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MODERNIZATION OF SINGLE- AND TWO-MASS RESONANT VIBRATION MACHINES WITH INERTIAL DRIVE

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Abstract. The article addresses excessive energy consumption in resonance vibration machines with inertial drives, widely used in mechanical engineering, construction, chemical, metallurgical, and mining industries. Conventional design approaches limit energy efficiency, motivating the modernization of one- and two-mass resonance systems into three-mass inter-resonance configurations. Using a method for determining inertia–stiffness parameters, the study ensures synchronous inter-resonance oscillatory modes, enhancing dynamic amplification and reducing drive power. Analytical modeling of the three-mass system, comprising active, interediate, and reactive masses connected by elastic and damping elements, yields closed-form expressions for steady-state amplitudes and stiffness parameters. A phase-synchronization criterion is applied to determine the reactive mass, enabling convergence of resonance peaks and maximal dynamic gain. The proposed methodology provides a unified framework for upgrading existing resonance machines, achieving significant energy savings-p to an order of magnitude-hile maintaining required oscillation amplitudes. These results offer a practical tool for energy-efficient modernization of industrial vibratory machinery with inertial drives.

Keywords: harmonic oscillations, pre-resonant mode, inertial parameters, stiffness parameters, force parameters, dynamic coefficient, amplitude–frequency characteristic.

Introduction

In the energy-intensive industries of our country, such as machine building, construction, mining, chemicals, and metallurgy, large-scale and powerful vibration installations are utilized. The most common type of drive for the vast majority of such machines is the inertial drive based on unbalanced vibration exciters due to their compactness, high disturbance force, relative ease of manufacture, and operational simplicity. The main methods for calculating such vibration in technological equipment were developed in the last century. Although contemporary scientific approaches fully ensure the implementation of a wide range of vibration equipment for various purposes, in most cases, it remains energy-intensive (drive power consumption from several tens of kW), and the calculation methods are technologically outdated. The introduction of energy-efficient design principles for vibration machines will have a system-wide impact on the technological modernization of various industries, providing a substantial economic benefit at the national scale.

Problem Statement

In practice, there is often a need to modernize existing vibration technological equipment by increasing its operating efficiency, which we understand as reducing the specific power consumption of the drive required to operate the mechanical oscillation system (MOS) while ensuring the necessary technological performance indicators of the vibration equipment. Therefore, it is highly advisable not to spend substantial resources on introducing new equipment into production, but to modernize existing baseline designs, converting them into high-efficiency (energy-saving) configurations of vibration machines.

Review of Modern Information Sources on the Subject of the Paper

The literature review demonstrates a significant progression in the design, modeling, and modernization of vibratory machines, particularly those utilizing electromagnetic drives and dual-frequency resonances. The development of modern vibrating technological equipment has been the subject of extensive research over recent decades. Using existing analytical, numerical, and experimental methods, engineers and researchers focus on several key directions aimed at improving the efficiency, reliability, and controllability of vibration systems.

A significant body of research is devoted to the design and creation of new configurations of vibrating technological equipment, taking into account structural, dynamic, and operational constraints [1]. In parallel, considerable attention is paid to the modernization and enhancement of existing vibration systems, primarily through the identification and implementation of optimal operating regimes that ensure increased productivity and reduced energy consumption [2].

Another important research direction concerns the development and optimization of motion trajectories of the working bodies of vibrating machines. Properly selected trajectories enable more efficient interaction between the working body and the processed material, thereby improving technological performance and reducing wear of system components [3]. Closely related to this is the task of ensuring required amplitude–frequency characteristics of oscillatory systems, which directly affect the stability, efficiency, and adaptability of vibrating equipment under varying load conditions [4].

Extensive studies have been carried out on the dynamic processes occurring in inertial vibration exciters, which represent one of the most widely used excitation mechanisms in industrial vibration systems [5], [6]. These investigations form the basis for further improvement of exciter design and performance characteristics, including vibration stability, synchronization accuracy, and energy efficiency [7]. In this context, mathematical modeling of vibration equipment plays a crucial role, enabling the prediction of system behavior, assessment of parameter sensitivity, and optimization of design solutions at early stages of development [8].

In addition, researchers have conducted in-depth analyses of the dynamic behavior of vibrating systems, particularly under resonant and near-resonant operating conditions, where nonlinear effects and complex interactions between system components become significant [9], [10]. A separate and highly important research area is the study of synchronization phenomena in inertial vibration exciters, which directly influence the uniformity of vibration fields and the operational reliability of multi-exciter systems [11].

Finally, recent studies increasingly focus on the development of advanced control strategies for inertial vibration exciters, including adaptive, feedback-based, and energy-efficient control approaches. Such strategies aim to ensure stable operation under variable technological loads, suppress undesirable dynamic effects, and expand the functional capabilities of vibrating machines [12], [13].

In [14], the authors present an implementation of dual-frequency resonant vibratory machines with pulsed electromagnetic drives, emphasizing the enhancement of operational efficiency through precise frequency tuning. The study highlights the advantages of dual-frequency resonance in increasing the amplitude and stability of vibrations while reducing energy consumption, which is a recurring theme in contemporary vibratory machine research.

Earlier studies by Gursky and Lanets [15] focused on the modernization of high-frequency vibratory tables equipped with electromagnetic drives. The authors propose a comprehensive theoretical framework for modeling electromagnetic excitation and analyzing its influence on the resonant characteristics of vibratory systems. Special attention is paid to the interaction between electromagnetic forces and the mechanical oscillatory response of the working body. Their approach enables the simulation of key operational parameters, such as excitation frequency, vibration amplitude, and dynamic stiffness, which are critical for optimizing machine performance. As a result, the proposed methodology contributes to increasing productivity while simultaneously reducing mechanical wear and energy losses. By integrating electromagnetic field theory with classical vibration analysis, this work establishes a systematic and physically grounded basis for modernizing and enhancing the performance of existing industrial vibratory equipment.

Building upon these theoretical foundations, Despotović et al. [16] further develop the analysis of electromagnetic vibratory systems by combining mathematical modeling with experimental validation. Their research focuses on resonant linear vibratory conveyors driven by electromagnetic exciters, emphasizing the importance of accurately capturing resonance phenomena and nonlinear effects inherent in such systems. The authors demonstrate that purely analytical or numerical models may be insufficient without experimental verification, particularly under conditions of varying load mass and excitation frequency. By correlating simulation results with experimental data, they confirm the adequacy of the proposed models and highlight the critical role of experimental feedback in refining theoretical assumptions. This integrated computational–experimental approach ensures the reliable prediction of system behavior and provides practical guidelines for designing and tuning resonant vibratory machines operating in industrial environments.

The analysis of electromagnetic vibratory systems has been a significant area of research due to its direct implications for the design, control, and optimization of industrial machinery. A critical contribution in this field is the harmonic analysis of electromagnetic vibrator currents, as addressed by Cherno [17]. This study provides an in-depth examination of the electrical characteristics that influence the dynamic behavior of vibratory machines. By identifying and characterizing harmonic components, the work enables engineers to design control strategies that minimize undesirable vibrations and reduce electrical losses, thereby enhancing the overall reliability and efficiency of the system.

Complementing this approach, Gursky and Kuzio [18] presented methodologies for the rational synthesis of dual-frequency resonance vibratory machines. Their work emphasizes structural optimization to achieve desired dynamic characteristics, while simultaneously ensuring mechanical robustness and operational stability. This research demonstrates the importance of integrating structural design considerations with dynamic performance requirements, highlighting the interplay between mechanical and vibratory parameters in the development of advanced vibration systems.

Further contributions by Lanets et al. [19] investigate the synthesis and operational evaluation of two-mass resonance vibrating tables equipped with electromagnetic drives. Their experimental studies offer valuable insights into the practical aspects of machine design, including the impact of structural damping, mass distribution, and component interactions on resonance efficiency. By bridging theoretical modeling with industrial application, this work provides actionable guidelines for the design, modernization, and optimization of vibratory machinery in real-world engineering contexts.

A foundational theoretical perspective is presented in [20], where the authors systematically analyze linear differential equations that describe the behavior of an inertial exciter operating near primary resonance frequencies. The study presents explicit formulations for amplitude-frequency characteristics, phase portraits, and the sensitivity of system response to the stiffness-to-mass ratio. This methodology enables direct computation of optimal tuning conditions, allowing engineers to achieve maximum amplitude amplification and thereby optimize system performance.

Finally, Lanets [21] provides a comprehensive textbook framework encompassing the fundamentals of vibratory machine analysis and design. This resource synthesizes theoretical, computational, and experimental methodologies developed in prior studies, offering a coherent foundation for both academic research and practical engineering applications. By integrating knowledge across these domains, the work serves as an essential reference for understanding, modeling, and designing advanced vibratory systems.

In summary, these studies collectively advance the understanding of vibratory machine dynamics, encompassing harmonic analysis, structural optimization, experimental evaluation, and theoretical modeling. The integration of these approaches provides a robust framework for the design, analysis, and operational enhancement of modern vibratory machinery.

Objectives and Problems of Research

As a basic method for calculating the inertial and stiffness parameters of mechanical vibration systems, which forms the foundation of the engineering methodology for the modernization of existing designs of resonant vibration machines with an inertial drive, the approach presented in [21] was chosen. This computational framework enables the unambiguous synthesis of parameters for the vibration machine under which the MOS acquires a qualitatively new ability to accumulate high dynamic potential in inter-resonant operating modes, thereby substantially reducing the required drive power.

Therefore, based on the well-known methodology [21], the authors in this article expand and refine the methodological basis for the modernization of single-mass and two-mass resonant vibration machines with an inertial drive.

Main Material Presentation

The three-mass design of a vibration machine (Fig. 1) is taken as the studied MOS, in which rectilinear oscillations are implemented and the dynamics of which occurs according to the three-mass scheme. Active 1, intermediate 2 and reactive 3 masses with inertial parameters, respectively m_a, m_n and m_p perform rectilinear oscillations along the vertical axis x in generalized coordinates, respectively x_1, x_2 and x_3. The active mass is set in motion due to the kinematic disturbance from the intermediate mass. The disturbance of forced oscillations occurs due to a sinusoidal force $P(t) = P_0 \sin(\omega t + \varepsilon)$ (here P_0, is the amplitude value of the disturbance force; t is time; ε is the force-displacement phase shift), which is applied to the reactive mass. The active and intermediate, intermediate and reactive masses are connected in pairs by elastic systems, respectively, 4 and 5 with stiffnesses c_1 and c_2 in the direction of motion, which in Fig. 1 are schematically depicted as coiled springs. The MOS of the vibration machine is mounted through an intermediate mass on vibration isolators with a stiffness of 6 c_3.

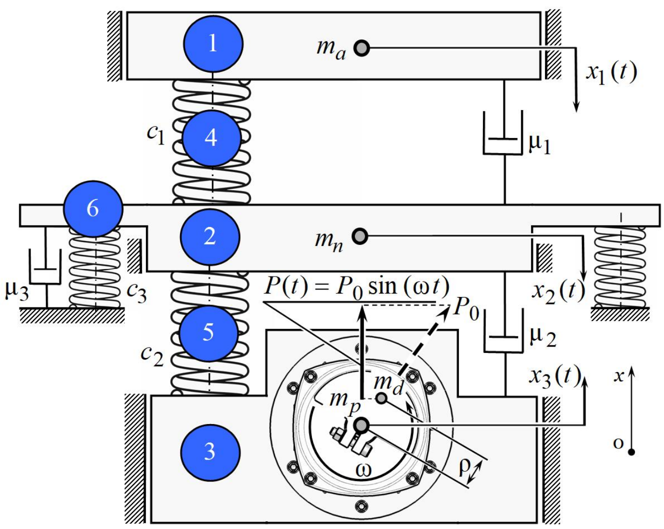

Fig. 1. Schematic diagram of a MOS three-mass vibration machine with an unbalanced vibration exciter n a reactive mass

We consider the motion of the oscillating system only in the vertical direction. We assume that the reactive mass 3 is acted upon by a sinusoidal the perturbation force in the vertical direction only, caused by circular motion with the frequency of forced oscillations ω of the unbalanced mass m_d at the eccentricity ρ.

It is assumed that dissipation occurs in the system, for which the viscous resistance coefficients μ_1, μ_2, μ_3, which are proportional to the velocity and reflect the phenomenon of hysteresis in elastic systems, respectively 4, 5, 6, are introduced into the dynamic model in the form of a damper.

The system of differential equations of motion in linear coordinates for a three-mass MOS will take the form:

$$\begin{cases} m_a\ddot{x}_1 + c_1(x_1 - x_2) + \mu_1(\dot{x}_1 - \dot{x}_2) = 0; \\ m_n\ddot{x}_2 + c_1(x_2 - x_1) + c_2(x_2 - x_3) + c_3 x_2 + \mu_1(\dot{x}_2 - \dot{x}_1) + \mu_2(\dot{x}_2 - \dot{x}_3) + \mu_3\dot{x}_2 = 0; \\ m_p\ddot{x}_3 + c_2(x_3 - x_2) + \mu_2(\dot{x}_3 - \dot{x}_2) = P_0 \cdot \sin(\omega t + \varepsilon) \end{cases} \quad (1)$$

The expressions for the amplitudes of oscillations of the active X_1, intermediate X_2 and reactive masses X_3 in steady-state modes are reduced to the form:

$$X_1 = \frac{-P_0 \cdot k_{12}k_{23}}{k_{12}k_{21}k_{33} - k_{11}k_{22}k_{33} + k_{11}k_{23}k_{32}};$$
$$X_2 = \frac{P_0 \cdot k_{11}k_{23}}{k_{12}k_{21}k_{33} - k_{11}k_{22}k_{33} + k_{11}k_{23}k_{32}}; \quad (2)$$
$$X_3 = \frac{-P_0 \cdot (-k_{12}k_{21} + k_{22}k_{11})}{k_{12}k_{21}k_{33} - k_{11}k_{22}k_{33} + k_{11}k_{23}k_{32}}.$$

where $k_{11} = c_1 - m_a\omega^2 + i\mu_1\omega$; $k_{12} = k_{21} = -c_1 - i\mu_1\omega$; $k_{13} = 0$; $k_{23} = k_{32} = -c_2 - i\mu_2\omega$;

$$k_{22} = c_1 + c_2 + c_3 - m_n\omega^2 + i(\mu_1 + \mu_2 + \mu_3)\omega;\ k_{33} = c_2 - m_p\omega^2 + i\mu_2\omega;\ k_{31} = 0.$$

Using the common denominator in expressions (2) (the determinant of the system of equations (1)), neglecting dissipation in the system and stiffness c_3:

$$-\left(c_1 - m_a\omega^2\right)\left(c_1 + c_2 - m_n\omega^2\right)\left(c_2 - m_p\omega^2\right) + \left(c_1 - m_a\omega^2\right)c_2^2 + c_1^2\left(c_2 - m_p\omega^2\right), \quad (3)$$

it is possible to determine the required values of the stiffnesses of the elastic elements 4 and 5 (Fig. 1), satisfying the resonance condition. Thus, the stiffness c_2 in analytical form is determined from (3), equating it to zero and taking into account the resonant tuning z of the MOS by replacing the value ω with ω/z:

$$c_2 = m_p\left(\frac{\omega}{z}\right)^2 \eta, \quad (4)$$

where η is a dimensionless coefficient, referred to as the fraction of stiffness c_2, and mathematically expressed as:

$$\eta = \frac{m_a m_n\left(\frac{\omega}{z}\right)^2 - c_1(m_a + m_n)}{m_a\left(\frac{\omega}{z}\right)^2(m_n + m_p) - c_1(m_a + m_n + m_p)}. \quad (5)$$

In the following, we specify the parameter η constructively, and $\eta \in [0\ldots1)$. From expression (5), we determine the stiffness value c_1:

$$c_1 = m_a\left(\frac{\omega}{z}\right)^2\left(\frac{m_p\eta + m_n(\eta - 1)}{(\eta - 1)(m_a + m_n) + m_p\eta}\right). \quad (6)$$

Analytical dependencies (4) and (6), which are connected through the parameter η, fully satisfy the conditions of the characteristic equation of the three-mass MOS and fix the second natural frequency ω_{n2} of the system with the value ω / z, and the first ω_{n1} is set depending on the constructive choice of the value η, which redistributes the inertial and stiffness parameters of the MOS. The perturbation of the system at the frequency $\omega = \Omega$ occurs in the interresonant zone, and $\omega_{n1} < \Omega < \omega_{n2}$ (Fig. 3).

The condition for ensuring in-phase oscillations assumes that the reactive mass 3 (Fig. 1), being in a force disturbance, will move as a single unit – in-phase together with the intermediate one, that is, their oscillations will be the same both in amplitude ($X_2 = X_3$) and in phase shift ($\varepsilon_2 = \varepsilon_3$) relative to the amplitude value of the disturbance force P_0. This condition is necessary when the two resonance peaks, namely the first non-working one, are brought as close as possible to the second working one. This results in the superposition of resonances, causing a significant increase in dynamic coefficients in the system. Equating the second and third expressions of system (2), the rational inertial m_p value of the reactive mass is determined taking into account (4) and (6):

$$m_p = \frac{m_n \left[(1-\eta)\left((m_a + m_n)(1-z^2)\right)\right]}{\eta \left[m_n (1-z^2) + m_a\right]}. \tag{7}$$

Let us introduce the parameter $\breve{D}$, which is defined as the ratio of the dynamic coefficients (or oscillation amplitudes) of the proposed systems relative to resonant two-mass systems with inertial perturbation from the reactive mass. In fact, the parameter $\breve{D}$ is an indicator of energy saving, since an increase in the dynamic coefficient in the system proportionally reduces the power consumption of the drive. Thus, using the analytical notation for the parameter $\breve{D}$, obtained from the ratio of the first expression of the system (2) to the expression $X_1 = P / \left(\omega^2 (1-z^2)(m_a + m_n)\right)$, which corresponds to two-mass systems with a perturbation from one mass, as is the case in resonant two-mass MOS with inertial perturbation from the reactive mass (Fig. 2, a), taking into account (7), we establish an expression for the stiffness fraction η:

$$\eta = \frac{m_a \breve{D}}{m_n (1-z^2) + m_a (1+\breve{D})}. \tag{8}$$

Development of an engineering methodology for the modernization of existing resonant vibration technological equipment based on an inertial drive. It is necessary to consider two cases: a) resonant two-mass MOS (Fig. 2, a; b) single-mass resonant MOS (Fig. 2, b), which operates at a low forced frequency and is tuned to a near-resonant mode of operation, where the function of resonant elastic elements is performed by supporting elastic packages. Such a scheme is rarely used and mainly in large-sized low-frequency screens, where it is necessary to achieve high amplitudes of oscillations.

Let us consider the structural diagram (Fig. 2, a). Substituting (7) into (5), we determine one of the masses, the inertial value of which must be corrected according to the dependence:

$$m_a = \frac{c\, m_n}{m_n \omega^2 - c}; \tag{9}$$

or

$$m_n = \frac{c\, m_a}{m_a \omega^2 - c}. \tag{10}$$

Thus, if the mass that will perform the active (kinematically perturbed) function in a three-mass system is lighter, it is necessary to use expression (9). If the mass that will perform the intermediate function in a three-mass system is lighter, it is necessary to use expression (10).

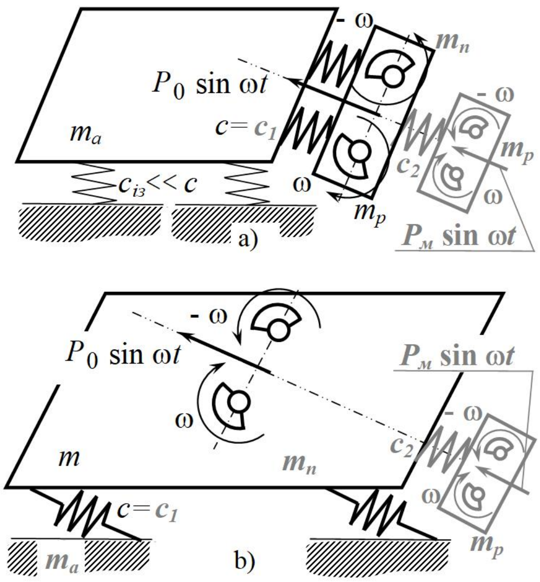

Fig. 2. Structural representation of the modernization of a two- (a) and single-mass (b) MOS vibration machine with an inertial drive (for which the contours and designations are shown in black) into a highly efficient three-mass system (for which the structural inputs and designations are in gray)

The method of modernization of resonant two-mass inertial equipment (Fig. 2, a) is as follows. According to (10), the smaller mass is incrementally increased in the direction of the larger mass, i.e., the reactive mass of the basic two-mass MOS, which in the newly created system performs the function of the intermediate mass. It should be loaded approximately by $10\,\%$, depending on the resonant tuning z incorporated in the basic two-mass system. Constructively setting the parameter of additional dynamic amplification of oscillations $\breve{D}$, we determine according to (8) the stiffness fraction η. Using expression (7), we calculate the inertial value of the reactive mass of the newly created system, which will be much smaller than the active and intermediate ($m_p << m_a$, $m_p << m_n$). According to (4), the required stiffness value c_2 ($c_2 << c_1$) is determined. Stiffness $c_1 = c$ can be verified using (6). In this case, the vibration machine following the three-mass scheme will consume substantially $\breve{D}$ less power compared to the basic model while providing the same amplitudes of mass oscillations.

Consider the structural diagram in Fig. 2, b. In reality, such a system is two-mass, since the second mass forms the foundation. Modernization of such equipment is carried out similarly to the method described above. On the basis of a single-mass resonant screen, modernization is required to create a highly efficient three-mass system. During the analysis of the basic model, it was established that the mass of the working body is approximately $m = 20000$ kg. Resonant tuning of the system (mandatory condition $z < 1$) $z = 0.96$, frequency of forced oscillations $\nu = 12\ Hz$ ($\omega = 75.4$ rad/s) must be considered. Thereore, the stiffness of the elastic system according to $c = m_{agg}(\omega / z)^2$, where $m_{agg} = m_a m_p /(m_a + m_p)$ – the reduced mass; z – resonant tuning MOS, is: $c = 1.23 \cdot 10^8$ N/m, when $m_a \to \infty$.

It is assumed that the coefficient of additional dynamic amplification of vibrations is $\breve{D} = 10$. Then, according to (8) the stiffness fraction is equal to $\eta = 0.91$, according to (10) the inertial value of the intermediate mass of the newly created system should be $m_n = 21700$ kg, i.e. the mass of the working body should be burdened by 1700 kg. Taking into account that $m_n << m_a$, formula (10) can be rewritten as:

$$m_n = \lim_{m_a \to \infty} \left(\frac{c\, m_a}{m_a\, \omega^2 - c} \right) = \frac{c}{\omega^2} = \frac{m}{z^2}. \tag{11}$$

To the intermediate mass of the three-mass system through the elastic system by stiffness $c_2 = 9.54 \cdot 10^5$ N/m (according to (4)) a reactive mass is attached $m_p = 170$ kg (according to (7)), on which low-power vibration exciters are installed. Vibration exciters on the newly formed intermediate mass are certainly no longer needed. Note that $c_1 = c = 1.23 \cdot 10^8$ N/m, which can be verified by calculating according to (6). In this case, the screen according to the three-mass scheme will consume $\breve{D} = 10$ times less power compared to the base model, providing the same amplitudes of mass oscillations (Fig. 3). The main results of the modernization are shown in Table 1.

Let us set the maximum possible value $\breve{D}$ while observing the clear requirements of the two resonant tunings. If the resonant tuning of the MOS by the second main peak is clearly set by the value z (expressions (4) and (6)), then the tuning by the first peak is not regulated. The first natural frequency of oscillations ω_{n1} can be determined according to:

$$\omega_{n1} = \sqrt{\frac{1}{2}\left(\frac{c_1}{m_a} + \frac{c_2}{m_p} + \frac{c_1 + c_2}{m_n}\right) - \frac{1}{2}\sqrt{\left(\frac{c_1}{m_a} + \frac{c_2}{m_p} + \frac{c_1 + c_2}{m_n}\right)^2 - 4\frac{c_1 c_2 (m_a + m_n + m_p)}{m_a m_n m_p}}}. \quad (12)$$

Let us introduce the resonant tuning $\tilde{z}$ of the first peak of the natural frequency ω_{n1} relative to the frequency of forced oscillations $\omega = \Omega$:

$$\tilde{z} = \omega_{n1} / \omega. \quad (13)$$

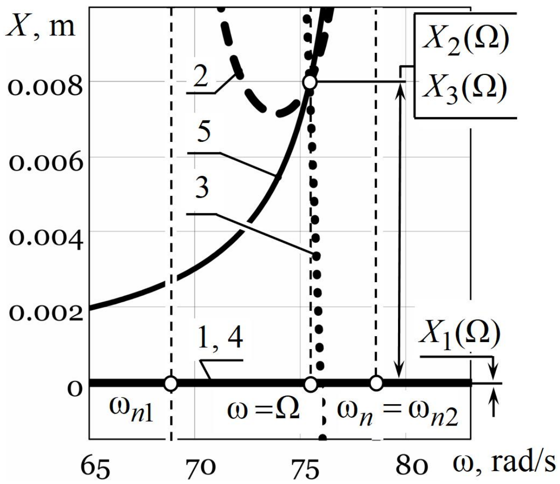

Fig. 3. Amplitude-frequency characteristics of the systems: 1, 2, 3 – modernized three-mass MOS, for which the amplitude value of the disturbance force is $P_M = 10$ kN ; 4, 5 – basic single-mass, for which the amplitude value of the disturbance force is $P_0 = 100$ kN , where 1, 4 – active masses (foundation), 2 – intermediate mass (working body); 3 – reactive mass; 5 – reactive mass (working body)

Table 1

Comparison of the main parameters of the basic and modernized vibration screens

Parameter	*Basic two-mass*	*Modernized three-mass*
Amplitude value of the disturbance force, kN	$P_0 = 100$	$P_M = 10$
Amplitude of oscillations X_2 of the working body, mm	8	

Solving analytical expressions (4), (6)-(8), (12) and (13) as a system of equations, the maximum possible value of the parameter of additional dynamic amplification of oscillations $\breve{D}$ is determined as:

$$\left[\breve{D}\right] \le \frac{m_a \tilde{z}^2 - m_n (1-z^2)(1-\tilde{z}^2)}{m_a (1-\tilde{z}^2)} . \tag{14}$$

The limiting value η for MOS with inertial drive on a reactive mass, substituting (14) into (8), is defined as:

$$[\eta] = \frac{m_a \tilde{z}^2 - m_n (1-z^2)(1-\tilde{z}^2)}{m_a} . \tag{15}$$

For inertial vibration technological equipment with disturbance from reactive mass, the maximum possible value of the parameter of additional dynamic amplification of vibrations is $\left[\breve{D}\right] = 49$, when $z = \tilde{z} = 0.99$, regardless of the ratio of inertial parameters of the active and intermediate masses. The limiting value of the parameter η is $[\eta] = 0.98$.

Analytical dependencies for determining the first natural frequency of oscillations ω_{n1} of systems with inertial drive on a reactive mass, expressed in terms of the parameter $\breve{D}$ and derived from (12), taking into account expressions (4) and (6)-(8), will accordingly take the form:

$$\omega_{n1} = \omega \sqrt{\frac{m_a \breve{D} + m_n \left(1-z^2\right)}{m_a \left(1+\breve{D}\right) + m_n \left(1-z^2\right)}} \tag{16}$$

or, as for the scheme in Fig. 2, b:

$$\lim_{m_a \to \infty} \left(\omega \sqrt{\frac{m_a \breve{D} + m_n \left(1-z^2\right)}{m_a \left(1+\breve{D}\right) + m_n \left(1-z^2\right)}} \right) = \omega \sqrt{\frac{\breve{D}}{1+\breve{D}}} . \tag{17}$$

Let us analyze expression (8). By imposing the condition $m_a \to \infty$:

$$\eta = \lim_{m_a \to \infty} \left(\frac{m_a \breve{D}}{m_n \left(1-z^2\right) + m_a \left(1+\breve{D}\right)} \right) = \frac{\breve{D}}{1+\breve{D}} . \tag{18}$$

Therefore, for the systems in Fig. 2, b, the following dependence can be written:

$$\eta = \breve{D} / \left(1+\breve{D}\right). \tag{19}$$

Expression (16) in this case will be rewritten as:

$$\omega_{n1} = \omega \sqrt{\eta} . \tag{20}$$

Using the parameters of the system given above, according to (19) $\eta = 10/(1+10) = 0.91$, which is consistent with the result according to formula (8). According to (20) $\omega_{n1} = 2 \cdot \pi \cdot 12 \sqrt{0.91} = 71.9$ rad/s, which is consistent with the data in Fig. 3. After analyzing the numerical results given by expressions (19) and (20), it was found that they are acceptable for analyzing a wide range of three-mass interresonant systems with perturbation from inertial vibration exciters. The calculation error is up to 1 %.

Similarly for expressions (14) and (15):

$$\left[\breve{D}\right] \le \lim_{m_a \to \infty} \left(\frac{m_a \tilde{z}^2 - m_n (1-z^2)(1-\tilde{z}^2)}{m_a (1-\tilde{z}^2)} \right) = \frac{\tilde{z}^2}{1-\tilde{z}^2} ; \tag{21}$$

$$[\eta] = \lim_{m_a \to \infty} \left(\frac{m_a \tilde{z}^2 - m_n (1-z^2)(1-\tilde{z}^2)}{m_a} \right) = \tilde{z}^2 . \tag{22}$$

Conclusions

A comprehensive methodology has been formulated for upgrading two-mass and single-mass resonant systems. For a two-mass system, modernization is achieved by increasing the mass designated to serve as the intermediate mass in the new three-mass configuration and by adjusting the stiffness of elastic elements according to the analytical relations presented. For single-mass systems, particularly large low-frequency industrial screens, the transition to a three-mass configuration involves introducing additional intermediate and reactive masses and relocating the inertial exciters to the reactive mass. In both cases, the analytical formulas provided define the required inertial and stiffness parameters necessary to achieve maximum dynamic amplification.

The obtained theoretical results clearly demonstrate the feasibility of applying the developed approaches and calculations during the modernization of existing resonant structures of vibration technological equipment with an inertial drive. The newly formed, modernized structures can consume an order of magnitude less electrical energy.

The significance of this article lies in the fact that the presented material provides a systematic methodology for the modernization of single- and dual-mass resonant vibration machines with an inertial drive. The methodology is practically applicable and can be employed by engineers in the energy-efficient modernization of various types of vibration technological equipment.

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