_{12}(p) \\ w_{21}(p) & w_{22}(p) \end{bmatrix},$$

where $w_{11}(p) = -\frac{a_{12}b_{21}}{\Delta(p)}$; $w_{12}(p) = \frac{b_{12}p - (a_{22}b_{12} + a_{12}b_{22})}{\Delta(p)}$; $w_{21}(p) = \frac{b_{21}p - a_{11}b_{21}}{\Delta(p)}$; $w_{22}(p) = \frac{b_{22}p - (a_{11}b_{22} + a_{21}b_{12})}{\Delta(p)}$; $\Delta(p) = p^2 - (a_{11} + a_{22})p + a_{11}a_{22} - a_{12}a_{21}$.

For the control system to be autonomous, it is necessary to have a diagonal transfer matrix of the open system. It is known that the sum and product of diagonal matrices also give a diagonal matrix, and the operation of inverting a diagonal matrix also generates a diagonal matrix. Indeed, if the matrix $W(p)$is diagonal, then the matrix will also be diagonal $W_{yu_d}(p)$.

It was assumed that $W_{cl}(p)$is a diagonal matrix. For the matrix $W(p)$to be also diagonal, the following condition must be met [2]:

$$W_{yu}(p)W_{eq}(p) = \mathrm{diag}W_{yu}(p),$$

where $\mathrm{diag}W_{yu}(p)$is the diagonal matrix obtained from the matrix $W_{yu}(p)$after zeroing off-diagonal elements.

From the last equation we find

$$W_{eq}(p) = W_{yu}^{-1}(p)\mathrm{diag}W_{yu}(p). \quad (3)$$

Formula (3) defines the matrix transfer function of the compensator and shows that the value $W_{eq}(p)$ depends only on the matrix transfer function of the object.

Let us calculate the matrix transfer function of the compensator. First, we find

$$W_{yu}^{-1}(p) = \frac{1}{\Delta_W(p)}\begin{bmatrix} w_{22}(p) & -w_{12}(p) \\ -w_{21}(p) & w_{11}(p) \end{bmatrix},$$

where $\Delta_w(p) = w_{11}(p)w_{22}(p) - w_{12}(p)w_{21}(p)$; $diagW_{yu}(p) = \begin{bmatrix} w_{11}(p) & 0 \\ 0 & w_{22}(p) \end{bmatrix}$.

Then

$$W_{eq}(p) = \frac{1}{\Delta_w(p)} \begin{bmatrix} w_{22}(p) & -w_{12}(p) \\ -w_{21}(p) & w_{11}(p) \end{bmatrix} \cdot \begin{bmatrix} w_{11}(p) & 0 \\ 0 & w_{22}(p) \end{bmatrix}.$$

After multiplying the matrices, we get

$$W_{eq}(p) = \frac{1}{\Delta_w(p)} \begin{bmatrix} w_{11}(p)w_{22}(p) & -w_{12}(p)w_{22}(p) \\ -w_{11}(p)w_{21}(p) & w_{11}(p)w_{22}(p) \end{bmatrix}. \quad (4)$$

From the matrix equation (4) we find the transfer functions of individual signal transmission channels from the compensator input to its output, i.e.

$$w_{11}^{(eq)}(p) = \frac{w_{11}(p)w_{22}(p)}{\Delta_w(p)}; \; w_{12}^{(eq)}(p) = -\frac{w_{12}(p)w_{22}(p)}{\Delta_w(p)};$$

$$w_{21}^{(eq)}(p) = -\frac{w_{21}(p)w_{11}(p)}{\Delta_w(p)}; \; w_{22}^{(eq)}(p) = w_{11}^{(eq)}(p).$$

Let's find the order of the compensator's transfer functions. To do this, we write the object's transfer functions in the following form: $w_{ij}(p) = \frac{r_{ij}(p)}{\Delta(p)}$, $i,j = 1,2$. Then $w_{11}^{(eq)}(p) = w_{22}^{(eq)}(p) = \frac{1}{\Delta_w(p)} \cdot \frac{r_{11}(p)r_{22}(p)}{\Delta^2(p)}$; $w_{12}^{(eq)}(p) = -\frac{1}{\Delta_w(p)} \cdot \frac{r_{12}(p)r_{22}(p)}{\Delta^2(p)}$; $w_{21}^{(eq)}(p) = -\frac{1}{\Delta_w(p)} \cdot \frac{r_{21}(p)r_{11}(p)}{\Delta^2(p)}$.

Since $\Delta_w(p) = \frac{1}{\Delta^2(p)}\left(r_{11}(p)r_{22}(p) - r_{12}(p)r_{21}(p)\right)$, then

$$w_{11}^{(eq)}(p) = w_{22}^{(eq)}(p) = \frac{r_{11}(p)r_{22}(p)}{D(p)}; \; w_{12}^{(eq)}(p) = -\frac{r_{12}(p)r_{22}(p)}{D(p)}; \; w_{21}^{(eq)}(p) = -\frac{r_{21}(p)r_{11}(p)}{D(p)}, \quad (5)$$

where $D(p) = r_{11}(p)r_{22}(p) - r_{12}(p)r_{21}(p)$.

Thus, the order of the transfer functions of the compensator is determined by the order of the products of the polynomials $r_{ij}(p)r_{kl}(p)$, $i,j,k,l \in \{1,2\}$. Since the order of each of the polynomials $r_{ij}(p)$ is not more than one, the order of the product of two polynomials $r_{ij}(p)r_{kl}(p)$ does not exceed number two. The analysis allows us to state that for each transfer function $w_{ij}^{(eq)}(p)$, $i,j = 1,2$ the condition $M \le N$ is satisfied. Here $M$ is the order of the polynomial of the numerator and $N$ is the order of the polynomial of the denominator of the corresponding transfer function. Then the condition of the physical implementation of the cross-coupling compensator is satisfied.

For the transfer functions of the cross-coupling compensator, we have the following values: $M = 1$ for $w_{11}^{(eq)}(p)$, $w_{22}^{(eq)}(p)$ and $w_{21}^{(eq)}(p)$; $M = 2$ for $w_{12}^{(eq)}(p)$; $N = 2$ for all transfer functions $w_{ij}^{(eq)}(p)$, $i,j = 1,2$.

Conducted research [1] on modeling the NTS process as an automatic control object made it possible to obtain the following transfer functions:

$$w_{11}(p) = \frac{0.1106}{p^2+49.42p+0.8282}; \; w_{12}(p) = \frac{0.2484p+12.27}{p^2+49.42p+0.8282};$$

$$w_{21}(p) = \frac{1994p+36.05}{p^2+49.42p+0.8282}; \; w_{22}(p) = \frac{4.396p+289.9}{p^2+49.42p+0.8282}.$$

So, we have: $r_{11}(p) = 0.1106$; $r_{12}(p) = 0.2484p + 12.27$; $r_{21}(p) = 1994p + 36.05$; $r_{22}(p) = 4.396p + 289.9$.

Using software developed in the Matlab environment, the transfer functions of the cross-coupling compensator were obtained, i.e.

$$w_{11}^{(eq)}(p) = w_{22}^{(eq)}(p) = -\frac{0.9812\cdot 10^{-3}p+0.064711}{p^2+49.42p+0.8282};\ w_{12}^{(eq)}(p) = \frac{0.2204\cdot 10^{-2}p^2+0.2543p+7.1811}{p^2+49.42p+0.8282};\ w_{21}^{(eq)}(p) = \frac{0.4451p+0.0080462}{p^2+49.42p+0.8282}.$$

Let us now find the transfer function of the closed-loop automatic control system of the LTS process, considering the transfer function of the compensator (3). To do this, first we find the transfer function of the open-loop system

$$W(p) = \mathrm{diag}W_{yu}(p)W_{cl}(p).$$

We substitute $W(p)$ into formula (1). As a result, we obtain

$$W_{yu_d}(p) = \left(I + \mathrm{diag}W_{yu}(p)W_{cl}(p)\right)^{-1} \mathrm{diag}W_{yu}(p)W_{cl}(p).$$

The resulting matrix equation in expanded form will be as follows:

$$W_{yu_d}(p) = \begin{bmatrix} 1 + w_{11}(p)w_{11}^{(cl)}(p) & 0 \\ 0 & 1 + w_{22}(p)w_{22}^{(cl)}(p) \end{bmatrix}^{-1} \begin{bmatrix} w_{11}(p)w_{11}^{(cl)}(p) & 0 \\ 0 & w_{22}(p)w_{22}^{(cl)}(p) \end{bmatrix}.$$

After performing the operations of rotating the diagonal matrix and multiplying the matrices, we obtain

$$W_{yu_d}(p) = \begin{bmatrix} \frac{w_{11}(p)w_{11}^{(cl)}(p)}{1+w_{11}(p)w_{11}^{(cl)}(p)} & 0 \\ 0 & \frac{w_{22}(p)w_{22}^{(cl)}(p)}{1+w_{22}(p)w_{22}^{(cl)}(p)} \end{bmatrix}.$$

Thus, we obtained the matrix transfer function of the closed-loop system, which is diagonal, which means that the two-dimensional system can be considered as two independent systems with the following transfer functions:

$$w_{11}^{(yu)}(p) = \frac{w_{11}(p)w_{11}^{(d)}(p)}{1+w_{11}(p)w_{11}^{(d)}(p)}; \tag{6}$$

$$w_{22}^{(pu)}(p) = \frac{w_{22}(p)w_{22}^{(c)}(p)}{1+w_{22}(p)w_{22}^{(cl)}(p)}. \tag{7}$$

Since the transfer functions $w_{11}(p)$ and $w_{22}(p)$ of the object (separator) are known, the synthesis of an autonomous control system is reduced to determining the transfer function of the compensator, choosing the control law (PI or PID law) and determining the tuning parameters for the regulators.

## 4. Calculation of the tuning parameters for the regulators of the autonomous control system

For the first and second control loops, we will choose PI control laws

$$w_{11}^{(cl)}(p) = \frac{C_0^{(1)}p+C_1^{(1)}}{p}, \tag{8}$$

$$w_{22}^{(c)}(p) = \frac{C_0^{(2)}p+C_1^{(2)}}{p}, \tag{9}$$

where $C_0^{(i)}$, $C_1^{(i)}$ are the controller settings ($i$ = 1, 2).

In formulas (6) and (7) the transfer functions of the object are known. Considering the transfer functions of the regulators of the first and second circuits (8) and (9), we write the transfer function of the closed-loop system for the first and second circuits

$$w_{11}^{(yu)}(p) = \frac{b_1^{(1)}C_0^{(1)}p+b_1^{(1)}C_1^{(1)}}{p^3+a_1p^2+(b_1^{(1)}C_0^{(1)}+a_2)p+b_1^{(1)}C_1^{(1)}}, \quad (10)$$

$$w_{22}^{(yu)}(p) = \frac{b_0^{(2)}C_0^{(2)}p^2+(b_0^{(2)}C_1^{(2)}+b_1^{(2)}C_0^{(2)})p+b_1^{(2)}C_1^{(2)}}{p^3+(a_1+b_0^{(2)}C_0^{(2)})p^2+(b_0^{(2)}C_1^{(2)}+b_1^{(2)}C_0^{(2)}+a_2)p+b_1^{(2)}C_1^{(2)}}. \quad (11)$$

The parameters for setting the controllers will be determined by the combined method [7], [8], the essence of which is that on the $p$ -plane (on the plane of poles) certain values of the poles in the left part of the $p$ -plane are selected to achieve the desired properties of the automatic control system.

Vieta's theorem establishes the relationship between the roots of the characteristic equation of the system and its coefficients. Since for the first and second circuits the characteristic equations of the closed system have the same orders ($n = 3$), we will have [7]:

$$\begin{cases} p_1 + p_2 + p_3 = -\frac{\alpha_1}{\alpha_0}, \\ p_1p_2 + p_1p_3 + p_2p_3 = \frac{\alpha_2}{\alpha_0}, \\ p_1p_2p_3 = -\frac{\alpha_3}{\alpha_0}. \end{cases} \quad (12)$$

The coefficients of the characteristic equation for the first circuit are as follows: $\alpha_0 = 1$, $\alpha_1^{(1)} = a_1$, $\alpha_2^{(1)} = a_2 + b_1^{(1)}C_0^{(1)}$, $\alpha_3^{(1)} = b_1^{(1)}C_1^{(1)}$, where $b_1^{(1)} = a_{12}b_{21}$, $a_1 = -(a_{11} + a_{22})$, $a_2 = a_{11}a_{22} - a_{12}a_{21}$. For the second circuit: $\alpha_0 = 1$, $\alpha_2^{(2)} = b_0^{(2)}C_1^{(2)} + b_1^{(2)}C_0^{(2)} + a_2$, $\alpha_3^{(2)} = b_1^{(2)}C_1^{(2)}$.

Since $\alpha_0 = 1$, we have the following system of equations for the first circuit:

$$\begin{cases} p_1 + p_2 + p_3 = -\alpha_1, \\ p_1p_2 + p_1p_3 + p_2p_3 = \alpha_2, \\ p_1p_2p_3 = -\alpha_3. \end{cases}$$

Let the roots of the characteristic equation of the first circuit be as follows: $p_1 = -\pi_1 + j\zeta_1$, $p_2 = -\pi_1 - j\zeta_1$.

From the first equation of the system of equations (12) we find $p_3 = -(p_1 + p_3) - \alpha_1$. Considering the values of $p_1$ and $p_2$ we have $p_3 = 2\pi_1 - \alpha_1^{(1)}$.

Considering the value of $p_3$, the second and the third equation of system (12) will take the following form:

$$\pi_1^2 + \zeta_1^2 - 2\pi_1(2\pi_1 - \alpha_1^{(1)}) = \alpha_2^{(1)},$$
$$(\pi_1^2 + \zeta_1^2)(2\pi_1 - \alpha_1^{(1)}) = -\alpha_3^{(1)}.$$

Let us determine the degree of oscillation of the first circuit of the system $\mu_1 = \frac{\zeta_1}{\pi_1}$. From the last equality we find $\zeta_1 = \pi_1\mu_1$. Considering the values of $\zeta_1$, $\alpha_1^{(1)}$, $\alpha_2^{(1)}$ and $\alpha_3^{(1)}$, we will have

$$\pi_1^2r_1 - 2\pi_1(2\pi_1 - a_1) = b_1^{(1)}C_0^{(1)} + a_2,$$
$$\pi_1^2r_1(2\pi_1 - a_1) = -b_1^{(1)}C_1^{(1)},$$

where $r_1 = \mu_1^2 + 1$.

From the resulting system of equations, we find

$$C_0^{(1)} = \frac{1}{b_1^{(1)}}(\pi_1^2 r_1 - 2\pi_1(2\pi_1 - a_1) - a_2), \tag{13}$$

$$C_1^{(1)} = -\frac{1}{b_1^{(1)}}\pi_1^2 r_1(2\pi_1 - a_1), \tag{14}$$

where $b_1^{(1)} = a_{12}b_{21}$.

Therefore, the tuning parameters of the primary circuit regulator will be calculated using formulas (13) and (14), which are functions of the real part of the roots $p_1$ and $p_2$

For the stability of a closed system, the condition $p_3 < 0$ must be fulfilled or considering the value of $p_3$ we will have

$$0 < \pi_1 < \frac{a_1}{2}. \tag{15}$$

In addition to condition (15), it is necessary that the PI controller tuning parameters be positive numbers, i.e. $C_0^{(1)} > 0$ and $C_1^{(1)} > 0$.

Now we will find the tuning parameters of the PI controller as a function of the real part $\pi_2$ of the characteristic equation of the second closed loop. We have

$$\begin{cases} p_1 + p_2 + p_3 = -\alpha_1^{(2)}, \\ p_1p_2 + p_1p_3 + p_2p_3 = \alpha_2^{(2)}, \\ p_1p_2p_3 = -\alpha_3^{(2)}. \end{cases}$$

where $p_1 = -\pi_2 + j\zeta_2$; $p_2 = -\pi_2 - j\zeta_2$; $\mu_2 = \frac{\zeta_2}{\pi_2}$.

From the first equation of the obtained system of equations we find $p_3 = 2\pi_2 - \alpha_1^{(2)}$. Considering the value of $\alpha_1^{(2)}$ we have

$$p_3 = 2\pi_2 - a_1 - b_0^{(2)}C_0^{(2)}, \tag{16}$$

If we now consider the roots of the characteristic equation $p_1$, $p_2$ and the root $p_3$, which is determined by formula (16), and the values of $\alpha_2^{(2)}$ and $\alpha_3^{(2)}$, we obtain the following result:

$$\begin{cases} (2\pi_2 b_0^{(2)} - b_1^{(2)})c_0^{(2)} - b_0^{(2)}c_1^{(2)} = a_2 - \pi_2^2(r_2 - 4) - 2\pi_2 a_1, \\ \pi_2^2 r_2 b_0^{(2)} C_0^{(2)} - b_1^{(2)} C_1^{(2)} = \pi_2^2 r_2(2\pi_2 - a_1), \end{cases} \tag{17}$$

where $b_0^{(2)} = b_{22}$; $b_1^{(2)} = a_{21}b_{12} - a_{11}b_{22}$; $r_2 = \mu_2^2 + 1$.

The system of linear algebraic equations (17) can be represented in the following form:

$$\begin{cases} \chi_{11}C_0^{(2)} + \chi_{12}C_1^{(2)} = q_1, \\ \chi_{21}c_0^{(2)} + \chi_{22}c_1^{(2)} = q_2. \end{cases} \tag{18}$$

From (18) we find

$$C_0^{(2)} = \frac{\chi_{22}q_1 - \chi_{12}q_2}{\Delta}, \tag{19}$$

$$C_1^{(2)} = \frac{\chi_{11}q_2 - \chi_{21}q_1}{\Delta}, \tag{20}$$

where $\chi_{11} = 2\pi_2 b_0^{(2)} - b_1^{(2)}$; $\chi_{12} = -b_0^{(2)}$; $\chi_{21} = \pi_2^2 r_2 b_0^{(2)}$; $\chi_{22} = -b_1^{(2)}$; $q_1 = a_2 - \pi_2^2(r_2 - 4) - 2\pi_2 a_1$; $q_2 = \pi_2^2 r_2(2\pi_2 - a_1)$; $\Delta = \chi_{11}\chi_{22} - \chi_{12}\chi_{21}$.

The stability of the second circuit of the autonomous control system will occur when the following conditions are met: $C_0^{(2)} > 0$, $C_1^{(2)} > 0$ and $p_3 < 0$. From formula (16) we find that

$$0 < \pi_2 < \frac{1}{2}(a_1 + b_0^{(2)} C_0^{(2)}). \tag{21}$$

Therefore, for both the first and second circuits, the tuning parameters of the PI controllers are functions of the real part $\pi_1$and $\pi_2$ the roots $p_1$and $p_2$.

The values of $\pi_1$and $\pi_2$ shall be defined in such a way that the generalized integral criterion takes a minimum value. Therefore, we will minimize

$$J = \int_0^\infty (\varepsilon^2(t) + \tau^2 \dot{\varepsilon}^2(t))\, dt, \tag{22}$$

where $\varepsilon(t)$ is the amount of inconsistency (control error); τ is a constant value that determines the influence of the component $\dot{\varepsilon}^2(t)$ on the quality of the control process.

We can rewrite criterion (22) in the following form:

$$J = J_1 + \tau^2 J_2, \tag{23}$$

where $J_1 = \int_0^\infty \varepsilon^2(t)dt$; $J_2 = \int_0^\infty \dot{\varepsilon}^2(t)dt$.

The values of the components $J_1$and $J_2$can be calculated using tables [7] if the transfer functions of the control error and its derivative with respect to the controller task are known.

The control error transfer function for the first loop was calculated using the following formula:

$$W_{\varepsilon\mu}^{(1)}(p) = \frac{1}{1 + W_{os}^{(1)}(p)},$$

where $W_{os}^{(1)}(p)$ is the transfer function of the open system.

Considering formula (6), we can write $W_{os}^{(1)}(p) = w_{11}(p) w_{11}^{(el)}(p)$. Considering the values of $w_{11}(p) = \frac{b_1^{(1)}}{a_0^{(1)}p^2 + a_1^{(1)}p + a_2^{(1)}}$ and $w_{11}^{(el)}(p) = \frac{C_0^{(1)}p + C_1^{(1)}}{p}$, we obtain

$$W_{Eu}^{(1)}(p) = \frac{(a_0^{(1)}p^2 + a_1^{(1)}p + a_2^{(1)})p}{a_0^{(1)}p^3 + a_1^{(1)}p^2 + (a_2^{(1)} + b_1^{(1)}C_0^{(1)})p + b_1^{(1)}C_1^{(1)}}. \tag{24}$$

With a single step input $u_d^{(1)}$ (Fig. 1), the Laplace representation of the control error will be as follows:

$$\mathrm{E}_1^{(1)}(p) = \frac{a_0 p^2 + a_1 p + a_2}{\alpha_0^{(1)}p^3 + \alpha_1^{(1)}p^2 + \alpha_2^{(1)}p + \alpha_3^{(1)}}, \tag{25}$$

where $\alpha_0^{(1)} = a_0 = 1$.

Now we find the Laplace representation of the derivative of the control error: $\mathrm{E}_1^{(1)}(p) = L[\dot{\varepsilon}(t)]$.

Since $L[\dot{\varepsilon}(t)] = p\mathrm{E}^{(1)}(p) - \varepsilon(0)$, then, using the theorem on the initial value of the function, we obtain: $\varepsilon(0) = \lim_{p\to\infty} p\mathrm{E}^{(1)} = 1$.

Considering the found value of $\varepsilon(0)$, we obtain

$$E_2^{(1)}(p) = \frac{\beta_0 p^2+\beta_1 p+\beta_2}{\alpha_0^{(1)} p^3+\alpha_1^{(1)} p^2+\alpha_2^{(1)} p+\alpha_3^{(1)}}, \tag{26}$$

where $\beta_0 = 0$; $\beta_1 = -b_1^{(1)} C_0^{(1)}$; $\beta_2 = -b_1^{(1)} C_1^{(1)}$.

For the second circuit, using the same algorithm, we determined the Laplace representation of the control error and its derivative. Therefore,

$$\mathrm{E}_1^{(2)} = \frac{a_0 p^2+a_1 p+a_2}{\alpha_0^{(2)} p^3+\alpha_1^{(2)} p^2+\alpha_2^{(2)} p+\alpha_3^{(2)}}, \tag{27}$$

$$\mathrm{E}_2^{(2)} = \frac{\varphi_0 p^2+\varphi_1 p+\varphi_2}{\alpha_0^{(2)} p^3+\alpha_1^{(2)} p^2+\alpha_2^{(2)} p+\alpha_3^{(2)}}, \tag{28}$$

where $\varphi_0 = a_1 - \alpha_1^{(2)}$, $\varphi_1 = a_2 - \alpha_2^{(2)}$, $\varphi_2 = -\alpha_3^{(2)}$.

The analysis of formulas (24) – (28) shows that the difference between the polynomials of the numerators and denominators is equal to one. Therefore, using the known values of $\mathrm{E}_1^{(i)}$, $\mathrm{E}_2^{(i)}$, $\mathrm{i} = 1{,}2$ from the table in [7], for the first circuit we find:

$$J_{N,1}^{(1)} = a_0^2 \alpha_2^{(1)} \alpha_3^{(1)} + (a_1^2 - 2a_0 a_2)\alpha_0^{(1)} \alpha_3^{(1)} + a_2^2 \alpha_0^{(1)} \alpha_1^{(1)}, \tag{29}$$

$$J_{N,2}^{(1)} = \beta_0^2 \alpha_2^{(1)} \alpha_3^{(1)} + (\beta_1^2 - 2\beta_0 \beta_2)\alpha_0^{(1)} \alpha_3^{(1)} + \beta_2^2 \alpha_0^{(1)} \alpha_1^{(1)}, \tag{30}$$

$$J_D^{(1)} = 2\alpha_0^{(1)} \alpha_3^{(1)} (\alpha_1^{(1)} \alpha_2^{(1)} - \alpha_0^{(1)} \alpha_3^{(1)}). \tag{31}$$

Since $\beta_0 = 0$, then

$$J_{N,2}^{(1)} = \beta_1^2 \alpha_0^{(1)} \alpha_3^{(1)} + \beta_2^2 \alpha_0^{(1)} \alpha_1^{(1)}. \tag{32}$$

For the second control circuit we find

$$J_{N,1}^{(2)} = a_0^2 \alpha_2^{(2)} \alpha_3^{(2)} + (a_1^2 - 2a_0 a_2)\alpha_0^{(2)} \alpha_3^{(2)} + a_2^2 \alpha_0^{(2)} \alpha_1^{(2)}, \tag{33}$$

$$J_{N,2}^{(2)} = \varphi_0^2 \alpha_2^{(2)} \alpha_3^{(2)} + (\varphi_1^2 - 2\varphi_0 \varphi_2)\alpha_0^{(2)} \alpha_9^{(2)} + \varphi_2^2 \alpha_0^{(2)} \alpha_1^{(2)}, \tag{34}$$

$$J_D^{(2)} = 2\alpha_0^{(2)} \alpha_3^{(2)} (\alpha_1^{(2)} \alpha_2^{(2)} - \alpha_0^{(2)} \alpha_3^{(2)}). \tag{35}$$

Formulas (27) – (35) determine the generalized quadratic criterion for both the first and second control loops, which we present in the following form:

$$J^{(i)} = \frac{1}{J_D^{(i)}} (J_{N,1}^{(i)} + \tau_i^2 J_{N,2}^{(i)}), \ \ i = 1{,}2 \tag{36}$$

The analysis of formulas (27) – (35) shows that the generalized quadratic criterion (36) for both the first and second control loops is a function of the position of the roots $P_1$and $P_2$on the *p* -plane, which is determined by the values of $\pi_1$and $\pi_2$.

Let us set the following problem: find such values of $\pi_1$and $\pi_2$that minimize the generalized quadratic criteria $J^{(1)}$ and $J^{(2)}$, that is

$$\min_{\pi_1^{(1)} \le \pi_i \le \pi_1^{(2)}} : J^{(i)}(\pi_i), \ i = 1{,}2, \tag{37}$$

where $\pi_i^{(1)}$, $\pi_i^{(2)}$, are the beginning and end of the interval of the local minimum of the functions $J^{(i)}(\pi_i)$, $i = 1{,}2$.

The interval of the local minimum of the functions $J^{(i)}(\pi_i)$ is chosen based on the stability requirement of the control system, which is determined by the conditions $C_0^{(i)} > 0$ and $C_1^{(i)} > 0$, $i = 1,2$, as well as relations (15) and (26).

Using the developed method, software was created in the Matlab environment, with the help of which graphs of the dependences $J^{(1)}(\pi_1)$ and $J^{(2)}(\pi_2)$ were constructed (Fig. 2).

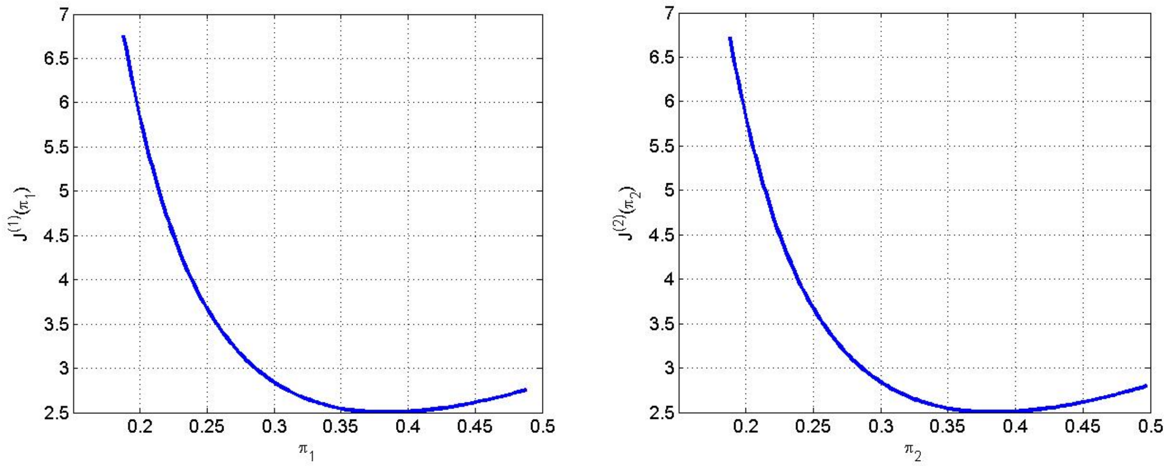


Fig. 2. Dependence graphs of $J^{(1)}(\pi_1)$ and $J^{(2)}(\pi_2)$.

From the graphs (Fig. 2) we found the intervals of local minima $\pi_1 \in [0.3; 0.45]$ and $\pi_2 \in [0.3; 0.45]$, which contain the minima of the functions $J^{(1)}(\pi_1^*)$ and $J^{(2)}(\pi_2)$. The solution of problem (37) gave the following results: $\pi_1^* = 0.38389$, $J^{(1)}(\pi_1^*) = 2.5007$; $\pi_2^* = 0.3880$, $J^{(2)}(\pi_2^*) = 2.5071$. The values of $\pi_1^*$ and $\pi_2^*$ made it possible to determine the optimal tuning parameters $C_1^{(i)}$ and $C_0^{(i)}$, $i = 1,2$ of the PI controllers for the first control loop according to formulas (13), (14) and for the second control loop according to formulas (19), (20) (Table 1).

To assess the quality of the control process, using formulas (10) and (11), graphs of transient processes were constructed for the first and second control loops (Fig. 3), from which the quality indicators of the control process were determined (Table 1).

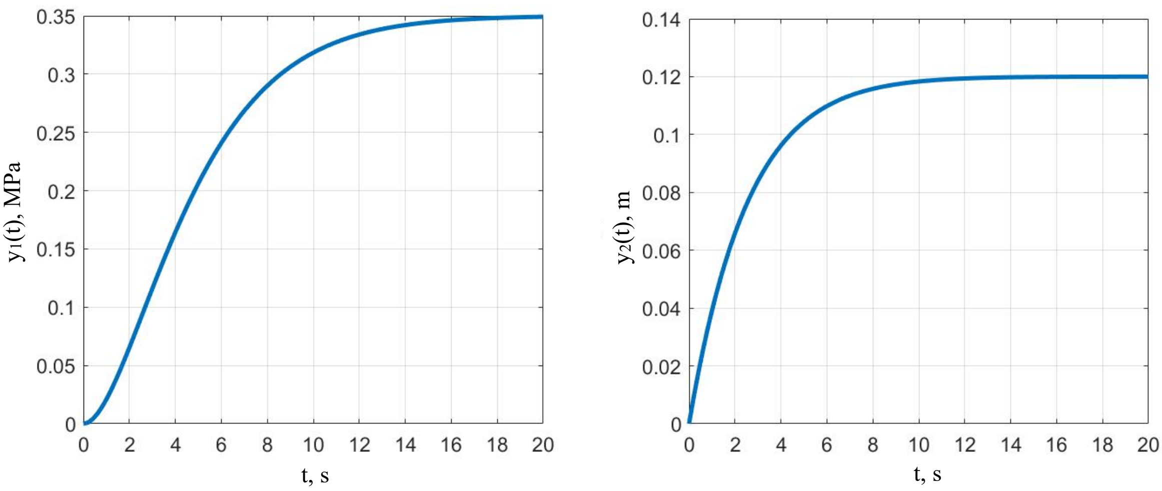


Fig. 3. Transient processes for the first and second control loops.

Table 1. Results of calculations of PI controller tuning parameters and control process quality indicators for the first and second control loops.

| | $C_0$ | $C_1$ | $t_c$, s | $\sigma$, % |
|---|---|---|---|---|
| First circuit | 12.901 | 5.0406 | 15.6 | 0 |
| Second circuit | 0.33871 | 0.13233 | 10.6 | 0 |

The calculation of the controller tuning parameters and the quality indicators of the control process were performed using software developed in the Matlab environment. Thus, the obtained indicators of the quality of the control process are satisfactory, since there is no overshoot in both the first and second circuits, and the control time is 15.6 and 10.6 s, respectively.

## 5. Presentation and discussion of research results

As a result of mathematical modeling of the low-temperature separation process as an object of automatic control, it was established that there are cross-links between control actions and output values (pressure and liquid level in the separator). The presence of cross-links reduces the efficiency of single-loop systems for automatic control of the low-temperature separation process.

The synthesized compensator makes it possible to eliminate cross-coupling and obtain two single-loop independent automatic control systems with PI regulators.

The tuning parameters of the PI controllers of both the first and second control loops of the autonomous system are determined using the developed method of placing the roots of the characteristic equations on the complex $p$ -plane, provided that the stability and minimum of the generalized quadratic control criterion are ensured.

Computer simulation of the autonomous system for automatic control of the low-temperature separation process confirmed the high quality of the control process. There is no overshoot in the system. The control time is 15.6 s for the first circuit and 10.6 s for the second one.

The direction of future research will be aimed at determining the structure of the cross-coupling compensator with the aim of its implementation on industrial microprocessor automation equipment.

## 6. Conclusion

The transfer functions of the cross-link compensator are determined, based on the condition that the matrix transfer function of the closed-loop control system must be diagonal. This condition makes it possible to "eliminate" cross-links and obtain two independent control loops with respect to the task influences.

A method for determining the tuning parameters of PI controllers of an autonomous control system for the low-temperature separation process by placing the roots of the characteristic equation on the p-plane has been developed. The choice of the placement of the roots of the characteristic equations of closed-loop systems of the first and second control loops was carried out by minimizing the generalized quadratic criterion of the quality of the control process and under the condition of the stability of the autonomous control system.

Based on the software developed in Matlab environment, the PI controller tuning parameters were calculated (using the combined method) and quality indicators of the control process were defined. There is no overshoot in the system. The control time is 15.6 s for the first circuit and 10.6 s for the second one.

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# Синтез автоматичної системи керування низькотемпературним сепаратором

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**Анотація**

Добутий газ із свердловини під високим тиском поступає на установку комплексної підготовки, де відбувається вилучення твердих домішок і води. Очищений природний газ має у своєму складі такі цінні компоненти як конденсат, а також важкі вуглеводні бутан і пропан. Для вилучення із газу попутних компонентів (конденсату і важких вуглеводнів) використовують низькотемпературну сепарацію Температурний режим в сепараторі підтримується за рахунок енергії стисненого газу. При проходженні газу через дросель внаслідок ефекту Джоуля-Томсона відбувається зниження тиску і температури. Технологічний режим в сепараторі забезпечується одноконтурними системами автоматичного керування – тиску і рівня конденсату. Як показали дослідження, виконані авторами статті, низькотемпературній сепарації як об'єкта керування притаманні внутрішні перехресні зв'язки. Їх наявність значно знижує ефективність одноконтурних систем керування. З метою підвищення якості процесу керування в роботі синтезована автономна система керування процесом низькотемпературної сепарації. У контур керування такої системи включений компенсатор перехресних зв'язків, внаслідок чого отримали дві незалежні одноконтурні системи автоматичного керування. На основі розробленої математичної моделі синтезована передавальна функція компенсатора та розроблений метод визначення параметрів ПІ-регуляторів. Суть методу у тому, що на комплексній площині коренів характеристичного рівняння визначається положення коренів, які повинні забезпечити бажану якість процесу керування. Розміщення коренів вибирається із умови мінімуму узагальненого квадратичного критерію якості процесу керування.

**Ключові слова:** низькотемпературна сепарація; математична модель; автономна система; компенсатор; ПІ-регулятор; квадратичний критерій.