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Roman Kapalo[1], Oleksii Lanets[2], Oleksii Vambol[3], Iryna Derevenko[4]

[1] Department of Design Machine and Automotive Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Roman.V.Kapalo@lpnu.ua, ORCID 0009-0003-1652-8491
[2] Department of Aviation and Manufacturing Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Oleksii.S.Lanets@lpnu.ua, ORCID 0009-0005-9631-717X
[3] Department of Aviation and Manufacturing Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Oleksii.O.Vambol@lpnu.ua, ORCID 0000-0002-1719-8063
[4] Department of Strength of Materials and Structural Mechanics, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Iryna.A.Derevenko@lpnu.ua, ORCID 0000-0003-0132-8035

PARAMETER DEVELOPMENT AND ANALYSIS OF THE OSCILLATORY SYSTEM OF A TWO-MASS RESONANT VIBRATING TABLE WITH AN INERTIAL DRIVE

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Abstract. The widespread use of vibrating tables in industry motivates researchers to develop new, efficient designs that can increase production profitability. For this purpose, the authors present a new schematic design of a vibrating table with an inertial drive, proposed to be powered by a hydraulic coupling. The principle of operation is as follows: the driving shaft of the hydraulic coupling is rotated by an electric motor, while its driven shaft is connected to an unbalanced mass. In this case, it is assumed that the rotor of the induction electric motor reaches its nominal operating mode, and the unbalanced speed “locks” near the resonant peak due to processes associated with the Sommerfeld effect. This enables the automatic maintenance of the forced oscillation frequency near the pre-resonant regime, thereby implementing energy-saving operating modes in the vibration system of the vibrating table, without the need for expensive control systems.

A problem arises because the implementation of such a design requires a clear justification of the inertial, stiffness, and force parameters of the oscillatory system. In addition, it is necessary to analyze the dynamic characteristics of the oscillatory system. This will enable a priori verification of the design's operability with the required technological parameters.

This article addresses the issues discussed. For this purpose, the authors present the methodology for determining the inertial, stiffness, and force parameters of a two-mass resonant vibrating table with an inertial drive. For the specified operating mode, analytical expressions are given for determining the amplitudes of mass oscillations, stiffness coefficients of elastic elements, the amplitude of the excitation force, and unbalance parameters. The dynamic parameters of such an oscillatory system, namely the dynamic coefficients of the oscillating masses, are analytically derived and analyzed. A specific example demonstrates the application of the proposed approach to calculating a two-mass oscillatory system with a force disturbance from a reactive mass. The amplitude-frequency characteristic of the oscillatory system is constructed and analyzed.

The value of this article lies in presenting a comprehensive methodology for calculating the vibration systems mentioned. A distinctive feature of this methodology is that it provides refined analytical expressions for setting the parameters of the vibration system. The material is accompanied by clear visualization. The presented methodology can be used by engineers when designing vibration technological equipment of this type.

Keywords: harmonic oscillations, pre-resonant mode, Sommerfeld effect, inertial parameters, stiffness parameters, force parameters, dynamic coefficient, amplitude-frequency characteristic.

Introduction

In industry, two-mass vibrating machines with unbalanced vibration exciters are widely used, whose operating mode is pre-resonant with respect to the natural frequency of the system's oscillations. This mode allows for significantly reduced power consumption of the drive, as much less energy is spent on setting the vibrating system into motion, in proportion to the dynamic coefficient of the system. In such vibrating machines, expensive control systems are used to stabilize the unbalanced rotation frequency in the vicinity of the resonance peak, which significantly increases the cost of such structures and their maintenance.

Problem Statement

As the capacity of vibration installations increases, the cost of control systems increases disproportionately. Therefore, eliminating such a component as the control system will make vibration technological equipment cheaper and significantly reduce the cost of its maintenance and repair.

It is the implementation of a new concept for driving an oscillatory system, in which there will be no control system, that will enable the solution of the scientific and applied problem of creating inexpensive and reliable vibration technology equipment.

Review of Modern Information Sources on the Subject of the Paper

Two-mass resonant vibration machines with inertial drives are widely used in material processing, compaction, feeding, screening, and testing systems due to their high energy efficiency and the ability to operate near natural frequencies while maintaining large output amplitudes at low energy input. The period from 2015 to 2025 has shown substantial progress in analytical modeling, nonlinear dynamic analysis, multi-parameter optimization, and numerical–experimental validation of two-mass resonant systems.

To date, the analytical methods for calculating mechanical oscillating systems are generally sufficiently described in educational literature [1]. Many years of work by scientists have been included in the reference literature for engineers and scientists, for example [2].

With the use of existing methods, engineers and scientists are engaged in developing new designs of vibrating technological equipment [3]; improving existing ones by providing optimal operating modes [4]; implementing optimal trajectories of the movement of the working body [5]; ensuring the necessary amplitude-frequency characteristics of oscillating systems [6]; investigating dynamic processes in inertial vibration exciters [7, 8] and improving their characteristics [9]; analyzing mathematical models of vibration equipment [10]; delving deeply into issues of dynamics [11, 12] and synchronization of inertial vibration exciters [13]; improving the control processes of inertial vibration exciters [14, 15].

Between 2015 and 2018, several European research groups expanded classical formulations of linear two-degree-of-freedom resonant systems. A foundational work is [16]. Their analysis formalized linear differential equations for an inertial exciter operating in frequency ranges close to the primary resonance. The authors derived explicit expressions for amplitude–frequency characteristics, phase portraits, and sensitivity to stiffness–mass ratios. Their method enabled the direct computation of optimal tuning conditions, allowing for maximum amplitude amplification.

In industry, two-mass vibration machines with unbalanced vibration exciters have become widespread, operating in a pre-resonant mode relative to the natural frequency of the system's oscillations [17]. This mode enables a significant reduction in the drive's power consumption. In such vibration machines, to stabilize the unbalanced rotation frequency in the vicinity of the resonant peak, control systems [14] or complex inertial drive systems [9] are used, which increase the cost of the structures and their maintenance.

Objectives and Problems of Research

Based on the materials [17, 18], it is proposed to develop a vibrating table with an inertial drive using a hydraulic coupling. For this purpose, it is proposed that the driving shaft of the hydraulic coupling be rotated by an electric motor. The driven shaft of the hydraulic coupling is connected to an unbalanced mass. In this case, it is assumed that the rotor of the induction electric motor will reach its nominal operating mode, and the unbalance speed will "lock" near the resonant peak due to processes associated with the Sommerfeld effect. This will enable the system to automatically maintain the frequency of forced oscillations near the pre-resonant tuning, thereby implementing energy-saving operating modes in the vibration system of the vibrating table. Expensive control systems are not required.

The proposed vibrating table (Fig. 1) contains a reactive body 5 with a mass m_2, to which an unbalanced unit 4 with a mass m_d is attached below. The working body 7 with a mass m_1 is connected to the reactive body 5 through flat resonant springs 6, the stiffness coefficient of which is c_{12}. The structure is mounted on a fixed base through vibration isolators 8, the stiffness coefficient of which is c_{iz}. The vibration isolators are attached to the working body 7.

The oscillatory system is set in motion as follows. The rotor of the induction electric motor 1 starts to rotate after it is turned on. Its motion is described by the torque of perturbation M and the angle of rotation θ. The induction motor drives the drive shaft of the hydraulic coupling 2. The rotation from the driven shaft of the hydraulic coupling, described by the angle of rotation φ, is transmitted through the flexible coupling 3 to the unbalanced mass 4. The alternating force disturbance of the forced oscillations in the system occurs due to the forced rotation with the circular frequency ω of the unbalanced mass m_d, the center of mass of which is located with an eccentricity r relative to the axis of rotation of the shaft. The centrifugal force F_i that arises is the cause of the alternating force disturbance of the oscillating mass along the x-axis. The oscillatory motion of the mass m_2 through the flat resonant springs 6 is transmitted to the mass m_1. The masses m_1 and m_2 oscillate in antiphase, with amplitudes X_1 and X_2, respectively. It is assumed that the oscillations of the masses along the y-axis will be small, and therefore these motions are neglected.

Due to the fact that the unbalanced shaft is located at a certain distance from the center of mass of the system c_m, a disturbance moment arises that rotates the oscillatory system in the xy-plane along the independent coordinate γ.

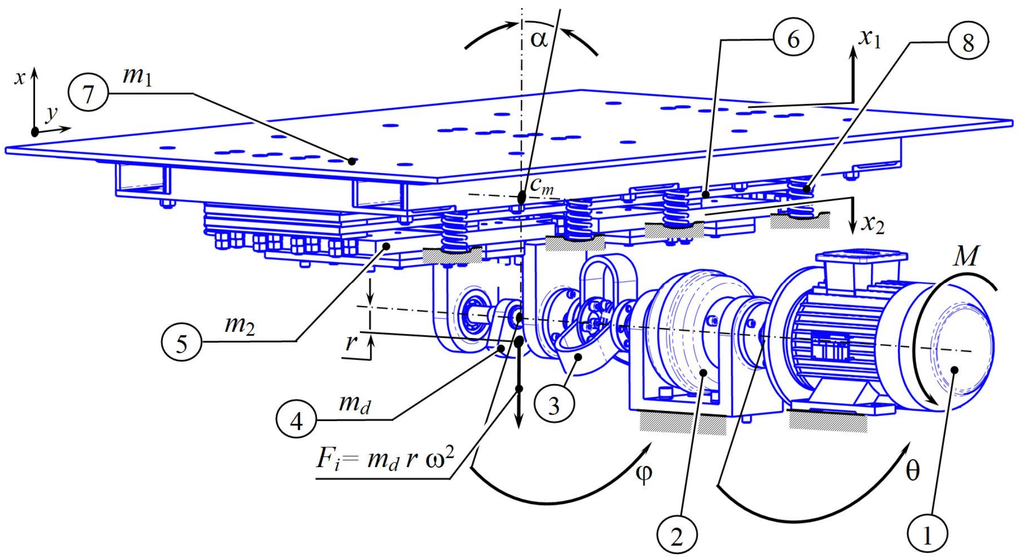

Fig. 1. Schematic diagram of a resonant vibrating table with a hydraulic coupling, where: 1 – induction electric motor; 2 – hydraulic coupling; 3 – flexible leaf coupling; 4 – unbalanced mass; 5 – reactive mass (reactive body); 6 – resonant elastic system; 7 – active mass (working body); 8 – vibration isolators.

Before proceeding with the development of the proposed design, it is necessary to clearly substantiate the inertial, stiffness, and force parameters of the oscillatory system. In addition, it is necessary to analyze the dynamic characteristics of such a system in near-resonant operating modes. It is the inertial, stiffness, and force parameters that are decisive during the operation of the design and determine its operability. The authors address these issues in this article. At this stage, the authors are interested in establishing the parameters of the two-mass resonant oscillatory system. The parameters of the fluid coupling and electric motor are not the object of these studies.

Main Material Presentation

Consider a two-mass oscillatory system with an inertial drive subjected to a disturbance from the reactive mass (Fig. 2). This schematic representation corresponds to the vibrating table shown in Fig. 1. Force perturbation from the mass m_2 is the most appropriate. The system has viscous resistance coefficients μ_{12} and μ_{iz}, which reflect the dissipation of energy in the resonant elastic unit and vibration isolators, respectively. The parameter μ_{iz} additionally takes into account the viscous resistance caused by the action of the loading medium on the working body (mass m_1).

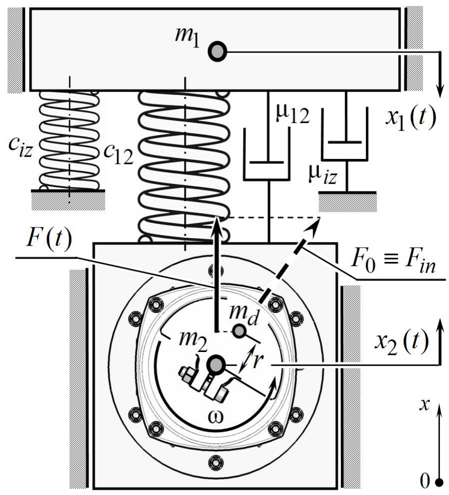

Fig. 2. Two-mass oscillatory system with inertial drive

The perturbation force

$$F(t) = F_0 \sin(\omega t) = m_d\, r\, \omega^2 \sin(\omega t)$$

is applied only to one mass, and its amplitude F_0 depends on the frequency, since it is generated by the rotational motion of the unbalance with a circular frequency ω. This determines the features of these systems. Therefore, the system of equations in complex values has the form:

$$\begin{cases} -m_1 \overline{X}_1 \omega^2 + c_{12}(\overline{X}_1 - \overline{X}_2) + c_{iz}\overline{X}_1 + i\omega\mu_{iz}\overline{X}_1 + i\omega\mu_{12}(\overline{X}_1 - \overline{X}_2) = 0; \\ -m_2 \overline{X}_2 \omega^2 + c_{12}(\overline{X}_2 - \overline{X}_1) + i\omega\mu_{12}(\overline{X}_2 - \overline{X}_1) = \overline{F}_0. \end{cases} \tag{1}$$

or

$$\begin{cases} \overline{X}_1[-m_1\omega^2 + c_{12} + c_{iz} + i\omega(\mu_{iz} + \mu_{12})] + \overline{X}_2[-c_{12} - i\omega\mu_{12}] = 0; \\ \overline{X}_1[-c_{12} - i\omega\mu_{12}] + \overline{X}_2[-m_2\omega^2 + c_{12} + i\omega\mu_{12}] = \overline{F}_0, \end{cases} \tag{2}$$

and in matrix form:

$$\begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} \begin{bmatrix} -m_1\omega^2 + c_{12} + c_{iz} + i\omega(\mu_{iz} + \mu_{12}) & -c_{12} - i\omega\mu_{12} \\ -c_{12} - i\omega\mu_{12} & -m_2\omega^2 + c_{12} + i\omega\mu_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{F}_0 \end{bmatrix}, \tag{3}$$

The complex amplitudes of $\overline{X}_1$ and $\overline{X}_2$ take the form:

$$\begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} = \begin{bmatrix} -m_1\omega^2 + c_{12} + c_{iz} + i\omega(\mu_{iz} + \mu_{12}) & -c_{12} - i\omega\mu_{12} \\ -c_{12} - i\omega\mu_{12} & -m_2\omega^2 + c_{12} + i\omega\mu_{12} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \overline{F}_0 \end{bmatrix}. \tag{4}$$

The modules of the complex amplitudes $\overline{X}_1$ and $\overline{X}_2$ will be written as:

$$\mathrm{X}_1 = \frac{-\mathrm{F}_0\sqrt{\mathrm{c}_{12}^2 + \mu_{12}^2\omega^2}}{\mathrm{D}_\mathrm{o}} = \frac{-\mathrm{m}_\mathrm{d}\,\mathrm{r}\,\omega^2\sqrt{\mathrm{c}_{12}^2 + \mu_{12}^2\omega^2}}{\mathrm{D}_\mathrm{o}}; \tag{5}$$

$$\mathrm{X}_2 = \frac{\mathrm{m}_\mathrm{d}\,\mathrm{r}\,\omega^2\sqrt{\mathrm{m}_1^2\omega^4 - \omega^2[2\mathrm{m}_1(\mathrm{c}_{12} + \mathrm{c}_{iz}) - (\mu_{12} + \mu_{iz})^2] + (\mathrm{c}_{12} + \mathrm{c}_{iz})^2}}{\mathrm{D}_\mathrm{o}}, \tag{6}$$

where

$$\mathrm{D}_\mathrm{o} = \sqrt{\begin{pmatrix} [\omega^2(\mathrm{m}_1 + \mathrm{m}_2)(\mathrm{c}_{12} - \mathrm{m}_{ag}\,\omega^2) - \mathrm{c}_{iz}(\mathrm{c}_{12} - \mathrm{m}_2\omega^2) + \mu_{12}\,\mu_{iz}\omega^2]^2 + \\ +[\omega\,\mu_{12}(\omega^2(\mathrm{m}_1 + \mathrm{m}_2) - \mathrm{c}_{iz}) - \omega\,\mu_{iz}(\mathrm{c}_{12} - \mathrm{m}_2\omega^2)]^2 \end{pmatrix}} \tag{7}$$

is the modulus of the determinant of the coefficient matrix from equation (3); $m_{ag} = m_1 m_2 /(m_1 + m_2)$ is the reduced mass.

Setting the frequency and stiffness parameters. We will use the determinant D_o (expression (7)) of the matrix of coefficients. By equating it to zero and neglecting the viscous resistance, we obtain the expression that sets the dependence of the natural frequencies on system parameters. Writing

$$D_1 = (c_{12} + c_{iz} - m_1 \omega^2)(c_{12} - m_2 \omega^2) - c_{12}^2 = 0, \tag{8}$$

we determine where the function $D_1(\omega)$ crosses the frequency axis ω. Equation (8) can also be written as

$$D_1 = \omega^4 - \left(\frac{c_{12} + c_{iz}}{m_1} + \frac{c_{12}}{m_2} \right) \omega^2 + \frac{c_{12}\, c_{iz}}{m_1 m_2} = 0 \cdot \tag{9}$$

Since during resonance the frequency of forced oscillations Ω coincides with the natural frequency Ω_n, assuming $\omega = \Omega_n$, the natural frequencies of the system are obtained as

$$\Omega_{n1,\,2} = \sqrt{\frac{1}{2}\left(\frac{c_{12} + c_{iz}}{m_1} + \frac{c_{12}}{m_2} \right) \mp \sqrt{\frac{1}{4}\left(\frac{c_{12} + c_{iz}}{m_1} + \frac{c_{12}}{m_2} \right)^2 - \frac{c_{12}\, c_{iz}}{m_1 m_2}}} \,. \tag{10}$$

As we see, two roots appear, since the system is two-mass and mounted on vibration isolators of stiffness c_{iz}. The signs (–) and (+) correspond to the first and second natural frequencies, respectively. According to (10), the natural frequencies can be determined if the inertial and stiffness parameters are known.

In fact, the system in Fig. 2 is a three-mass system if the foundation mass m_f is considered. We assume the foundation mass is infinitely large and does not participate in oscillations; therefore, it is omitted. The two-mass system installed on soft vibration isolators does not "feel" them at higher frequencies, because it operates far above the first resonant peak caused by the isolators. Thus, its working mode can be regarded as resonant with respect to the first resonance. If it is necessary to take into account the mass of the foundation m_f, the natural frequencies can be determined from

$$\Omega_{n1,\,2} = \sqrt{\frac{1}{2}\left(\frac{c_{12} + c_{iz}}{m_1} + \frac{c_{12}}{m_2} + \frac{c_{iz}}{m_f} \right) \mp \sqrt{\frac{1}{4}\left(\frac{c_{12} + c_{iz}}{m_1} + \frac{c_{12}}{m_2} + \frac{c_{iz}}{m_f} \right)^2 - \frac{c_{12}\, c_{i3}}{m_1 m_2 m_f}(m_1 + m_2 + m_f)}} \,. \tag{11}$$

Applying the limit as $m_f \to \infty$, (11) reduces to (10).

When designing vibrating machines, the inertial and stiffness parameters must be chosen to provide strictly specified natural frequencies. The following approach is used. Assuming vibrating masses m_1 and m_2 are already defined, the natural frequencies Ω_{n1} and Ω_{n2} are assigned from conditions:

$$\Omega_{n1} = \Omega_{iz} = \frac{\Omega}{z_{iz}}; \quad \Omega_{n2} = \frac{\Omega}{z}, \tag{12}$$

where

$$z = 0.94...098 \tag{13}$$

– resonant tuning of a two-mass oscillatory system;

$$z_{iz} = 3...5 \tag{14}$$

– resonant tuning of vibration isolators; Ω – frequency of forced oscillations. The unknown parameters remain c_{12} and c_{iz}. Using (8), we form a system of equations by substituting (12):

$$\begin{cases} (c_{12} + c_{iz} - m_1 (\Omega / z_{iz})^2)(c_{12} - m_2\ (\Omega / z_{iz})^2) - c_{12}^2 = 0; \\ (c_{12} + c_{iz} - m_1 (\Omega / z)^2)(c_{12} - m_2\ (\Omega / z)^2) - c_{12}^2 = 0. \end{cases} \tag{15}$$

Solving (13), we obtain:

$$c_{iz} = (m_1 + m_2)\left(\frac{\Omega}{z_{iz}} \right)^2 Z_{iz}, \tag{16}$$

where

$$Z_{iz} = \frac{z_{iz}^2 \left[(z^2 - z_{iz}^2) + \sqrt{(z^2 - z_{i3}^2)^2 - 4\frac{m_2}{m_1} z^2 z_{i3}^2} \right]}{z^2 \left[(z^2 - z_{iz}^2) - 2 z_{iz}^2 \frac{m_2}{m_1} + \sqrt{(z^2 - z_{iz}^2)^2 - 4\frac{m_2}{m_1} z^2 z_{iz}^2} \right]}; \tag{17}$$

$$c_{12} = \left(\frac{m_1 m_2}{m_1 + m_2} \right) \left(\frac{\Omega}{z} \right)^2 Z_p, \tag{18}$$

where

$$Z_p = \frac{(z^2 + z_{iz}^2) + \sqrt{(z^2 - z_{i3}^2)^2 - 4\frac{m_2}{m_1} z^2 z_{iz}^2}}{2 z_{iz}^2}, \tag{19}$$

and the equality holds

$$Z_p = \frac{1}{Z_{iz}}, \quad \text{or} \quad Z_p \cdot Z_{iz} = 1. \tag{20}$$

Inserting the necessary parameters into (16) and (18), we determine the stiffnesses c_{12} and ensure that the required natural frequencies and operating modes are obtained. If stiffness c_{12} is sought from a known parameter c_{iz}, then, using (8) in the form

$$(c_{12} + c_{iz} - m_1 (\Omega / z)^2)(c_{12} - m_2 (\Omega / z)^2) - c_{12}^2 = 0, \tag{21}$$

we obtain a simpler expression than (18):

$$c_{12} = \left(\frac{m_1 m_2}{m_1 + m_2} \right) \left(\frac{\Omega}{z} \right)^2 \left(\frac{\left(\frac{\Omega}{z} \right)^2 - \frac{c_{iz}}{m_1}}{\left(\frac{\Omega}{z} \right)^2 - \frac{c_{iz}}{m_1 + m_2}} \right). \tag{22}$$

Let us note that (16), (18), and (22) are rarely used in practice. To establish c_{12}, a simplified notation is usually applied, neglecting the stiffness of vibration isolators. If $c_{iz} \to 0$, determinant (8) becomes $m_1 m_2 (\Omega / z)^2 - c_{12} (m_1 + m_2) = 0$, from where

$$c_{12} = m_{ag} \left(\frac{\Omega}{z} \right)^2, \tag{23}$$

where the aggregate mass m_{ag} is

$$m_{ag} = \frac{m_1 m_2}{m_1 + m_2}. \tag{24}$$

Using (8) again, write

$$(c_{12} + c_{iz} - m_1 (\Omega / z_{iz})^2)(c_{12} - m_2 (\Omega / z_{iz})^2) - c_{12}^2 = 0,$$

where

$$c_{iz} = (m_1 + m_2) \left(\frac{\Omega}{z_{iz}} \right)^2 \left(\frac{\left(\frac{m_1 m_2}{m_1 + m_2} \right) \left(\frac{\Omega}{z_{iz}} \right)^2 - c_{12}}{m_2 \left(\frac{\Omega}{z_{iz}} \right)^2 - c_{12}} \right). \tag{25}$$

Since $c_{iz} << c_{12}$ (typical in practice), we apply the condition $c_{12} \to \infty$, obtaining

$$c_{iz} = m_{vm} \left(\frac{\Omega}{z_{iz}} \right)^2, \tag{26}$$

where $m_{vm} = m_1 + m_2$ – the total vibrating mass. Thus, the first natural frequency of the system on vibration isolators can be determined with high accuracy as

$$\Omega_{n1} = \frac{\Omega}{z_{iz}} \approx \sqrt{\frac{c_{iz}}{m_{vm}}} = \sqrt{\frac{c_{iz}}{m_1 + m_2}} . \tag{27}$$

So, if the condition is met $c_{iz} << c_{12}$ – the first resonant peak at the frequency $\omega = \Omega_{n1}$ is caused almost only by the presence of vibration-isolating elastic elements. The softer they are, the more this peak is directed towards $\omega = 0$, and their stiffness c_{iz} will have a negligible effect on the value of the second natural frequency Ω_{n2} of the system.

In fact, the operation of the vibration machine at the first resonance peak is reduced to a single-mass system formed by a conditionally isolated mass on vibration isolators with a certain stiffness c_{iz}. Stiffness $c_{12} >> c_{iz}$, so the system at the frequency Ω_{n1} will oscillate as a whole without relative displacement of the masses m_1 and m_2.

The second resonant peak at the frequency Ω_{n2}, which is the working one for us, is formed almost only by the resonant two-mass oscillatory system formed by the masses m_1, m_2 and the elastic node with stiffness c_{12}, connecting them. Thus, using (23)

$$\Omega_{n2} = \frac{\Omega}{z} \approx \sqrt{\frac{c_{12}}{m_{ag}}} = \sqrt{\frac{c_{12}}{\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)}} . \tag{28}$$

Using determinant (7), the phase shift γ between displacement amplitude and disturbance force is obtained as

$$\gamma = \text{atan}\left(\frac{\omega\, \mu_{12}\,(\omega^2 (m_1 + m_2) - c_{iz}) - \omega\, \mu_{iz}\,(c_{12} - m_2\, \omega^2)}{\omega^2 (m_1 + m_2)\left(c_{12} - m_{ag}\, \omega^2\right) - c_{iz}\,(c_{12} - m_2 \omega^2) + \mu_{12}\, \mu_{iz} \omega^2} \right), \tag{29}$$

The required amplitude of the disturbance force F_0 (static moment of inertia of the unbalance $m_d\, r$) to ensure a specified oscillation amplitude (X_1 or X_2) is derived from (5) and (6):

$$F_0 = \frac{X_1\, Д_о}{\sqrt{c_{12}^2 + \mu_{12}^2\, \omega^2}} = \frac{X_2\, Д_о}{\sqrt{m_1^2\, \omega^4 - \omega^2 [2 m_1 (c_{12} + c_{iz}) - (\mu_{12} + \mu_{iz})^2] + (c_{12} + c_{iz})^2}} . \tag{30}$$

$$m_д\, r = \frac{X_1\, D_o}{\omega^2 \sqrt{c_{12}^2 + \mu_{12}^2\, \omega^2}} = \frac{X_2\, D_o}{\omega^2 \sqrt{m_1^2\, \omega^4 - \omega^2 [2 m_1 (c_{12} + c_{iz}) - (\mu_{12} + \mu_{iz})^2] + (c_{12} + c_{iz})^2}} . \tag{31}$$

For such an oscillatory system, using expressions (5) and (6), the ratio of oscillation amplitudes is

$$\frac{X_1}{X_2} = \sqrt{\frac{c_{12}^2 + \mu_{12}^2\, \omega^2}{m_1^2\, \omega^4 - \omega^2 [2 m_1 (c_{12} + c_{iz}) - (\mu_{12} + \mu_{iz})^2] + (c_{12} + c_{iz})^2}} . \tag{32}$$

Under the boundary condition $\omega = 0$ expressions (5) and (6) become:

$$X_{1(\omega=0)} = 0 ; \quad X_{2(\omega=0)} = 0, \tag{33}$$

and if $\omega \to \infty$

$$X_{1(\omega\to\infty)} = 0 ; \quad X_{2(\omega\to\infty)} = m_\partial\, r / m_2 . \tag{34}$$

Let us analyze the results obtained in (33) and (34) for their application in establishing the dynamic coefficients of the system. The complexity of this situation lies in the fact that the static deflection is zero, because for $\omega = 0$ we obtain $X_{1(\omega=0)} = X_{2(\omega=0)} = 0$. Indeed, when the unbalance does not rotate, there is no inertial force and displacement. But understanding that static deflection is a displacement under the action of a force without additional reinforcements, we can consider the far-resonant case, when the dynamic coefficient in systems with an unbalance drive is equal to unity. For the mass, m_2 the displacement in the far-resonant mode is determined according to the second expression from (34). Indeed, this expression indicates the displacement of the mass only under the action of the disturbance force, because it can be represented in the form

$$X_{2(\omega\to\infty)} = \frac{m_d\, r}{m_2} = \frac{m_d\, r\,\omega^2}{m_2\omega^2} = \frac{F_0}{m_2\omega^2}, \quad \text{or} \quad F_0 = m_2\omega^2 X_{2(\omega\to\infty)}. \tag{35}$$

That is, the disturbance force is counteracted only by the inertial force from the mass movement. There are no stiffnesses that would indicate the presence of elastic nodes, and therefore, no reinforcement is required either. Although the mass m_1 To establish its displacement in a non-dynamic mode, we will adhere to the above approach kinematically. This is due to the fact that, although the disturbance force F_0 It is not directly applied to the mass; it is still transmitted through the elastic elements. We predict what the displacement of the mass under the action of the disturbance force would be if it were applied directly to it. Thus, the hypothetical static displacement is

$$\delta_{1\,st} = \frac{F_0}{m_1\omega^2} = \frac{m_d\, r\,\omega^2}{m_1\omega^2} = \frac{m_d\, r}{m_1}. \tag{36}$$

So, dividing expressions (5) and (6) into the corresponding expressions from (34) and (36), we have the dependences for the mass dynamic coefficients m_1 and m_2:

$$\lambda_1 = \frac{X_1}{\delta_{1st}} = \frac{m_1\omega^2\sqrt{c_{12}^2 + \mu_{12}^2\,\omega^2}}{Д_о}; \tag{37}$$

$$\lambda_2 = \frac{X_2}{\delta_{2st}} = \frac{m_2\,\omega^2\sqrt{m_1^2\,\omega^4 - \omega^2[2m_1(c_{12}+c_{iz}) - (\mu_{12}+\mu_{iz})^2] + (c_{12}+c_{iz})^2}}{D_o}, \tag{38}$$

where D_o is established according to (7). At the limits, expressions (37) and (38) take on the following values:

$$\lambda_{1(\omega=0)} = 0;\ \lambda_{2(\omega=0)} = 0; \quad \lambda_{1(\omega\to\infty)} = 0;\ \lambda_{2(\omega\to\infty)} = 1. \tag{39}$$

Case with viscous resistance only in the resonant elastic node. The corresponding diagram is shown in Fig. 3. For such a case, the complex amplitudes of oscillations, taking (4) as a basis, can be determined as follows:

$$\begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix} = \begin{bmatrix} -m_1\,\omega^2 + c_{12} + i\,\omega\,\mu_{12} & -c_{12} - i\,\omega\,\mu_{12} \\ -c_{12} - i\,\omega\,\mu_{12} & -m_2\,\omega^2 + c_{12} + i\,\omega\,\mu_{12} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \bar{F}_0 \end{bmatrix}, \tag{40}$$

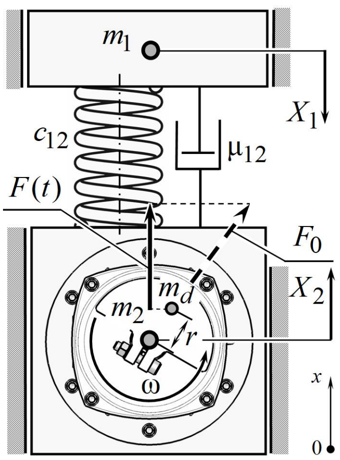

Fig. 3. Idealized diagram of a two-mass oscillatory system with an unbalanced drive

The solution to (40) will be the following modules of the oscillation amplitudes:

$$X_1 = -\frac{F_0}{\omega^2 (m_1 + m_2)} \sqrt{\frac{c_{12}^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{зв}\,\omega^2)^2 + (\mu_{12}\omega)^2}} = -\frac{m_d\, r}{m_1 + m_2} \sqrt{\frac{c_{12}^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,; \quad (41)$$

$$X_2 = \frac{F_0}{\omega^2 (m_1 + m_2)} \sqrt{\frac{(c_{12} - m_1\,\omega^2)^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}} = \frac{m_d\, r}{m_1 + m_2} \sqrt{\frac{(c_{12} - m_1\,\omega^2)^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,. \quad (42)$$

For (41) and (42) at the limiting frequencies, the results obtained for the previous case are valid. The amplitude values of the disturbance force and the static moments of unbalance, recorded through the system parameters, are elementary established using expressions (41) and (42). Thus:

$$F_0 = X_1\omega^2 (m_1 + m_2) \sqrt{\frac{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}{c_{12}^2 + (\mu_{12}\omega)^2}} = X_2\omega^2 (m_1 + m_2) \sqrt{\frac{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}{(c_{12} - m_1\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,. \quad (43)$$

$$m_d\, r = X_1 (m_1 + m_2) \sqrt{\frac{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}{c_{12}^2 + (\mu_{12}\omega)^2}} = X_2 (m_1 + m_2) \sqrt{\frac{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}{(c_{12} - m_1\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,. \quad (44)$$

Dividing expressions (41) and (42) into the corresponding expressions from (34) and (36), we will determine the mass dynamic coefficients using the following expressions:

$$\lambda_1 = \frac{m_{ag}}{m_2} \sqrt{\frac{c_{12}^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,; \quad (45)$$

$$\lambda_2 = \frac{m_{ag}}{m_1} \sqrt{\frac{(c_{12} - m_1\,\omega^2)^2 + (\mu_{12}\omega)^2}{(c_{12} - m_{ag}\,\omega^2)^2 + (\mu_{12}\omega)^2}}\,. \quad (46)$$

For expressions (45) and (46) at the limit frequencies, the results obtained for the dynamic coefficients in the previous case are valid. The natural frequency of the system will be determined by the simplified expression (28), and the stiffness value c_{12} – according to (23). The critical value of the resistance coefficient μ_{12k}, at which no peak will be observed, is established as follows [17]:

$$\mu_{12\,k} = \sqrt{2 c_{12}\, m_{ag}}\,. \quad (47)$$

Using the denominator under the root in expressions (45)-(46), the phase shift between the mass displacement m_2 and the disturbance force vector F_0 is found according to the expression

$$\gamma = \operatorname{atan}\left(\frac{\omega\,\mu_{12}}{c_{12} - m_{ag}\,\omega^2}\right). \quad (48)$$

The resonant frequency at which the maximum of the oscillation amplitude and the dynamic coefficient is observed is determined by equating the first derivative of ω expressions (45) and (46) to zero. Therefore, for the mass, m_1 it will be observed at the circular frequency

$$\Omega_{p1} = \frac{c_{12}}{\mu_{12}} \sqrt{\sqrt{1 + \frac{2\,\mu_{12}^2}{c_{12}\, m_{ag}}} - 1}\,, \quad (49)$$

and for mass m_2 we will have

$$\Omega_{p2} = \sqrt{\left(1 + \sqrt{\left(\frac{m_1 - m_{ag}}{m_1 + m_{зв}}\right)^2 + \frac{2\,\mu_{12}^2}{c_{12}\,(m_1 + m_{ag})}}\right) \Bigg/ \left(\frac{1}{c_{12}}\left(\frac{2\, m_1\, m_{ag}}{m_1 + m_{ag}} - \frac{\mu_{12}^2}{c_{12}}\right)\right)}\,. \quad (50)$$

It is interesting that for such oscillatory systems, the resonance peaks of the active and reactive masses are observed at close but different frequencies. Substituting (49) and (50) into (41), (45) and (42), (46), respectively, one can determine the maximum values of the oscillation amplitudes and dynamic coefficients at the resonance frequencies for the masses m_1 and m_2, respectively.

Let for the scheme in Fig. 4, in which the oscillating masses $m_1 = 200$ kg and $m_2 = 100$ kg are driven by an unbalanced vibration exciter at the frequency of forced oscillations $\omega = \Omega = 150$ rad/s, it is necessary to select the stiffness c_{12} of the elastic unit and set the static moment of unbalances $m_d\, r$ so that the working body (mass m_1) develops the oscillation amplitude $X_1 = 2\ mm$. We assume that the inertial value of the mass m_1 as a working body already takes into account the mass fraction of the conditionally attached loading medium. Using (13), we assume that the resonant tuning of the system $z = 0.96$. Using (24)

$$m_{ag} = m_1 m_2 /(m_1 + m_2) = 200 \cdot 100/(200+100) = 66.7\ \text{kg}.$$

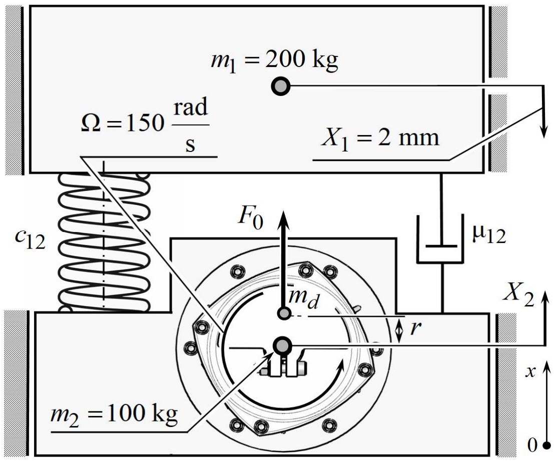

Fig. 4. Calculation scheme of a two-mass oscillatory system with an unbalanced drive with mass disturbance m_1

Then the stiffness c_{12} of the elastic node connecting the oscillating masses m_1 and m_2, according to (23), is equal to

$$c_{12} = m_{ag}\left(\frac{\Omega}{z}\right)^2 = 66.7 \cdot \left(\frac{150}{0.96}\right)^2 = 1.63 \cdot 10^6\ \frac{\text{N}}{\text{m}},$$

and the viscous drag coefficient μ_{12}, using (47), is

$$\mu_{12} = 0.05\sqrt{2 c_{12}\, m_{ag}} = 0.05 \cdot \sqrt{2 \cdot 1.63 \cdot 10^6 \cdot 66.7} = 737\ \ \text{N}\cdot\text{s/m}.$$

Taking into account (44), the static moment of unbalance, established through the amplitude of oscillations X_1 of the working body, will be

$$m_d\, r = X_1 (m_1 + m_2)\sqrt{\frac{(c_{12} - m_{ag}\,\Omega^2)^2 + (\mu_{12}\Omega)^2}{c_{12}^2 + (\mu_{12}\Omega)^2}} =$$

$$= 0.002 \cdot (200+100) \cdot \sqrt{\frac{(1.63 \cdot 10^6 - 66.7 \cdot 150^2)^2 + (737 \cdot 150)^2}{(1.63 \cdot 10^6)^2 + (737 \cdot 150)^2}} = 0.062\ \ \text{kg}\cdot\text{m}. \qquad (51)$$

To ensure parameter (51), the unbalance can be made with a mass $m_d = 3$ kg, and its center of mass should be located on the radius $r = 0.02$ m $= 2$ cm relative to the axis of rotation. To determine whether the calculated parameters will provide the technologically necessary amplitude of oscillations, we will construct the amplitude-frequency characteristic of the system (Fig. 5a), using expressions (41) and (42). For our case $F_0(\omega) = m_d\, r\,\omega^2 = 0.062\,\omega^2$ and the circular frequency varies within $\omega \in 0...250$ rad/s. From the graphical dependence, we observe that the amplitude of oscillations of the working body at the frequency of forced oscillations $\Omega = 150$ rad/s is $X_1 = 2$ mm. Indeed, taking the second expression from (41) as a basis, we obtain

$$X_1 = -\frac{m_d\, r}{m_1 + m_2}\sqrt{\frac{c_{12}^2 + (\mu_{12}\,\Omega)^2}{(c_{12} - m_{ag}\,\Omega^2)^2 + (\mu_{12}\,\Omega)^2}} = -\frac{0.062}{200+100}\cdot\sqrt{\frac{(1.63\cdot 10^6)^2 + (737\cdot 150)^2}{(1.63\cdot 10^6 - 66.7\cdot 150^2)^2 + (737\cdot 150)^2}} =$$

$$= -0.002 \text{ m},$$

and if we use the second expression from (42), the amplitude of the oscillations of the reactive mass will be

$$X_2 = \frac{m_d\, r}{m_1 + m_2}\sqrt{\frac{(c_{12} - m_1\,\Omega^2)^2 + (\mu_{12}\,\Omega)^2}{(c_{12} - m_{ag}\,\Omega^2)^2 + (\mu_{12}\,\Omega)^2}} = \frac{0.062}{200+100}\cdot\sqrt{\frac{(1.63\cdot 10^6 - 200\cdot 150^2)^2 + (737\cdot 150)^2}{(1.63\cdot 10^6 - 66.7\cdot 150^2)^2 + (737\cdot 150)^2}} =$$

$$= 0.0035 \text{ m}.$$

Note that in a system with an inertial drive $X_2 / X_1 \neq m_1 / m_2 = 200/100 = 2$, the relation of the amplitudes of mass oscillations is not straightforward. The real ratio of the amplitudes of mass oscillations can be obtained by dividing expressions (41) and (42) by each other, taking into account (23). We will have

$$\frac{X_2}{X_1} = -X_1\sqrt{\frac{(c_{12} - m_1\,\Omega^2)^2 + (\mu_{12}\,\Omega)^2}{c_{12}^2 + (\mu_{12}\,\Omega)^2}} = \sqrt{\frac{(1.63\cdot 10^6 - 200\cdot 150^2)^2 + (737\cdot 150)^2}{(1.63\cdot 10^6)^2 + (737\cdot 150)^2}} = 1.75. \qquad (52)$$

The phase shift between the mass displacement and the disturbance force vector F_0 is determined according to (48)

$$\gamma = \operatorname{atan}\left(\frac{\omega\mu_{12}}{c_{12} - m_{ag}\,\omega^2}\right) = \operatorname{atan}\left(\frac{150\cdot 737^2}{1.63\cdot 10^6 - 66.7\cdot 150^2}\right) = 0.714 \text{ rad} \equiv 41^\circ.$$

For the mass m_1 the resonant peak will be observed at the circular frequency (see expression (49))

$$\Omega_{p1} = \frac{c_{12}}{\mu_{12}}\sqrt{\sqrt{1 + \frac{2\,\mu_{12}^2}{c_{12}\,m_{ag}}} - 1} = \frac{1.63\cdot 10^6}{737}\cdot\sqrt{\sqrt{1 + \frac{2\cdot 737^2}{1.63\cdot 10^6\cdot 66.7}} - 1} = 156.06\,\frac{\text{rad}}{\text{s}},$$

and for the mass m_2 we have (see expression (50))

$$\Omega_{p2} = \sqrt{\frac{1 + \sqrt{\left(\frac{m_1 - m_{ag}}{m_1 + m_{ag}}\right)^2 + \frac{2\,\mu_{12}^2}{c_{12}\,(m_1 + m_{ag})}}}{\frac{1}{c_{12}}\left(\frac{2\,m_1\,m_{ag}}{m_1 + m_{ag}} - \frac{\mu_{12}^2}{c_{12}}\right)}} = \sqrt{\frac{1 + \sqrt{\left(\frac{200 - 66.7}{200 + 66.7}\right)^2 + \frac{2\cdot 737^2}{1.63\cdot 10^6\cdot(200 + 66.7)}}}{\frac{1}{1.63\cdot 10^6}\left(\frac{2\cdot 200\cdot 66.7}{200 + 66.7} - \frac{737^2}{1.63\cdot 10^6}\right)}} = 156.6\,\frac{\text{rad}}{s}.$$

Substituting the obtained values Ω_{p1} and Ω_{p2} into the corresponding expressions (41)-(42) and (45)-(46), we obtain that the maximum values of the amplitude of oscillations and the dynamic coefficient for the mass m_1 at the resonant frequency $\Omega_{p1} = 156.06$ rad/s are:

$$X_1(\Omega_{p1}) = X_{1\max} = -2.94 \text{ mm}; \quad \lambda_1(\Omega_{p1}) = \lambda_{1\max} = 9.46,$$

and for the mass m_2 at the resonant frequency $\Omega_{p2} = 156.6$ rad/s:

$$X_2(\Omega_{p2}) = X_{2\max} = 5.87 \text{ mm}; \quad \lambda_2(\Omega_{p2}) = \lambda_{2\max} = 9.46.$$

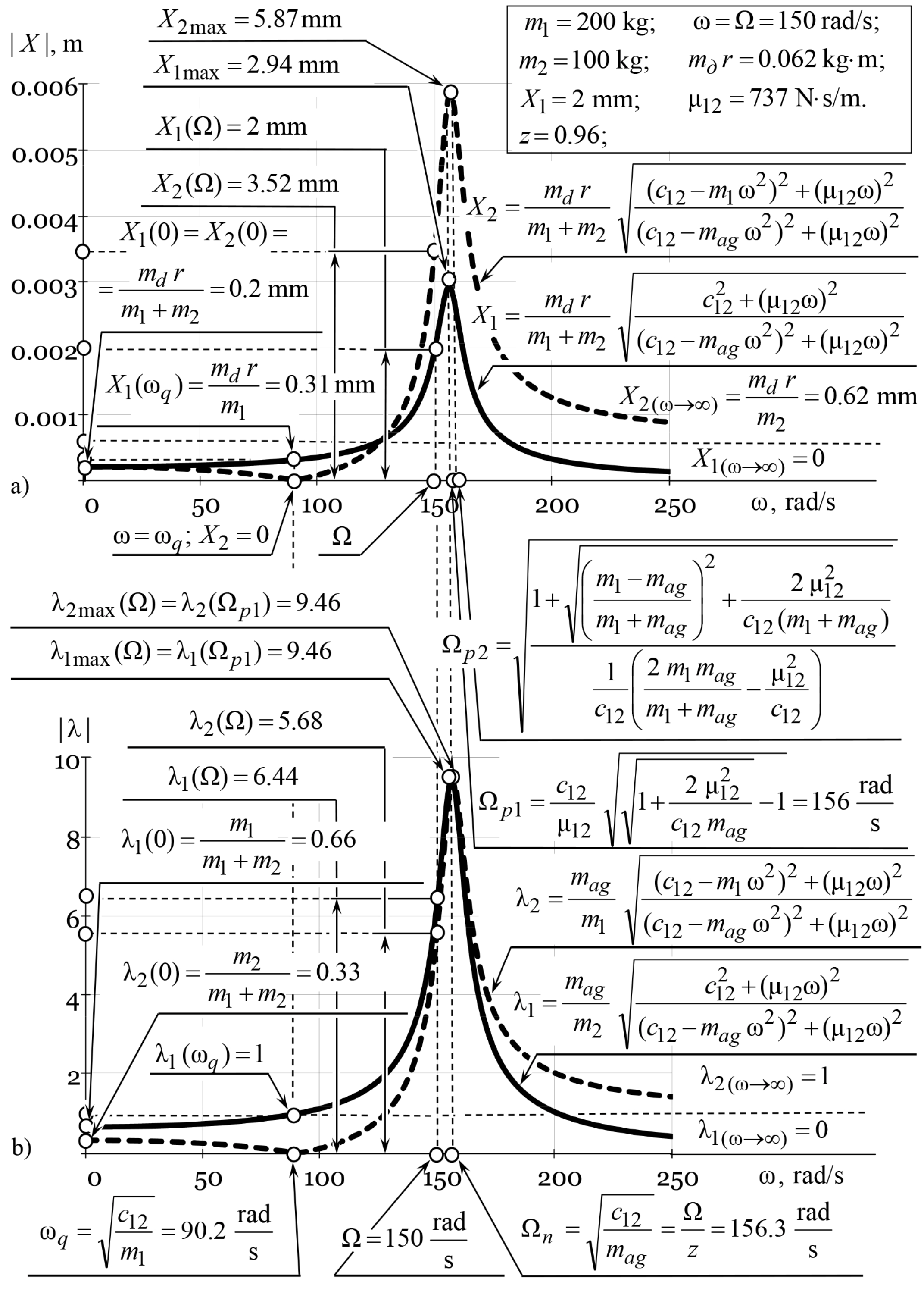

Fig. 5. Frequency-domain dependences of the amplitudes of mass oscillations (a) and dynamic coefficients (b) for the simplified calculation scheme of a two-mass oscillatory system with an unbalanced drive, shown in Fig. 4.

Let us establish the engine power required to ensure the nominal amplitudes of oscillations at the forced frequency Ω. To do this, we use the expression from [17], having first neglected the load mass. So we get

$$N = \frac{\Omega^3 \cdot \sqrt{6}}{4\eta}\left(\frac{m_1 X_1^2}{\lambda_1} + \frac{m_2 X_2^2}{\lambda_2}\right). \tag{53}$$

Let us establish the components of expression (53). The dynamic coefficients of oscillating masses (Fig. 5, b), if we use expressions (45) and (46), will be as follows:

$$\lambda_1 = \frac{m_{ag}}{m_2}\sqrt{\frac{c_{12}^2+(\mu_{12}\omega)^2}{(c_{12}-m_{ag}\,\omega^2)^2+(\mu_{12}\omega)^2}} = \frac{66.7}{100}\cdot\sqrt{\frac{(1.63\cdot 10^6)^2+(737\cdot 150)^2}{(1.63\cdot 10^6-66.7\cdot 150^2)^2+(737\cdot 150)^2}} = 6.44\,;$$

$$\lambda_2 = \frac{m_{ag}}{m_1}\sqrt{\frac{(c_{12}-m_1\,\omega^2)^2+(\mu_{12}\omega)^2}{(c_{12}-m_{ag}\,\omega^2)^2+(\mu_{12}\omega)^2}} = \frac{66.7}{200}\cdot\sqrt{\frac{(1.63\cdot 10^6-200\cdot 150^2)^2+(737\cdot 150)^2}{(1.63\cdot 10^6-66.7\cdot 150^2)^2+(737\cdot 150)^2}} = 5.68\,.$$

If we assume an engine efficiency of $\eta = 0.8$, then the drive power consumption will be

$$N = \frac{150^3\cdot\sqrt{6}}{4\cdot 0.8}\left(\frac{200\cdot 0.002^2}{6.44}+\frac{100\cdot 0.0035^2}{5.68}\right) = 1.42\ \text{kW}\,\cdot$$

When choosing the type of electric motor, it is necessary to ensure that its power is not less than the calculated value. This condition is satisfied by an asynchronous electric motor 4A80B4У3 with a power of $N = 1.5$ kW, with a synchronous rotational speed of $\omega_s = 157$ *рад / с*. However, it is always better to choose a motor with a safety margin in 1.1...1.3 times to avoid operation in critical modes. Therefore, we take the electric motor model 4A90L4У3 with a power of $N = 2.2$ kW. In any case, excess power will not harm the operation of the vibrating machine.

Conclusions

In this article, the authors present the methodology for establishing the inertial, stiffness, and force parameters of a two-mass resonant vibrating table with an inertial drive. For the established operating mode, analytical expressions are given for determining the amplitudes of mass oscillations, stiffness coefficients of elastic elements, the amplitude of the disturbance force, and unbalance parameters. The dynamic parameters of such an oscillatory system, namely the dynamic coefficients of the oscillating masses, are analytically derived and analyzed. A specific example demonstrates the application of the proposed approach to calculating a two-mass oscillatory system with a force disturbance from a reactive mass. The amplitude-frequency characteristic of the oscillatory system is constructed and evaluated.

The value of this article lies in demonstrating a comprehensive methodology for calculating the mentioned vibration systems. A feature of this methodology is that it provides refined analytical expressions for setting the parameters of the vibration system. The material is accompanied by clear visualization. The presented methodology can be used by engineers when designing vibration technological equipment of this type.

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